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Using a Quadratic Model to Theoretically Describe the Nature of Equilibrium Shorerise Profiles

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ABSTRACT

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Given that the assumptions of a quadratic model are true, the results of this study reveal new insights about the nature of equilibrium shorerise profiles. When an equilibrium state has been achieved between wave forcings and the shape of a shorerise profile, the quadratic model assumes that along a traverse of shoaling waves the acceleration rate of onshore wave energy expenditure on slope bottom areas per unit of horizontal and longshore distance remains constant. Given this constant onshore energy condition, it is mathematically shown that a quadratic function predicts the shape of a shorerise profile in the form of $z_l = ax_l^2 + bx_l + c$, where z_l is the relief of a shorerise point above the origin located at the shorerise-ramp juncture, x_l is the horizontal shoreward distance from the origin, and a,b,c are empirical coefficients. Using this quadratic function, observed z_l values were correlated with corresponding x_l values for 37 shorerise profiles stemming from Long Island, New York, and 37 profiles from Mustang-Padre Island, Texas. For the 74 profiles originating from two regions with diverse geomorphic histories, hence diverse degrees of profile curvature, the coefficients of determination (r^2) ranged between 0.95 and 0.99. For each nine Duck, North Carolina, shorerise profiles taken at the same line from July, 1994 through December, 1995, and with varying degrees of curvature caused by varying wave conditions, r^2 was 0.99. No r^2 value was recorded at unity; therefore, no profile was interpreted as being at equilibrium. Profiles were regarded as moving closer to an equilibrium state as r^2 increased.

Because the results show that a quadratic function effectively predicts the shape of shorerise profiles that varied over space and time, it follows that the geometric property of a bottom slope increasing onshore at a constant rate of $2a$ ($d^2z_l/dx_l^2 = 2a$) is invariant over space and time. Therefore, when r^2 is equal to unity, the geometric property of a bottom slope increasing onshore at a constant rate of $2a$ may be the signature of all shorerise profiles at equilibrium with waves that deliver a constant acceleration rate of onshore wave energy expenditure over slope bottom areas per unit of horizontal and longshore distance. Because wave conditions frequently vary, profiles should rarely achieve equilibrium. Using Airy wave theory and assuming that an open system operates across a shorerise, a discussion is presented in an attempt to explain why shoaling waves should maintain a profile with a slope that increases onshore at a constant rate.

ADDITIONAL INDEX WORDS: *Equilibrium profiles, Long Island, Mustang-Padre Island, nearshore, ramp, shoreface, shorerise.*

INTRODUCTION

The concept of an equilibrium shoreface profile has been with us for decades (CORNAGLIA, 1889; FENNEMAN, 1902). For a given wave-climatic regime, wave-current processes acting on transportable bottom sediments eventually construct a cross-shore profile shape that on average is maintained through time; the geographic position of such a profile may change with time, but the its average shape does not (FENNEMAN, 1902). The equilibrium profile concept has been used to predict the rate of beach erosion when sandy shores are subjected to a relative sea-level rise (BRUUN, 1962; DUBOIS, 1995, 1997). On the other hand, developing a theoretical-mathematical model that describes an equilibrium cross-shore profile and can reasonably determine if an existing profile is at or near equilibrium has been problematic. BRUUN (1954) and DEAN (1977) have attempted to formulated such a model, but their models have raised concerns regarding the underlying model assumptions and whether the models actually predict an equilibrium profile (PILKEY *et al.,* 1993; RIGGS *et al.,* 1995; THIELER *et al.,* 1995; DUBOIS, 1999). The purpose of this paper is to (a) review Dean's model, and to (b) offer a quadratic function as a viable model for describing the shape of an equilibrium shorerise profile. The review portion of this paper focuses solely on Dean's model because it has been generally accepted by coastal engineers, including the U.S. Army Corps of Engineers, as being the correct expression for describing equilibrium profiles. For a review of other cross-shore profile models including Dean's model, consult KOMAR (1998); additional models have been constructed by KEULEGAN and KRUMBEIN (1949), LEE (1994), and WANG and DAVIS (1998).

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SHORE PROFILE MODELS

Shore Terminology

The submarine portion of a shore consists of a shoreface and ramp (Figure 1). In turn, the shoreface is composed of a

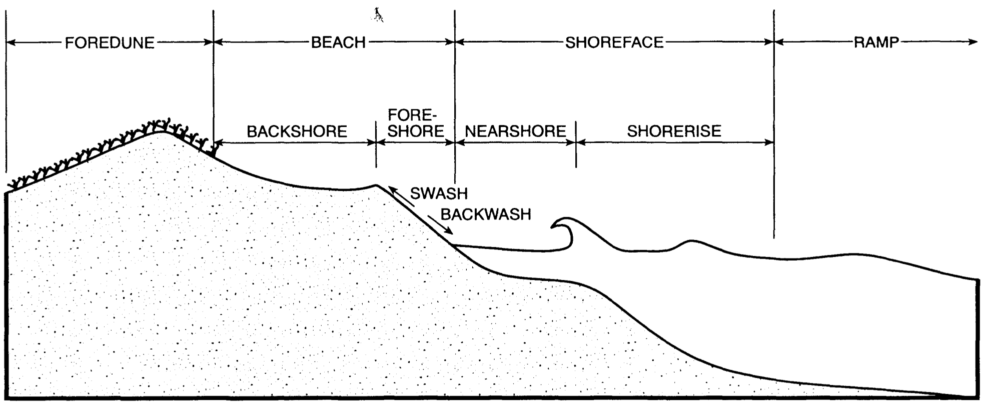

Figure 1. A definition sketch of shore terms used in this paper.

nearshore and shorerise. In a marine setting, a nearshore extends seaward from a shoreline to a water depth of about 2 to 6 m where storm waves break. A shorerise extends from the seaward edge of the nearshore to a water depth of about 10 to 15 m where the limited depth of wave sediment transport is generally reached (Figure 1). A shorerise toe is here defined as the seaward point where a shorerise ends and a ramp begins. The shape of a shorerise is concave upward with curvature increasing onshore. A ramp begins at a shorerise toe and extends in the offshore direction as a linear plane dipping gently seaward. The water depth (h) over the ramp is related to offshore distance (x) in the form

$$h = a_r + b_r x, \tag{1}$$

where a_r is the intercept of h at a shoreline and b_r is the slope tangent of the ramp (EVERTS, 1978).

Dean's Model

Using shallow-water linear wave theory, DEAN (1977) theoretically concluded that if a shore profile is subjected to a uniform energy dissipation per unit water volume, then water depth increases with increasing offshore distance following the form

$$h = Ax^m, \tag{2}$$

where A is regarded as a scale coefficient and m as a shape coefficient equal to ⅔. In turn A is predicted as

$$A = \left[\frac{24D(\mathrm{D})}{5\gamma\sqrt{g}k^2}\right]^{2/3}, \tag{3}$$

where D is the rate at which energy is dissipated for a given size of bottom sediment (D), γ is the specific weight of water, g is the gravitational constant, and k is the ratio of spilling breaker height to water depth and is taken as a constant of 0.78. DEAN (1977) further showed that D was a direct function of D; thus, A was reduced to vary directly as a function of D (MOORE, 1982), or of the fall velocity of a particle size (DEAN, 1987). In this paper the combination of (2) with m set at ⅔ and (3) is henceforth regarded as Dean's model of an equilibrium shore profile. Dean's model was designed for the surf zone, but its use has been extended to include upper segments of a shorerise (DEAN, 1977).

More complex variations of (2) with m set at ⅔ are found in KRIEBEL *et al.* (1991), LARSON (1991), and DEAN *et al.* (1993).

Concerns About Dean's Model

Several concerns have been raised in regards to the properties and accuracy of the assumptions that govern Dean's model (PILKEY *et al.,* 1993; DUBOIS, 1999), and alternative models have been proposed (BOWEN, 1980; BODGE, 1992; INMAN *et al.,* 1993; KOMAR and MCDOUGAL, 1994; WANG and DAVIS, 1998). The following three concerns are raised by this writer.

1. It seems reasonable to assume that all profiles at equilibrium should each have a common geometric profile property that identifies such a profile as being in an equilibrium state. In Dean's model, the exponent value of ⅔ serves as the index that characterizes all equilibrium profiles. However, this value does not represent any common geometric property among shore profiles. Cross-shore profiles drawn with varying A values and m set at ⅔ will all have different degrees of curvature with no one common geometric property, other than being concave upward. If m was equal to 1 or 2, then it could be said of equilibrium profiles that they have a constant slope or that they have a constant rate of slope decrease in the seaward direction, respectively. As it stands, Dean's model shows no geometric profile property that is constant among equilibrium profiles.
2. Dean's model assumes that for an equilibrium shore profile, m is constant at ⅔ while A is independent of m. However, when observed A and m values are plotted against each other for nearshore or shorerise profiles within a given shore region, the results reveal that A values are inversely dependent on m values and that m is not constant at any value (DUBOIS, 1999). The A coefficient is related to the m coefficient in the form of

$$A = ae^{-bm}, \tag{4}$$

where e is the base of the natural logarithm, a and b are empirical coefficients that vary from one shore region to another DUBOIS (1999). The inverse relationship between A and m appears to be a function of the geometry of shore profiles. Note that in Dean's model the A value is actually the water depth at a unit distance from a shoreline. Given the range of profile shapes within a shore region, the water depth at a unit distance from shore is shallower for a straight profile ($m = 1$) than it is for a concave one ($m < 1$) (DUBOIS, 1999).

3. Dean's model assumes that only the mean particle will influence the shape of an equilibrium profile. The model excludes the initial shore slope from which an equilibrium profile is eventually sculptured by waves, and this may be the reason why m is not constant but varies from profile-to-profile within a shore region and among shore regions (DUBOIS, 1999). Based on wave tank experiments, RECTOR (1954) and EAGLESON *et al.* (1961) reported that the shape of an equilibrium shore profile is not only dependent on wave action and on the physical properties of bottom sediment, but also on the initial bottom slope as reflected by the original volume of shore sediments. Waves with identical physical properties traversing across significantly different initial slopes resulting from an abundance of sediments or a lack there of will eventually construct equilibrium profiles with different shapes, hence different m values. In turn, the sediment volume of a shore segment is linked to the geomorphologic history of a referent shore region. Because shore regions usually have different geomorphologic histories, these different histories lead to major or minor spatial variations of shore sediment volume, which in turn contributes to the regional variations of m values (DUBOIS, 1999).

Based on the sum of issues raised in these three concerns, this writer believes that Dean's model may not be a correct theoretical model for predicting equilibrium cross-shore profiles.

Quadratic Model

The quadratic model, developed for a shorerise, is based on assumptions made from observations of fundamental properties of shorerise profiles. Beginning at a shorerise toe and traversing landward, it is observed that each consecutive slope segment rests on a preceding segment in response to gravity and that the bottom slope increases onshore suggesting that waves begin to sculpture a profile near a shorerise toe. Therefore, from a physical point of view, it is reasonable to assume that the origin of a shorerise profile is located at a shorerise toe. With the origin located at a shorerise toe, the profile coordinates of a shorerise point are given as z_l and x_l, the relief above the origin and the horizontal shoreward distance from the origin, respectively. As the bottom slope angle increases onshore, the bottom slope length (c) per unit horizontal distance (x) likewise increases (Figure 2) as does the bottom area (A_b) ($A_b = cy$, where y is a unit distance in the longshore direction). With the near bottom orbital velocities increasing as wave height increases during shoaling (KING, 1972), the amount of dissipated energy across A_b increases shoreward. For a shore profile to be in a state of equilibrium with wave action, some physical property of wave energy acting on a shore bottom must remain constant as this energy travels shoreward across a profile. If a shorerise slope increased shoreward at a constant value (K) or

$$d^2z_l/dx_l^2 = K, \tag{5}$$

then c would accelerate shoreward (Figure 2) at some constant value (K_c) or

$$d^2c/dx_l^2 = K_c. \tag{6}$$

In turn, the amount of energy dissipated in a shoreward direction across A_b (E) should likewise accelerate at some constant value (K_e) or

$$d^2E/dx_l^2 = K_e. \tag{7}$$

To further advance this discussion, (7) is here assumed to be true in the real world; for an equilibrium setting, it is this physical property of wave energy that remains constant as waves travel onshore. Given such a wave energy condition, an equilibrium shorerise profile would form with a bottom slope that increases shoreward at a constant rate (5). The equation for the coordinates of an equilibrium shorerise profile that begin at a shorerise toe is obtained as follows. Integrating (5) twice yields

$$z_l = (K/2)x_l^2 + bx_l + c, \tag{8}$$

where b and c are coefficients. Setting a equal to K/2, (8) becomes

$$z_l = ax_l^2 + bx_l + c. \tag{9}$$

Because a shorerise profile passes through the origin, c is equal to 0; thus, (9) reduces to

$$z_l = ax_l^2 + bx_l. \tag{10}$$

The value of the a coefficient reflects the degree of concavity in a profile; as a decreases the degree of concavity decreases, and the shape of a profile comes closer to being linear. If a is equal to zero, then the shape of a profile is linear with a constant slope equal to b.

Differentiating (10) yields

$$dz_l/dx_l = 2ax_l + b, \tag{11}$$

where b is the slope tangent at the origin. Because the origin

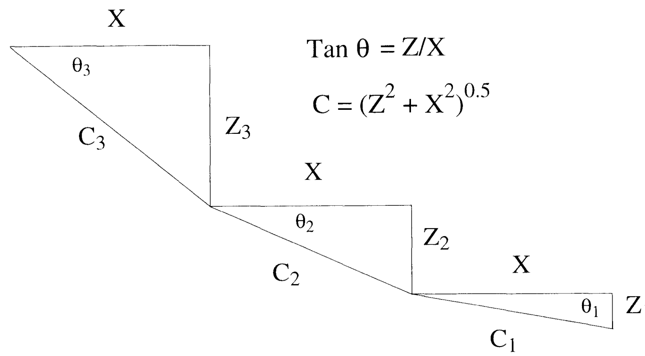

Figure 2. θ is the angle between sides x and c. With the horizontal distance (x) being constant, the hypotenuse (c) increases as slope (θ) increases in the onshore direction.

is located at the juncture of a shorerise and ramp, *b* should equal or nearly equal b_r in (1). For a shore region, it has been suggested that *b* is an index of the erodibility of bottom sediments (DUBOIS, 1999). Profiles with gentle slopes near a toe are associated with lesser amounts of shore sediments as compared to profiles with relatively steep slopes (DUBOIS, 1999). Also, *b* may vary from profile-to-profile as the degree of sediment resistance to transport changes from one profile to another.

Differentiating (11) yields

$$d^2z_l/dx_l^2 = 2a = K. \quad (12)$$

Equation (12) shows that the slope of a shorerise profile will increase onshore at a constant rate of twice the value of *a*. The coefficient *a* may be related to the same variables that influences m in (2). For equation (2), A is inversely related to m (4), and m is in part influenced by the width of a shoreface, which in turn reflects the geomorphologic history of a shore region (DUBOIS, 1999). Thus, *a* may be likewise influenced by the width of a shoreface.

The rest of this paper focuses on describing how equations (1) and (9) were tested against reality and on interpreting the results of the tests.

STUDY AREAS AND METHODOLOGY

The water depths (h) and corresponding offshore distances (x) used to test the accuracy of (1) and (9) in the field were obtained from bathymetric maps of Mustang-Padre Island, Texas, and of Long Island, New York. These two sites were selected because of their diverse geomorphic histories. Historically, the Gulf of Mexico basin has served as a sediment sink for fluvial transported materials (VAN ANDEL and POOLE, 1960). In west Texas, sediments transported by the Rio Grande and the Colorado-Brazos Rivers and deposited during the late Pleistocene produced deltas that were subsequently submerged by sea-level rises during the Holocene (MAZZULLO and WITHERS, 1984). During the Holocene sea-level rise, additional fluvial sediments were deposited onto the Pleistocene deltaic materials (SHIDELER, 1977; MAZZULLO and WITHERS, 1984). The net result of these depositional events along the Mustang-Padre Island has yielded a relatively wide nearshore and shorerise with mean widths of 415 and 1832 m, respectively (DUBOIS, 1999).

Conversely, the Atlantic Long Island region has undergone a different geomorphic history. The shorerise area is composed of Pleistocene glacial drift (TANEY, 1961). However, during the Holocene sea-level rise, there is no geological evidence suggesting that fluvial deposition was extensive along the shoreline. Rivers deposited their sediments in lagoons formed landward of barriers. Thus, the barrier shore, lacking an input of alluvium, transgressed as sea-level rose during the Holocene. Here, because of a relatively low volume of shore sediments, the mean widths of nearshore and shorerise are relatively small at 396 and 954 m, respectively (DUBOIS, 1999).

For Mustang-Padre Island and for Long Island, h and x values were recorded from the Baffin Bay and Corpus Christi, and from the Long Island East and West topographic-bathymetric maps, respectively. These maps, published by the United States Geological Survey and the National Ocean Services, have a scale of 1:100,000. Spaced at 2.5 km apart, the number of profiles taken along the Texas and New York shore was 37 and 42, respectively; however, for the New York shore, five of the 42 lines were excluded from analysis because each had a highly irregular ramp topography. The set of profile lines at the Texas site begins about 10 km south from Port Aransas and extends southward for 90 km (Figure 3). At the New York site, the set of profile lines originates approximately 9 km east from the western terminus of Fire Island and extends eastward for 105 km (Figure 4). The shorerise at Mustang-Padre Island extends from about 3 m to about 11 m of water depth; for Long Island, the range is from about 4 to 14 m (DUBOIS, 1995, 1999). Seaward of the nearshore the contour interval is 1 m for all maps. At both sites, profile coordinates were recorded out to a water depth of 20 m.

Once the depths and corresponding offshore distances were recorded for all profiles, cross-shore profiles were then plotted. Because a relatively sharp break occurred at the juncture between the nearshore and shorerise, determining the landward point of the shorerise was relatively simple. From the same plot, the position where a break in slope symmetry occurred between the concave shorerise and the linear seaward-dipping slope of a ramp was noted. By employing least-squares regression analysis, equation (1) was solved using all points seaward of the shorerise-ramp break (Figure 5); the purpose of solving (1) was to obtain b_r for each profile so that it could be compared with *b* in equation (9).

Locating the origin of the z_l and x_l axes was determined by trial-and-error. For the first trial, the isobath that visually marked the break-point between the shorerise and ramp was selected as the origin. At this point, the water depth (h_{max}) was recorded and set to zero ($h_{max} - h_{max}$). Then for each isobath (h) between the origin and the landward end of a shorerise profile, z_l was calculated as $h_{max} - h$, and x_l was measured as the horizontal distance between a referent isobath and the origin. Equation (9) was empirically fitted with the observed z_l and x_l valves by using least-squares regression, and the goodness-of-fit was determined by the coefficient of determination (r^2). After (9) was solved, it was often noted that the values of *b* and *c* in (9) did not come close to their expected values of b_r and zero, respectively. Therefore, assuming that the water depth changed linearly between two isobaths, the origin was slightly shifted either landward or seaward, and the procedure of refitting (9) with new z_l and x_l values was repeated. This trial-and-error process continued until the *b* and *c* coefficients reasonably matched their expected values while maintaining a high r^2 value (Figure 5).

To test the temporal applicability of the quadratic model, water depths and corresponding offshore distances along line 188 at the U.S. Army Corps of Engineers' Field Research Facility (FRF), Duck, North Carolina were downloaded from FRF's web site at http://www.frf.usace.army.mil. Since 1981 the US Army Corps of Engineers has surveyed biweekly profile lines down to a water depth of about 8 m (LEE *et al.*, 1998). Line 188 is located about 500 m south from the research pier. At each line, water depths have been measured at increments of about 1 m of horizontal distance. For further

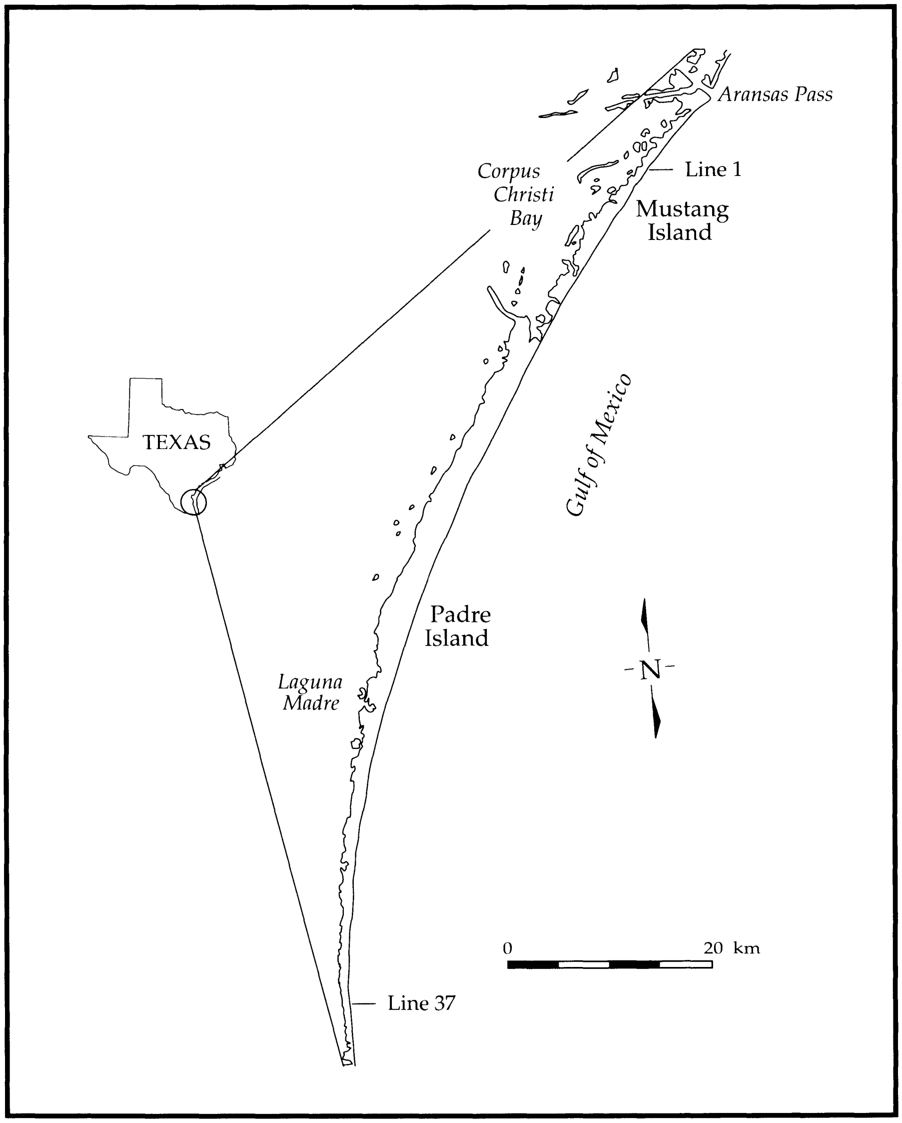

Figure 3. Location of the Texas study area (after DUBOIS, 1999).

information about this research station consult BIRKEMEIER *et al.* (1985). Between 29 July 1994 and 12 December 1995, bathymetric data for 12 surveys were posted on FRF's web site; of the 12 surveys, nine were selected for this study because collectively these surveys showed the widest range of bottom topography at line 188. The reason for selecting time frame between July 1994 and December 1995 is explained as follows. To test the quadratic model, bathymetric data extending down to water depths of about 20 m are needed. Surveys at FRF extend to only 8 m. However, at FRF during August of 1994, bathymetric data that spanned the survey area and extended beyond the depth of 20 m were collected and used to construct a bathymetric map with a scale of 1: 15,000 and a contour interval of one meter. Line 188 was located on a copy of this map, and along this line water depths and corresponding offshore distances were recorded at a one meter contour interval beginning and ending at the 8 and 20 m contour, respectively. Seaward of the 20 m contour, an irregular bottom topography existed owing to the presence of sand ridges (WRIGHT *et al.,* 1994). The bathymetric data obtained from the map were then joined at the 8 m contour with each of the nine data sets obtained from FRF's web site. From the end of July 1994 to mid-December 1995, it was assumed that the shape and elevation of the shore bottom below the 8 m contour would remain reasonably constant. This assumption was based on the work conducted by LARSON and KRAUS (1994). From 1981 through 1991, they found that the change in the shore bottom elevation of the 8 m contour line was relatively small as measured by a 0.025 m value of the standard deviation of water depth over that line (LARSON and KRAUS, 1994); at times, however, the 8 m contour line can vertically fluctuate by 40 cm (NICHOLLS *et*

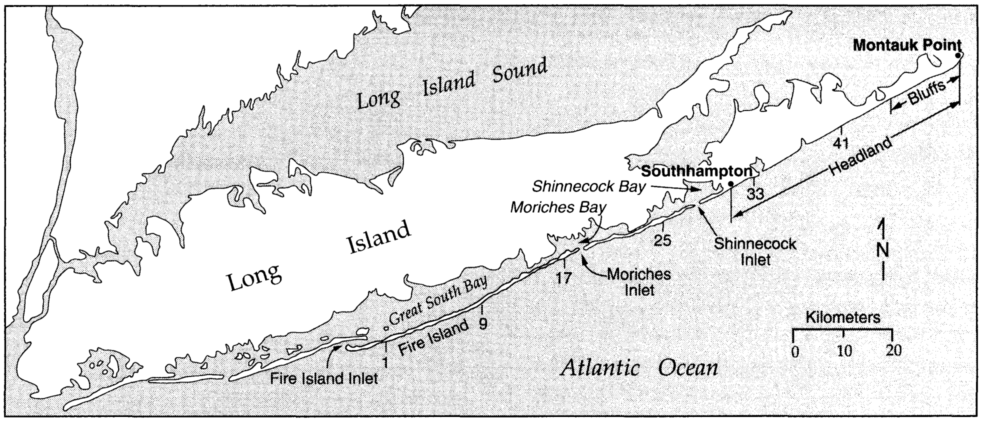

Figure 4. Location of the New York study area. Numbers along the shoreline are profile line numbers (after DUBOIS, 1995).

al., 1998). The largest frequency and magnitude of the bottom elevation changes occurred onshore of about 3 m of water depth (LARSON and KRAUS, 1994).

At Duck, N.C., the location of the shorerise toe was taken at the 16 m water depth contour where a break in slope was noted. A shorerise landward end was determined by trial-and-error using the same procedure as previously described for locating the seaward end of profiles from Long Island and Mustang-Padre Island. For the nine profiles, the shorerise landward end was located on the upper seaward flank of the inner bar in water depths between 1 and 2 m.

RESULTS AND DISCUSSION

Spatial Variation of the Ramp and Shorerise

The solution to equation (1) for each ramp profile off of Mustang-Padre Island and Long Island is shown in Table 1. For all profiles at both locations, r^2 is greater than 0.90;

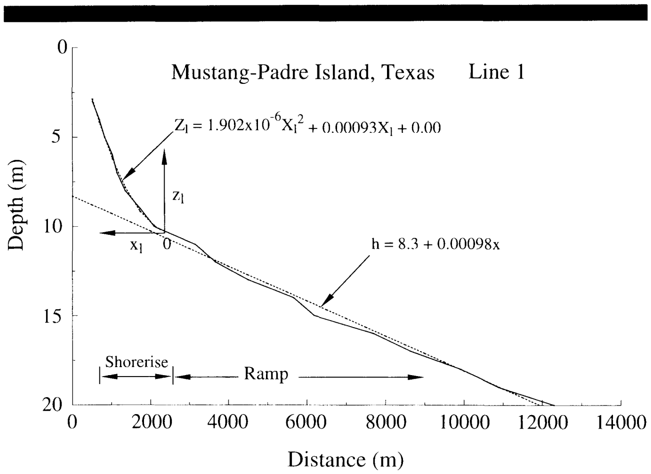

Figure 5. The observed profile (solid line) and predicted profiles (dash lines) for line 1 at Mustang-Padre, Texas. The vertical exaggeration is about 394×.

thus, the ramp in a cross-shore view can be regarded as a landform with a planar surface that dips gently seaward, which concurs with EVERT's (1978) observations. The trend line of a ramp slope intercepts a shoreline (a_r) at an average depth of 9 m at both study sites, whereas the average ramp slope (b_r) at Mustang-Padre Island is gentler than its counter part at Long Island (Table 1). All ramp surfaces are not nearly flat. Some ramps have highly irregular surfaces as was the case for five Long Island profiles not used in this study. No attempt is made to explain the reason for similar a_r values, diverse b_r values, or irregular topography because the geomorphic processes responsible for developing and maintaining a ramp are not clearly understood at this time.

The solution to equation (9) for each shorerise profile at Mustang-Padre Island and Long Island is shown in Tables 2 and 3. Based on the large r^2 values at both study sites, it appears that a quadratic function effectively predicts the shape of a shorerise profile when the *b* and *c* coefficients are adjusted so that they equal or nearly equal the ramp slope (b_r) and zero, respectively. Of the two study sites, the average r^2 value is slightly larger for the Texas profiles (Tables 2 and 3). The variation between the average r^2 values of both shore regions may be a function of the diversity in physical properties of the bottom materials and how differently the materials interact with coastal processes. For example at Long Island, the shorerise as it has transgressed has cut into a textural variety of materials ranging from fluvioglacial gravels to muddy estuarine deposits (SCHWAB *et al.,* 1999). It is reasonable to assume that as a shorerise cuts into strata with varying vertical physical properties, such as particle size and the degree of compaction or cementation, the sediment resistance strength against transport will also vary across a profile. For a given amount of wave power, slope segments across a profile will be relatively steep where sediment resistance is strong and gentle where resistance is weak. If resistance to sediment transport is significantly greater in the lower portion of a shorerise than it is in the upper portion (or vice

Table 1. *Parameters and solution to equation 1 for ramp profiles.*

Mustang-Padre Island, Texas						Long Island, New York					
L	R	No.	r^2	a_r	b_r	L	R	No.	r^2	a_r	b_r
1	10–20	11	0.99	8.3	0.00098	1	14–20	7	0.94	9.4	0.00301
2	10–20	11	0.99	8.3	0.00097	4	14–18	5	0.94	10.7	0.00235
3	10–20	11	0.99	8.3	0.00093	8	13–20	8	0.99	10.5	0.00222
4	11–20	10	0.99	9.0	0.00091	9	12–20	9	0.95	9.2	0.00300
5	12–20	9	0.97	9.7	0.00084	10	14–20	7	0.96	9.0	0.00354
6	11–20	10	0.99	9.5	0.00091	11	14–20	7	0.97	9.9	0.00314
7	11–20	10	0.99	9.3	0.00090	12	15–20	6	0.94	10.6	0.00311
8	11–20	10	0.99	9.1	0.00096	13	16–19	4	0.98	9.6	0.00397
9	11–20	10	0.99	9.4	0.00094	14	15–20	6	0.98	8.2	0.00451
10	11–20	10	0.99	9.3	0.00098	15	13–20	8	0.99	7.3	0.00498
11	11–20	10	0.98	9.3	0.00099	16	16–20	5	0.99	9.6	0.00381
12	11–20	10	0.98	9.4	0.00100	17	16–20	5	0.97	8.5	0.00434
13	12–20	9	0.99	9.7	0.00098	18	14–20	7	0.99	8.1	0.00467
14	11–20	11	0.99	8.8	0.00109	19	14–20	7	0.95	8.0	0.00484
15	10–20	11	0.99	8.8	0.00110	20	14–20	7	0.98	7.7	0.00477
16	13–20	8	0.99	8.9	0.00096	21	14–22	9	0.99	8.6	0.00421
17	12–20	9	0.99	9.6	0.00101	22	15–20	6	0.97	8.3	0.00443
18	12–20	9	0.99	9.3	0.00106	23	10–22	13	0.99	6.7	0.00499
19	12–20	9	0.99	9.5	0.00104	24	14–20	7	0.91	10.2	0.00314
20	12–20	9	0.99	9.3	0.00107	25	14–19	6	0.92	7.6	0.00474
21	13–20	8	0.98	9.7	0.00107	26	15–20	6	0.98	10.3	0.00332
22	11–20	10	0.99	9.0	0.00107	27	12–22	11	0.98	6.6	0.00512
23	11–20	10	0.99	8.9	0.00110	28	14–20	7	0.99	8.0	0.00483
24	11–20	10	0.99	8.8	0.00116	29	13–21	10	0.94	8.0	0.00429
25	11–20	10	0.99	8.8	0.00117	30	16–20	5	0.98	8.9	0.00389
26	10–20	11	0.99	8.4	0.00122	31	15–22	8	0.96	8.9	0.00396
27	11–20	10	0.99	8.8	0.00116	32	14–20	7	0.97	9.0	0.00367
28	11–20	10	0.99	8.8	0.00117	33	14–22	9	0.99	8.0	0.00425
29	12–20	9	0.99	9.2	0.00112	34	10–20	11	0.95	7.7	0.00434
30	10–20	11	0.99	8.9	0.00118	35	10–22	13	0.97	8.1	0.00411
31	10–20	11	0.99	8.7	0.00121	36	11–20	10	0.98	8.6	0.00379
32	12–20	9	0.99	8.9	0.00103	37	12–20	9	0.98	9.9	0.00345
33	11–20	10	0.97	9.7	0.00108	38	12–20	9	0.99	9.5	0.00334
34	11–20	10	0.97	9.8	0.00106	39	12–20	9	0.98	9.5	0.00309
35	12–20	9	0.98	10.3	0.00099	40	12–20	9	0.97	10.1	0.00285
36	14–20	7	0.97	11.3	0.00085	41	12–18	7	0.97	9.3	0.00317
37	11–20	10	0.97	9.8	0.00107	42	12–20	9	0.97	10.1	0.00296
Avg.			0.99	9.2	0.00104				0.97	8.9	0.00384
S.D.			0.01	0.6	0.00010				0.02	1.1	0.00078

Note: Columns L, R, and No. refer to line number, range of water depth (m), and number of observations, respectively; r^2 is the coefficient of determination; Avg. is the arithmetic mean and S.D. is the standard deviation.

versa), then the profile shape will appear as an S-curve (Figure 6) with lower r^2 values. On the other hand, the Mustang-Padre Island shorerise has been cut into strata composed of alluvial deposits that may have relatively homogeneous physical properties. Therefore, the curvature of a Texas cross-shore profile is relatively smooth, and a quadratic function is very effective at predicting z_l values for given observed x_l values (Table 2).

Within a shore region, the physical properties of shorerise strata may also influence the depth of water where a shorerise toe occurs. For Mustang-Padre Island shorerise, the water depth mean over the shorerise toes is 12 m and has a standard deviation of 1.0 m (Table 3). Along this Texas shore, wave action begins to effectively sculpture a shorerise at a relative uniform water depth; therefore, assuming that any given major weather event generates deepwater wave properties that are reasonably similar along the length of this 90 km study site, it appears from the relatively low standard deviation that the sediment strength to resist transport is reasonably uniform across the shorerise at a water depth of about 12 m. On the other hand, waves off of Long Island begin to sculpture the shorerise at various depths; the water depth at the shorerise toe has a mean of 14.5 m with a standard deviation of 2.3 m (Table 3). This relatively large variability of water depth where a shorerise profile begins may be a function of the spatial variation of the degree to which bottom sediments can resist transport; again, this assumes that any major weather event generates deepwater waves properties that are similar along the length of this 105 km shoreline.

As previously suggested by DUBOIS (1999), the reason why Long Island shorerise profiles begin on average at greater depths than Texas profiles is because deep-water waves that annually strike the Long Island shore may be larger. Although both coasts are occasionally affected by hurricane waves, the deeper location of Long Island shorerise toes may be directly related to large waves generated by extratropical storms that annually strike this shoreline during winter. Ex-

Table 2. *Parameters and solution to equation 10 for shorerise profiles at Mustang-Padre Island, Texas.*

L	R	No.	r^2	X_{lmax}	$a \times 10^{-6}$	b	c
1	3–10.15	9	0.99	1727	1.902	0.00093	0.00
2	3–10.20	9	0.99	1750	1.916	0.00090	−0.04
3	3–10.45	9	0.99	1807	1.896	0.00088	−0.14
4	3–11.10	10	0.99	1943	1.752	0.00092	−0.06
5	3–12.25	11	0.99	2334	1.389	0.00076	−0.08
6	3–11.20	10	0.99	2011	1.670	0.00091	−0.03
7	3–11.40	10	0.99	2146	1.529	0.00087	−0.14
8	3–11.10	10	0.99	1957	1.710	0.00089	−0.02
9	3–11.80	10	0.99	2231	1.369	0.00097	−0.08
10	3–11.25	10	0.99	1984	1.608	0.00094	0.01
11	3–11.50	10	0.99	2216	1.285	0.00094	−0.01
12	3–11.50	10	0.99	2180	1.360	0.00091	−0.03
13	3–12.15	11	0.99	2265	1.367	0.00092	0.09
14	3–11.10	10	0.99	1862	1.769	0.00111	−0.04
15	3–11.95	10	0.99	2042	1.593	0.00101	0.30
16	3–12.35	11	0.99	2289	1.339	0.00095	0.06
17	3–12.10	11	0.99	2149	1.524	0.00097	0.00
18	3–12.20	11	0.99	2266	1.356	0.00112	−0.15
19	3–12.80	11	0.99	2542	1.158	0.00104	0.00
20	3–12.20	11	0.98	2522	1.157	0.00102	−0.05
21	3–12.50	11	0.99	2368	1.261	0.00108	0.15
22	3–12.15	11	0.99	2287	1.350	0.00104	−0.04
23	3–12.75	11	0.99	2451	1.166	0.00110	0.09
24	3–11.10	10	0.99	1828	1.872	0.00116	−0.04
25	3–12.00	10	0.99	2254	1.288	0.00114	0.00
26	3–10.95	9	0.99	1867	1.741	0.00122	−0.17
27	3–11.70	10	0.99	2215	1.378	0.00109	−0.18
28	3–11.70	10	0.99	2087	1.468	0.00111	0.03
29	3–12.30	11	0.99	2456	1.184	0.00113	−0.14
30	3–12.25	11	0.99	2216	1.488	0.00117	−0.19
31	3–11.45	10	0.98	2061	1.541	0.00124	−0.12
32	3–13.20	12	0.98	2718	1.101	0.00105	−0.12
33	3–13.25	12	0.99	2683	1.067	0.00111	−0.02
34	3–13.75	12	0.99	2796	1.028	0.00108	0.04
35	3–14.25	13	0.99	3139	0.900	0.00090	0.00
36	3–14.15	13	0.99	3005	0.964	0.00089	0.01
37	3–13.50	12	0.99	2676	1.099	0.00107	0.11
Avg.	12.00		0.99	2252	1.420	0.00101	−0.03
S.D.	1.00		0.00	340	0.279	0.00011	0.10

Note: Columns L, R, and No. refer to line number, range of water depth (m), and number of observations, respectively; r^2 is the coefficient of determination; Avg. is the arithmetic mean and S.D. is the standard deviation.

Table 3. *Parameters and solution to equation 10 for shorerise profiles at Long Island, New York.*

L	R	No.	r^2	X_{lmax}	$a \times 10^{-6}$	b	c
1	4–14.15	13	0.99	1077	5.472	0.00314	0.05
4	4–16.50	14	0.99	1998	1.757	0.00265	−0.24
8	4–14.75	11	0.98	1468	3.397	0.00207	−0.12
9	4–12.50	9	0.98	794	9.381	0.00305	−0.12
10	4–14.35	11	0.98	1112	4.834	0.00348	−0.14
11	4–16.00	12	0.99	1507	2.853	0.00321	0.05
12	4–17.50	14	0.98	1799	2.200	0.00308	−0.14
13	4–18.00	14	0.99	1617	2.514	0.00355	0.15
14	4–17.50	14	0.99	1664	1.793	0.00475	−0.11
15	4–15.75	13	0.99	1285	3.187	0.00477	−0.05
16	4–18.00	14	0.99	1727	2.499	0.00350	0.02
17	4–18.00	14	0.98	1837	1.715	0.00463	−0.31
18	4–18.00	15	0.98	1801	1.535	0.00467	−0.12
19	4–14.75	11	0.97	1125	4.631	0.00437	−0.40
20	4–16.90	13	0.98	1516	1.251	0.00494	0.12
21	4–15.50	12	0.98	1326	3.014	0.00443	−0.12
22	4–16.30	13	0.98	1541	1.910	0.00459	−0.12
23	4–10.55	7	0.97	536	12.260	0.00482	−0.04
24	4–15.00	11	0.99	1270	4.183	0.00352	−0.16
25	4–14.60	10	0.98	1175	4.187	0.00422	−0.38
26	4–15.40	12	0.99	1098	6.570	0.00314	−0.15
27	4–12.25	9	0.98	598	13.022	0.00507	0.05
28	4–16.55	13	0.98	1476	2.489	0.00488	−0.12
29	4–13.00	10	0.98	1113	3.944	0.00445	−0.23
30	4–16.00	12	0.97	1400	2.787	0.00348	0.28
31	4–15.00	12	0.99	1379	2.687	0.00398	−0.02
32	4–14.00	10	0.98	954	5.829	0.00388	0.33
33	4–14.25	12	0.99	1090	3.933	0.00470	0.00
34	4–10.50	7	0.98	393	31.190	0.00401	0.00
35	4–10.25	7	0.99	375	32.514	0.00443	−0.02
36	5–11.50	7	0.99	420	27.751	0.00387	0.03
37	4–12.00	6	0.98	447	29.611	0.00382	0.10
38	4–12.50	8	0.99	515	24.500	0.00326	0.17
39	5–12.20	7	0.98	501	23.327	0.00330	−0.07
40	4–12.20	9	0.99	477	29.990	0.00300	0.05
41	4–12.25	8	0.95	639	17.359	0.00320	−0.18
42	4–12.00	8	0.97	419	32.204	0.00300	0.33
Avg.	14.50		0.98	1121	9.844	0.00386	−0.04
S.D.	2.33		0.01	493	10.793	0.00076	0.17

Note: Columns L, R, and No. refer to line number, range of water depth (m), and number of observations, respectively; r^2 is the coefficient of determination; X_{lmax} is the shorerise width (m); Avg. is the arithmetic mean and S.D. is the standard deviation.

tratropical storms that generate strong onshore winds are relatively less frequent in the Gulf of Mexico. Assuming shallow-water wave action is the primary process responsible for sculpturing shorerise profiles, an estimate can be made of the minimum significant deep-water wave height and period that control the water depth mean of shorerise toes in a referent region. Shallow-water depths begin where

$$h/L_o = 0.05 \quad (13)$$

(KOMAR, 1998); L_o is deep-water wave length and following linear wave theory is a function of wave period (T) in the form of

$$L_o = gT^2/2\pi. \quad (14)$$

Setting h to 12 and 14.5 m, the water depth means of shorerise toes (Tables 2 and 3), in (13) and solving for L_o yields 240 and 290 m for Mustang-Padre Island and Long Island, respectively. Substituting both wave length values in (14) and solving for T yields 12.4 and 13.6 sec for Mustang-Padre Island and Long Island, respectively. Assuming the T values represent significant wave periods and using a nomogram in KOMAR (1998) to predict significant deep-water wave conditions, periods of 12.4 sec at Mustang-Padre Island and 13.6 sec at Long Island are associated with minimum significant deep-water wave heights of 5.5 and 7 m, respectively.

The a values vary between regions, being on average smaller and with less variability for the Mustang-Padre Island profiles than for the Long Island profiles (Tables 2 and 3). It has been suggested that regional variations of profile curvature may be a function of the diverse geomorphologic history of shore sites (DUBOIS, 1999). To test this hypothesis, a values were correlated with the shorerise width (x_{lmax}) and the cross-section area (A_{rea}) beneath a profile, which was calculated by

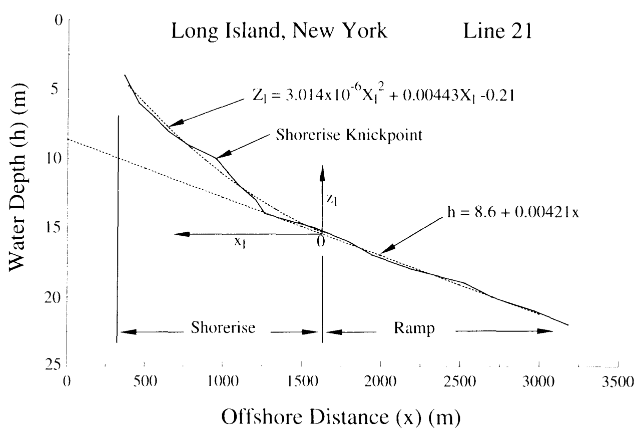

Figure 6. A knickpoint was found on most shorerise profiles of Long Island, New York. The vertical exaggeration is about 88×.

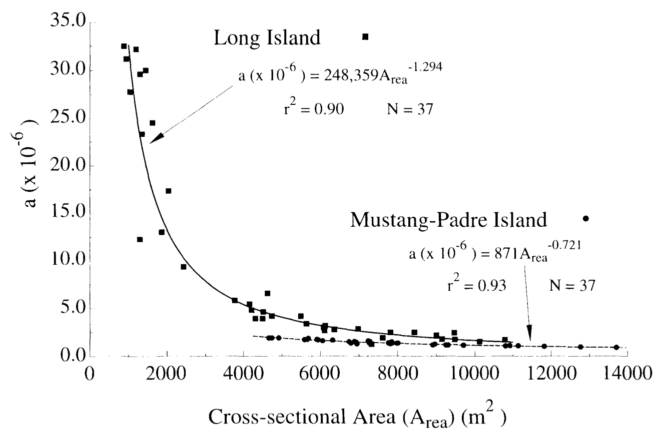

Figure 8. The relation between a values and cross-shore areas beneath profiles at Long Island, New York, and at Mustang-Padre Island, Texas.

integrating each quadratic function from zero to x_{lmax}; stated mathematically,

$$A_{rea} = \int_0^{x_{lmax}} (ax^2 + bx + c)\, dx. \tag{15}$$

Integrating (15) and algebraically simplifying the results yields

$$A_{rea} = [ax^3_{lmax}/3 + bx^2_{lmax}/2 + cx_{lmax}]. \tag{16}$$

The shorerise width and cross-sectional area are variables that should reflect the geomorphic history or the antecedent condition that influenced the development of a present day profile shape (RECTOR, 1954). For a given wave-climatic regime where width and area are large, waves will begin sculpturing profiles further from the shore causing flatter profiles to form (DUBOIS, 1999). The results of the correlation analysis confirms this hypothesis; as either shorerise width or cross-sectional area increases, curvature decreases (Figures 7 and 8). Thus, it appears that the geomorphic history of a shore region has an impact on the shape of a shorerise profile.

The variation of a values within a shore region also may be a function of the variation of geomorphic history within that region as well as the variability of bottom sediments to resist transport. Where resistance against transport is large, shorerise width and area should also be relatively large and a values should be relatively small.

Temporal Variation the Ramp and Shorerise

For the ramp at Duck, North Carolina, no temporal observations can be reported because the ramp was surveyed only once. As offshore distance (x) increased, water depth (h) increased arithmetically in the form of $h = 11.61 + 0.00174x$; r^2 was equal to 0.99 (Figure 9). The number of contour lines (N), from 16 through 20 m of water depth, was five.

The solution to equation (9) for the nine selected Duck profiles is given in Table 4. Based on the large r^2 values, it ap-

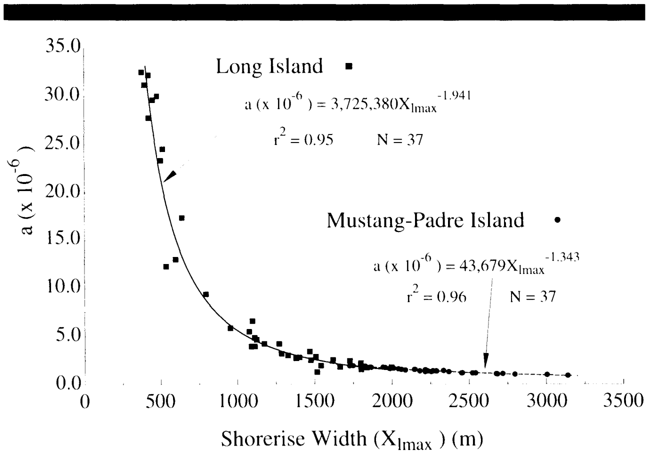

Figure 7. The relation between a values and shorerise widths for profiles at Long Island, New York, and at Mustang-Padre Island, Texas.

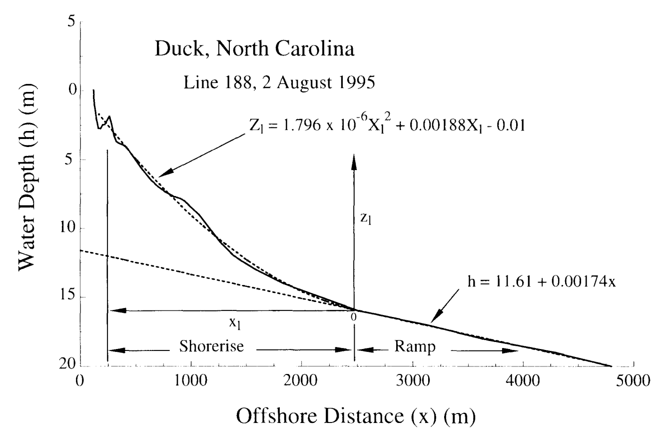

Figure 9. The observed profile (solid line) and predicted profiles (dash lines) for line 188, 2 August 1995. The vertical exaggeration is about 126×.

Table 4. *Parameters and solution to equation 10 for shorerise profiles at Duck, North Carolina.*

Date (yr/mo/dy)	R	No.	r^2	X_{lmax}	$a \times 10^{-6}$	b	c
94/07/09	2.16–16	15	0.99	2292	1.687	0.00211	−0.08
94/11/09	2.12–16	15	0.99	2381	1.859	0.00182	−0.01
94/11/21	2.62–16	15	0.99	2236	1.831	0.00184	−0.01
95/01/25	1.73–16	16	0.99	2293	1.844	0.00179	0.01
95/06/14	1.86–16	16	0.99	2322	1.741	0.00196	−0.03
95/08/02	1.05–16	16	0.99	2345	1.796	0.00188	−0.01
95/08/22	1.86–16	16	0.99	2254	1.991	0.00154	0.07
95/10/03	2.04–16	15	0.99	2235	1.918	0.00167	0.04
95/12/12	2.15–16	15	0.99	2256	1.825	0.00181	0.01
Avg.	1.94			2291	1.832	0.00181	0.00
S.D.	0.42			51	0.090	0.00016	0.04

Note: R is the range of water depth (m); No. is number of observations; r^2 is the coefficient of determination; X_{lmax} is the shorerise width; Avg. is the arithmetic mean; S.D. is the standard deviation.

pears that a quadratic function effectively predicts the varying shapes of shorerise profiles as caused by varying wave-climatic events. Figure 10A shows the varying position of the inner bar; storm waves drive the bar offshore whereas swells move the bar onshore (Figure 10B) (LEE *et al.,* 1998). No matter how waves reconfigure the upper profile portion of line 188, the curvature of a shorerise surface closely follows that of a quadratic function. The shape of a profile, however, does not follow a monotonic curve. Except for the 9 November 1994 profile which has an inflection at about the 8 m contour, the remaining profiles have an inflection at about the 4 m contour and another at the 8 m contour (Figures 9 and 10). The convex profile segment at the 4 m contour is a longshore sand bar that develops during major storm events; thereafter, persistent swells drive the bar onshore and lower the relief until the bar disappears (LEE *et al.,* 1998). The origin of the second convex segment at the 8 m contour (Figure 10B) is unclear to me at this time.

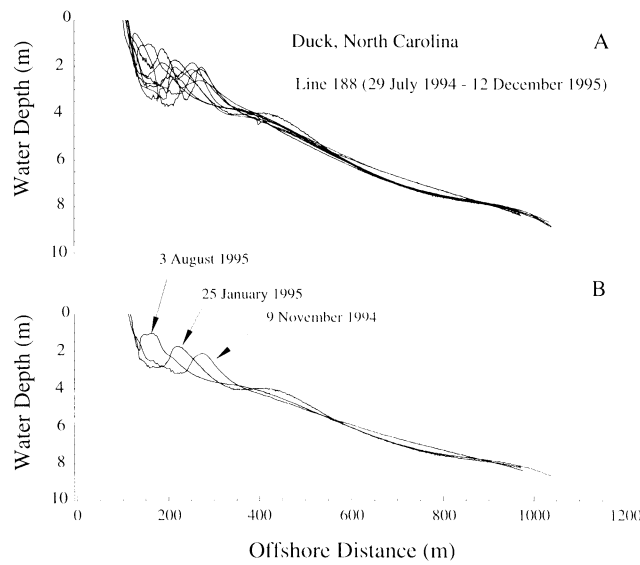

Figure 10. (A) Nine profiles of line 188 taken from 29 July 1994 through 12 December 1995; note the variability of the upper profile shapes. (B) Three of the nine profiles were selected to show more clearly the changing position of the inner bar. The vertical exaggeration is about 42×.

Concepts Inferred from the Quadratic Model

The quadratic model assumes that a shorerise profile achieves equilibrium when the acceleration rate of wave energy expenditure over slope bottom areas (A_b) is constant in the onshore direction, and the shape of a profile is predicted by a quadratic function. Although these assumptions have yet to be proven, the model did effectively described the shape of shorerise profiles and may indeed describe the shape of an equilibrium profile. It can be argued that if the state of an equilibrium profile could not be reasonably predicted by a quadratic function, then one or more of the 83 r^2 values should have been much lower than what was presented in Tables 2–4. A r^2 value of 1.00 was not recorded for any of the 83 profiles; therefore, no profile was interpreted as being at equilibrium, although 59 profiles had r^2 values of 0.99, and these could be regarded as being near equilibrium. It is unreasonable to expect the full length of a shorerise profile to be in equilibrium simply because forcings with sufficient magnitudes to alter the shape of a bottom vary temporally between storm waves and swells. As suggested by STIVE and DE VRIEND (1995), it may be best to think of profile shapes as being in a transient state rather than being in a very-near static one.

As a working hypothesis for future studies, it is here suggested that the signature of a profile at equilibrium is the geometric property of a slope increasing onshore at a constant rate of $2a$. As shown in this study, the degree of profile curvature varies over space and time, but the geometric property of onshore slope increase remains at some constant rate over space and time. Focusing on the temporal aspect, severe storms, which create forcings that can adjust the full range of a profile, occur infrequently while moderate storms, adjusting only the middle and upper portions of a profile to fit their forcings, happen more frequently. Small storms, capable of altering only the upper profile, have the largest frequency of occurrence. Following storm events, swells prevail and readjust a bottom profile to conform to their physical properties. Generalizing, the lower portion of a shorerise profile is frequently inactive while the upper portion is in a near constant stay of flux, and yet the geometric property of onshore slope increase remains at some near constant rate. In an onshore direction, a shorerise profile can be viewed as a composite of surface segments that progressively become younger, the past blending into the present. The profile property that connects surface segments through time is the near constant rate of slope increase in the onshore direction.

For a shorerise profile at or near equilibrium, the fact that the bottom slope is increasing onshore at a constant rate indicates that the dimensions of shoaling waves are changing in a similar fashion. For Airy waves in shallow water, where the ratio of water depth (h) over deepwater-wave length (L_o) is equal to 0.05 or less (KOMAR, 1998), wave length (L_s) and wave phase velocity (C_s) are directly dependent on water depth: $L_s = T\sqrt{gh}$ and $C_s = \sqrt{gh}$. If the bottom slope is increasing onshore at a constant rate, then water depth per unit of x will decelerate onshore at a constant rate, and therefore, L_s and C_s also will decelerate at a constant rate. Wave height in shallow water (H_s) is predicted as $H_s = H_o [0.5 (C_o/C_s)]^{0.5}$ where H_o and C_o are deepwater-wave height and deepwater-wave phase velocity, respectively (KOMAR, 1998); in this case, as C_s decelerates, wave height will accelerate onshore at some constant rate as will the amount of energy dissipated along a traverse of slope bottom areas (A_b). If a shore is recognized as an open system with inputs and outputs of energy and material, then such a shore system may incorporate a condition whereby shallow-water wave properties decelerate or accelerate onshore at constant rates in order to minimize to the lowest possible level the rate of onshore accelerating energy expenditure over a shorerise bottom. To maintain this system of minimized energy expenditure or efficiency, changing wave conditions adjust and readjust the curvature of a shorerise profile so that the onshore slope increases at a constant rate. Thus, a shorerise profile can have varying degrees of curvature and yet be near an equilibrium state with coastal processes.

CONCLUSION

A quadratic function is being offered as a model for describing shorerise profiles at or near equilibrium and for providing an insight into how the shape of an equilibrium profile and shoaling waves are interdependently connected. Based on high coefficients of determination, a quadratic function reasonably predicts z_i values for given x_i values and probably the shape of equilibrium shorerise profiles. Profile curvatures as reflected by the *a* coefficient are related to shorerise widths and to areas of cross-sectional profiles and therefore, are influenced to some degree by the geomorphic history of a referent shore region. Profile curvatures also vary with space and time, but the geometric property of slope increasing onshore at a constant rate of $2a$ is invariant over space and time. Thus, it is proposed that the mark of an equilibrium shorerise profile is the geometric property of a slope increasing onshore at some constant rate. If the bottom slope of an equilibrium profile is increasing onshore at a constant rate, then the water depth per unit of x will decelerate onshore at a constant rate. In turn, the dimensions of Airy shallow-water wave properties will likewise decelerate or accelerate onshore at a constant rate. Where shallow-water wave properties decelerate or accelerate onshore at constant rates, then such a condition may represent the most efficient way by which the accelerating rate of onshore energy expenditure over a shorerise bottom is minimized to its lowest possible value. To maintain this system of minimized energy expenditure, changing wave conditions adjust and readjust the curvature of a shorerise profile so that the onshore rate of slope increase is maintained at a constant value.

Finally, it should be recognized that although the quadratic model reasonably describes the shape of shorerise profiles, there is no proven theory that explains why a shorerise profile should be predicted by a quadratic function. For an equilibrium shorerise profile, the quadratic model assumes that the acceleration of energy expenditure over slope bottom areas (A_b) remains constant in the onshore direction; but is this assumption correct and does it correspond with an equilibrium state? Future research should focus on developing a theory that explains why a quadratic function predicts the shape of a shorerise profile.

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