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DISCUSSION

Discussion of: Maa, J.P.-Y.; Hsu, T.-W.; Tsai, C.H., and Juang, W.J., 2000. Comparison of Wave Refraction and Diffraction Models. *Journal of Coastal Research,* 16(4), 1073–1082.

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The authors have produced a valuable contribution to clarify the respective capabilities of different, some commercially available, numerical models for the Mild–Slope equation using finite differencing techniques. Given the increasing number of models available, such comparative studies is of utmost interest in order to help end users to select the most appropriate approach to use for a specific class of wave transformation problem and how to interpret the results for each model.

The writers would like to contribute to point out some issues arising on the paper, mainly centred on the application of such models to the study of wave transformation phenomena over large, complex coastal areas, typically from deep to shallow waters in order to analyse the degree of coastal exposure to storms, wave disturbance in harbours, wave-induced currents on beaches and resulting sediment transport and coastal evolution. The mild–slope equation is a good candidate to be used to perform such tasks because it includes most of the relevant phenomena of wave evolution, even if in a linear way only, and, very important from a practical point of view, it is applicable (at least in a mathematical sense) over the whole range of non-dimensional water depths. On the opposite, powerful specialised approaches like Boussinesq approximation based models are only valid on the shallow water area, and even if important advances have been performed during the last years to extend in a significant way its range of application, the computational effort required may be high and, basically being time-domain models, the very rich detail of results may require additional processing in order to obtain an approximation to what is required in some cases, and the mild slope equation gives in a natural way: an equilibrium solution for the dispersion of waves on a domain.

It is unfortunate then that the simpler direct solution of the elliptic problem posed by the mild-slope equation over large domains has been considered, even quite recently, an unfeasible task given the memory and computing time requirements involved. This fact has been stated in most of the papers cited by the authors in its literature survey dealing with iterative approximations to the equation, both through the use of conjugate–gradient derived methods and through transformed hyperbolic equation systems where time is to be regarded only as an iterator towards the convergence of the system with the elliptic equation, following a well known mathematical technique.

Nevertheless, but still limited to small computational domains, MAA *et al.* (1997) showed the feasibility of the direct solution with computing times that, according to the results from table 2 in the paper, compare favourably with the conjugate gradient approach—like model PBCG—for the domains used. This result is confirmed by the writers own experience, as is the fact, mentioned on the paper and perhaps not too well documented yet, of the dependence of the convergence rate for such iterative models when the domain becomes more complex, see *e.g.* PANCHANG, 1991. On the other hand, the hyperbolic based model EMS gives shorter computational times but it has to be considered that it uses a rather sophisticated variable time step scheme. The growth in computational time noticed by the authors in the EMS model when comparing the results for his Berkhoff-1 and Berkhoff-2 tests could also be due in part by some degradation on convergence rates. In contrast, the calculation time for the direct solution is quite predictable as a function of the number of computational nodes and the bandwidth of the coefficient matrix of the problem. However, this is not to say that the iterative methods have not its own positive aspects. For instance, it may be easier to accommodate weak nonlinearity effects in the calculation scheme as the solution progresses towards convergence.

Considering the promising results obtained with the direct solution by the authors, a question remains about the practical application over larger domains. The domain used by the authors for the comparison is only in the order of 20 × 20 wavelengths, and as they point out a problem may arise when dealing with domains larger than say 50 × 50 wavelengths and complicated coastal geography, strong diffraction and backscattered waves precluding the use of parabolic approaches. Such practical difficulty is enhanced by the fact that, for applied studies, one has to reproduce complex sea states described by a deepwater wave spectrum which has to be discretized, as is explained by the authors, over the frequency and directional domains into a number of simple monochromatic waves. Then the model has to be run of each component and the results linearly combined in order to get the spectral wave field over the domain. The number of discrete components needed to obtain an accurate representation may be rather large (*e.g.* GRASSA, 1990) and thus the whole computing time may be a major concern. Considering how the number of numerical operations grows with the dimensions of the domain, it may be estimated that a 500 × 500 nodes domain would take, with the RDE model, 33,600 sec for each single component and more than six days for a 1,000 × 1,000 nodes domain.

It is therefore quite interesting to research ways of reducing the computational time. When using the classical 5-point finite differencing scheme over a structured grid, a possibility may be to exploit the various levels of symmetries of the problem in order to reduce the number of equations to be solved simultaneously and the storage needs. Grassa, 2000 has presented a method for the basic mild–slope equation, but using only first order boundary conditions as those presented by the authors. When applied to the Berlhoff-2 test, such model requires only 9.7 sec. running on a 2-processors Pentium 450 MHz computer. FLORES, 1999 has used such model with very large grids for studying irregular wave propagation in the Algeciras Bay, south of Spain, where strong diffraction and back-scattering of reflected waves precludes the use of parabolic methods and the large water depths (more than 400 mt) within the bay and the size of the domain, 10 × 12 km, makes difficult to employ Boussinesq approaches for short–period waves. In this study, a test with 10 frequency components over a 452 × 653 nodes domain required 7,635 sec. on a Pentium II 350 MHz computer, that is, approximately 13 min. by component. Like RDE, the model uses out-of-core storage and has been trivially parallelized using Intel MathLib BLAS routines prepared for SMP pc computers.

There is however some limitations: the boundary conditions should preserve the symmetry of the coefficient matrix and while this is possible for the most common forms of such conditions, it isn't for other conditions such as those 2nd order conditions presented by the authors. Second, direct solution methods may be more sensitive to finite arithmetic error propagation than iterative methods and therefore, residual norms should be computed (this is a very cheap operation) in order to assess the quality of the results and eventually, to improve them.

As a conclusion, the writers would like to point out that elliptic mild slope modelling of waves over quite large, complex coastal areas up to at least 100 × 100 square wavelengths is already feasible and reasonable for applied study cases with nowadays personal computing technology. This may be an improvement in many situations where the restrictions of parabolic models precludes its usage, as it happens in presence of artificial works, very oblique waves on deep bays, etc.

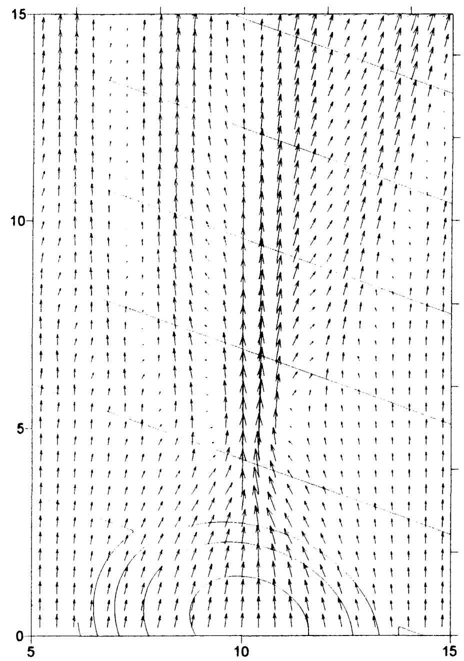

Figure 1. Calculated wave vectors after the elliptic shoal for the Berkhoff-1 test using a second order parabolic model equivalent to Ref-Dif1.

It is also an interesting fact that the authors have produced a comparison between different models not only in terms of resulting wave amplitudes but also including a less frequent comparative analysis of wave directions as resulting from each model. It is worth to note, however, that the method used for estimating the wave direction implies to assume that the wave field is represented by a plane wave. While the incident boundary conditions may more frequently be represented by a plane wave with constant amplitude, the potential wave field obtained as a result is not restricted, for every model used except RCPWAVE, to such a simple description. This is specially important in the test case selected by the authors. In fact, from a physical perspective, in the test case studied the wave field in the scattering area produced by the shoal is made up of the constructive and destructive interference produced by waves refracted on both sides of the shoal and is therefore difficult to assimilate such a field to a single progressive plane wave.

The solution potential obtained or approximated by the models used—except for RCPWAVE—can accommodate such interference phenomena and are not restricted to the assumption of a plane wave. It is not surprising therefore that the results given by RCPWAVE are the poorest for this case where the local wave direction has to be regarded as a multivalued function corresponding to the crossing wave trains.

An alternative, more general way of estimating a mean wave direction for such complex wave fields may be obtained using the velocity potential definition. Using the nomenclature of the authors, horizontal wave particle velocities for an arbitrary linear velocity potential may be estimated as:

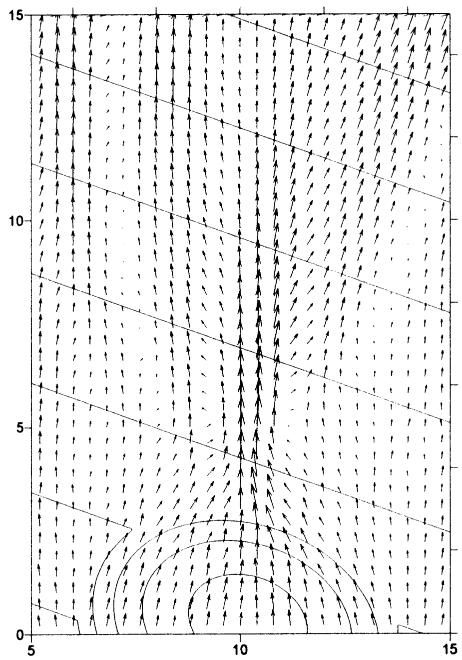

Figure 2. Calculated wave vectors after the elliptic shoal for the Berkhoff-1 test using a direct solution model for the basic mild slope equation, equivalent to RDE.

$$\vec{u}(x, y, z_0, t) = \mathrm{Re}\left\{\frac{g}{\sigma}\frac{\cosh(k(z_0 + h))}{\cosh(kh)}\nabla_h\phi(x, y)e^{i\sigma t}\right\} \quad (1)$$

Where the vector $\vec{u} = (u, v)$ represents the horizontal instantaneous wave velocity at a depth z_0, and the subscript 'h' means the horizontal part of the gradient for the reduced velocity potential ϕ solved by the Mild Slope equation.

It has to be stressed, as well, that even while parabolic wave potential models, such as Ref-Dif 1, approximate the mild-slope equation assuming a main propagation direction and considering only forward going waves, they are able to reproduce accurately the wave interference phenomena occurring in the scattering area for the tests presented by the authors. Second order parabolic models do reproduce correctly (that is to say with very small errors) wave propagation direction for up to 45 degrees from the main propagation direction, as is mainly the case in this case. In fact, figure 1 shows the calculated wave vectors for the Berkhoff-1 test, using a second order parabolic model equivalent to Ref-Dif 1 (GRASSA, 1990). This results compare very well with those obtained with the direct solution model for the basic mild–slope equation (GRASSA, 2000), showed in the figure 2. The writers believe therefore that some implementation-specific problem may have produced the results showed by the authors and do not share the conclusion on the lack of adequacy of such model class in terms of wave directions.

Finally, the writers would like to note that the knowledge of wave direction is not at all needed for the evaluation of components of the radiation stress tensor. Radiation stress for complex wave fields may be obtained directly from the linear velocity potential. MEI (1983) has given expressions for a horizontal bed and DINGEMANS (1997) has extended such expressions for variable beds. For the case of a model in terms of water surface elevation and horizontal fluxes, as is the case in hyperbolic approximations to the elliptic mild slope equation—EMS model in the survey made by the authors—MARUYAMA (1988) has also given the expressions for the radiation stress components. In fact, under complex wave fields, wave direction is not even a uniquely defined concept and it is clearly better to use the more correct expressions cited for calculating the radiation stress, whose variation is the fundamental driving force for wave-induced mean currents on beaches.

LITERATURE CITED

MAA, J.P.-Y. HWUNG, H.-H., 1997. A wave transformation model for harbour planning. *Proceedings, Waves '97 Conference.* IAHR.

MEI, C.C., 1983. The Applied Dynamics of Ocean Surface Waves. New York, Wiley, p. 464.

DINGEMANS, M.W., 1997. *Water Wave Propagation over Uneven Bottoms. Part 1—Linear Wave Propagation.* Singapore: World Scientific, pp. 211–215.

MARUYAMA, K., 1988. Computation of Nearshore Wave Field, pp. 252–253. *In:* Horikawa, K. (ed.), *Nearshore Dynamics and Coastal Processes. Theory, Measurement and Predictive Models.* University of Tokyo Press.

GRASSA, J.M., 1990. Random Wave Propagation on Beaches. *Proceedings, XIX International Coastal Engineering Conference* (Delft, The Netherlands), ASCE.

GRASSA, J.M., 2000. Un modelo sencillo de la ecuación de la pendiente suave para estudios de agitación y ondas largas en puertos. *Memorias, XIX IAHR*—Latin American División Conference, Cordoba, Argentina.

FLORES, J., 1999. Estudio de clima marítimo y agitación en la terminal de Crinavis, Algeciras. *CEDEX Report no. 24-499-9-121.*