# **A Rip Current Model Based on a Hypothesized Wave** / **Current Interaction**

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#### **ABSTRACTI**



MURRAY, A.B. and REYDELLET, G., 2001. A Rip Current Model Based on a Hypothesized Wave/Current Interaction. *Journal of Coastal Research,* 17(3), 517-530. West Palm Beach (Florida), ISSN 0749-0208 .

What causes strong, focused rip currents to develop on planar beaches? Observations of waves in the presence of rip currents suggest an interaction between waves and currents that causes wave dissipation. Ahypothesized mechanism that would cause such an interaction plays a key role in a rip current model. In this model rip currents can be selforganized, rather than being necessarily forced by bathymetric features or incident wave patterns. The variables in this cellular numerical modelinteract according to rules that are either direct applications ofequations, or abstractions of physical principles. Key processes in the model include: 1) the hypothesized wave-curren t interaction ; 2) onshore water transport by waves; 3) offshore flow caused by differences between local radiation-stress gradients and surfaceslope-generated pressure gradients, 4) alongshore flow driven by alongshore water surface slopes,5) alongshore mixing ofoffshore-current momentum, and 6) time and space variations in incident wave heights. Model results indicate that interactions and feedbacks between these processes offer a plausible explanation for why rip currents are often narrow and jet-like while also widely spaced, why they can occuron planar beaches as well those with alongshore bathymetric variations, and why they are generally dynamic rather than steady state phenomena.

ADDITIONAL INDEX WORDS: *Nearshore processes, surf-zone circulation, wave dissipation, hydrodynamics, planar beach*

## INTRODUCTION

Rip currents- discrete zones of offshore-directed flow in and near the surf zone-are often narrow, high-velocity, jetlike features. In an environment with a surf zone width on the order of 100 m, rip-current velocities can be nearly one *mls* while widths are on the order of 10 m (SMITH, 1995), an order of magnitude smaller than typical alongshore spacings on the order of a surf-zone width. This fascinating and dangerous phenomenon can be associated with bathymetric features such as cross-shore oriented depressions (rip channels) or gaps in an alongshore bar. However, strong rip currents also occur on alongshore-uniform beaches. Rip currents are not generally steady-state phenomena. Observations from the pier at Scripps Institution of Oceanography (SMITH, 1995) and on other beaches in the vicinity indicate that in this area an individual rip-current occurrence typically lasts for around 10 min.

Recent rip-current models (HAAS *et al.*, 1998; SORENSEN *et al.,* 1998) involve alongshore bathymetric variations that lead to the offshore flow by creating alongshore variations in set up, the offshore-sloping super elevation of the average water surface in the surf zone . The shoreward decrease in wave heights in the surf zone creates a gradient in wave momentum flux (the radiation stress) that supports this elevated surface. Alongshore bars with gaps illustrate how bathymetric features can create alongshore variations in set up. If waves are breaking and dissipating over an alongshore bar, the gradient in wave momentum flux tends to elevate the water surface behind the bar. If the waves are not breaking in the deeper water in a gap in the bar, no momentum gradient tends to elevate the water surface locally. Water will flow from behind the bar alongshore to where the water surface is lower, and then out through the gap in the bar (SAN-CHO, 1995). Other bathymetric configurations can, in a similar way, drive rip currents (SORENSEN *et al., 1998).*

SHEPARD (1950) suggested that incident wave patterns with areas in which the waves are relatively small could be responsible for alongshore variations in set up causing rip currents. Refraction over offshore bathymetry can create alongshore variations in wave height and therefore set up, but at larger scales than those of rip currents. Interference between multiple incident wave trains can produce smaller areas of transiently smaller waves. In the laboratory, generation of nonlinear waves (HAMMACK, 1991) or the interaction of incident waves and standing edge waves of the same frequency (BOWEN, 1969) can generate wave patterns with areas of smaller waves. However, our observations suggest that in the field the wave pattern on small scales generally changes on a time scale shorter than the rip-current scale.

*<sup>00176</sup> received and accepted in revision* 12 *February 2001.*

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It has more recently been suggested that instabilities in the alongshore current could be relevant to rip currents. A numerical model of this phenomenon (ALLEN, 1996; SLINN, 1998), which does not involve variations in the water surface elevation, predicts that under some circumstances, discrete areas of offshore flow will evolve from a strong alongshore flow. The offshore flows in this model are dynamic, and can occur in the absence of alongshore bathymetric variations, but it is not clear whether they can form the narrow, jet-like features often observed in nature. In addition, rip currents in nature do not appear to be restricted to situations with a strong alongshore current.

In a series of analytical models for rip currents on planar beaches, nearshore circulation cells are predicted to develop because of a feedback between waves and currents *(e.g.* DAL-RYMPLE and LOZANO, 1978; SASAKI *et al.,* 1988; SURIAMI-HARDJA and TSUCHIYA, 1996; FALQUES *et al.,* 1999). Alongshore differences in cross-shore currents bend (refract) the waves. In parts of the surf zone where the refracted waves converge, radiation-stress gradients drive currents that converge in the alongshore direction. The cross-shore return flow localized by this convergence, in turn, bends the waves. This instability requires wave refraction, and therefore alongshore gradients in the cross-shore currents, to occur over scales on the order of a surf-zone width. However, typical widths of intense rip currents are much narrower than a surf zone width (SMITH, 1995), as is the refraction caused by rip currents that we have observed on Southern California beaches.

In this paper we present an alternative model of rip currents. This model involves alongshore variations in set up, but these variations need not be imposed by the incident wave patterns or bathymetry. In this model, rip currents organize themselves, driven by a feedback involving a newly hypothesized interaction between waves and currents. In the second section, we qualitatively describe the hypotheses that motivated the model. In the third section we describe the modeling approach. In the next four sections we present the algorithms and results of the cellular model developed to test these hypotheses. In this model, we have parameterized some aspects of the hydrodynamical equations. We then discuss the results and future work.

## **QUANTITATIVE DESCRIPTION OF HYPOTHESES AND MODEL PROCESSES**

A hypothesized mechanism for wave-energy dissipation in the presence of a strong current plays a key role in the model. This hypothesis arises from observations: In a very strong rip current a gap in wave breaking can extend through the surf zone (SHEPARD, 1941), even when approximately uniform breaking at the same location shortly before and after the rip-current occurrence indicates alongshore-uniform bathymetry. We have observed that when this gap in breaking occurs, the waves are not generally larger when they reach the shore than they are in adjacent areas, suggesting that some process other than breaking dissipates wave energy in the presence of a rip current. This phenomenon could be explained by the following interactions: Waves are generally more disorganized in a rip current than in adjacent areas,



Figure 1. Schematic illustration of the hypothesized mechanism for dissipation of wave energy in the presence of a current. (a) The bending of a wave crest because of a current-generated eddy (gray), and the orbital velocities below the vicinity of the wave crest, onshore is dotted and offshore is dashed. We have shown an eddy with a vertical axis, but an eddy with a horizontal axis would have a similar effect, creating a strong vertical shear in wave orbital velocities, Offsets in a wave crest from refraction or diffraction would also have a similar effect, (b) Close up of the resultant shear in wave-orbital velocities, and the small-scale turbulent eddy generated at the expense of wave energy,

exhibiting offset and/or discontinuous wave crests. Wave crests can be offset or terminated by various processes, including current-generated turbulence, and refraction and diffraction caused by a strong, narrow rip current. Offsetting wave crests juxtaposes regions with very different wave-orbital velocities (Figure 1(a)). This gradient in orbital velocities and the associated shear will then generate small-scale turbulence at the expense of wave energy (Figure 1(b)). In a subsequent section we derive an expression for the crossshore rate of wave dissipation based on this mechanism.

Previously recognized interactions between waves and currents can have the opposite effect, increasing wave heights when waves enter an opposing current. The bending of wave crests by the current causes wave energy to be concentrated where the crests are concave in the propagation direction, As waves enter an opposing current and become shorter, in the absence of energy dissipation they become higher. In a laboratory experiment, such interactions caused waves entering a rip current to become steeper and higher, and to break farther from shore than they would in the absence of a current (HAAS *et al.,* 1998; HALLER *et al.,* 1997). However, in the field the ratio of current speed to wave propagation speed is generally lower than in this laboratory-scale experiment (SMITH, 1995), making these wave-current interactions less effective. We have occasionally observed waves peaking up and breaking when entering a rip current in nature, but this does not



Figure 2. Schematic illustration of hypothesized interactions. (a) A weak offshore flow begins to decrease wave heights locally, which allows the offshore slope to accelerate the current. (b) The removal of water from the surf zone locally creates alongshore surface slopes that drive alongshore currents feeding the rip current. (c) The surface elevations in the surf zone are lowered regionally, decreasing the offshore surface slope, making it more likely that the rip will decelerate. This allows wave heights to increase locally, causing further deceleration.

occur generally. When waves do break for this reason outside the surf zone, their energy will be partly dissipated before they enter the surf zone.

Regardless of the mechanism responsible for the dissipation of wave energy in the presence of a rip current, such an interaction could lead to an instability causing rip currents. Imagine starting with an alongshore-uniform set up-a hill of water in the surf zone supported by the momentum flux lost by waves as they break. In this case, the water transported into the surf zone by waves would be flowing back out as an alongshore-uniform undertow current in the steady state. However, for simplicity, imagine also starting with no currents. In this case, a perturbation that initiates a slight offshore flow, such as a transitory area of lower incident wave heights, would initiate a feedback: The offshore flow would decrease the wave heights locally, which would cause the flow to accelerate because the radiation-stress gradient would no longer balance the set up (Figure 2(a)). The increased current would in turn decrease the wave heights further, allowing the current to accelerate further. As the offshore flow lowers the water surface locally, alongshore pressure gradients would generate alongshore flows to feed the rip current (Figure  $2(b)$ ).

If the waves replenish the water in the region from which the alongshore currents are feeding the rip current as rapidly as it removes water from the surf zone, the currents could attain a steady state. If the rip current removes water from the surf zone faster than it is replenished in the region, then the offshore slope will decrease (Figure  $2(c)$ ). An instability opposite to the one that caused the rip to strengthen could cause it to weaken; If the radiation-stress gradient became greater than that needed to support the local set-up slope, the rip current would decelerate. This would allow the waves to become bigger. The resulting increase in the local radiation-stress gradients in the surf zone would further decelerate the rip current, possibly leading to its cessation.

In the context of these hypotheses, the fate of a rip current-steady state or limited duration-will depend on the configuration of the water surface in the region, which is not assumed to necessarily have a slope that will balance the radiation-stress gradient. This configuration will depend on the recent history of cross-shore and alongshore variations in rip, undertow and alongshore currents in the region. Important factors including the velocities and widths of rip currents are not prescribed. In the absence of conceptual or analytical predictions of rip-current behavior, we have constructed a model to facilitate investigation of the consequences of these processes interacting over an extended spatial domain.

The interactions described qualitatively above do not depend on the mechanism causing the wave dissipation suggested by the field observations, but only on the existence of such an interaction. However, to construct a numerical model to test whether such an interaction could produce the main features of rip currents on planar beaches, we must include a particular mechanism, and we treat the mechanism described above.

#### **MODELING APPROACH**

The goals of this model involve relatively long time scalesthose of multiple rip-current occurrences, on the order of hours, and ultimately those of bathymetric changes, on the order of days. The forces considered in the proposed model, such as radiation stress and pressure gradients averaged over a wave period, can change on time scales no faster than a wave period. Using a time step of one wave period is appropriate for treating current motions, and allows relatively rapid simulation of long time scales. The spatial discretization must be no coarser than the order of meters to resolve the smallest spatial scale addressed, that of rip-current widths.

Some other models of surf zone circulation employ much smaller time steps, simulating velocity and surface-elevation changes on time scales shorter than a wave period (SANCHO, 1995; SORENSEN *et al.,* 1998). Partial differential equations such as those employed by these models can not be solved using a grid size on the order of meters and a time step on the order of ten seconds (a wave period), respectively; numerical instabilities lead to artifacts such as diverging surface elevations. We employ an alternative method of treating interactions between wave heights, set up, and cross-shore

and alongshore pressure gradients that occur on time scales of a wave period and greater.

Cellular models, sometimes called coupled map lattices (KANEKO, 1993), evolved from the cellular automata developed in nonlinear-dynamics research. Cellular models have proven useful for determining the interactions that are important in producing the fundamental aspects of large-scale phenomena in spatially extended systems, including eolian dunes (WERNER, 1995), beach cusps (WERNER and FINK, 1993), arctic and alpine patterned ground (WERNER and HAL-LET, 1993), drainage-network development (CHASE, 1992; HOWARD, 1994; WILLGOOSE *et al.,* 1991), and braided streams (MURRAY and PAOLA, 1994; MURRAY and PAOLA, 1997). In such a model, including the one described here, the variables defined in each cell in a lattice interact according to rules that can be divided into three types: 1) direct application of equations; 2) parameterizations of processes occurring at smaller temporal and/or spatial scales; and 3) abstractions of physical principles, such as the conservation of mass and momentum.

Rather than attempting to construct a model that is as realistic as possible, in the sense of including as many of the processes in natural surf zones as possible, our approach is to include as few processes as possible, in an attempt to discover what is essential to produce the fundamental aspects of the phenomena. In this spirit, we will describe a basic model which includes the minimal set of processes necessary to produce narrow, isolated, non-bathymetrically driven rip currents in the context of our hypotheses. We then present an embellished version of the model, in which we add some processes that affect the main rip-current characteristics, such as their velocity and duration.

In this model, which has cell widths on the order of meters, some variables are defined from one iteration to the next in each cell. The most basic are water surface elevation, representing an average over a wave period, and bed elevation. Cross-shore current values are also passed on from iteration to iteration. During each iteration, which represents a wave period, several processes are treated sequentially, starting with the propagation of waves across the domain, and their dissipation. Then we compare the cross-shore gradient of radiation stress in the surf zone to the cross-shore surface slopes. In general these forces will not be balanced locally, producing an acceleration or deceleration of cross-shore currents during the iteration. (In the embellished version of the model, bed friction also decelerates cross-shore currents.) Next, water depths are adjusted according to the convergence and divergence of cross-shore currents during the iteration. Then, using the approximation that alongshore currents are in quasi steady state with the alongshore surface slopes (pressure gradients) on the time scale of a wave period, we calculate alongshore currents by balancing driving force and flow resistance. Finally we adjust the depths according to the convergence and divergence of alongshore currents. Depths and cross-shore currents calculated during the iteration are then used to determine wave heights during the next iteration. In general, the distributions of wave heights, depths, and cross-shore and alongshore currents changes from one

iteration to the next. In subsequent sections we describe the details of how we treat these processes.

## BASIC-MODEL ALGORITHM

## Wave Processes

Waves are parameterized by a period,  $T$ , that is held constant, a height,  $H$ , and whether or not the wave is breaking. Incident wave heights are assigned each iteration in the row of cells farthest offshore (at a depth of 8 m). During an iteration, the waves move through each cross-shore column of cells (propagating normal to the average trend of the shore) until they reach the shore, which is defined in each iteration as the location where the depth falls below a threshold, generally 0.1 m. The basic model treats only surf-zone processes. In this model, the waves do not change height until they break, after which their heights decrease for either of the two reasons discussed below.

## Breaking

Following a commonly used approximation, in the model breaking occurs when the height reaches 0.6 times the depth, D, and limits the height to that value:

$$
H = 0.6D, \tag{1}
$$

for breaking waves. STIVE and WIND (1982) show that using a proportionality constant of 0.6 in equation (1), and using linear theory for the radiation stress predicts set up elevations that are in rough agreement with the results of laboratory-scale experiments.

#### Wave-Current Interaction

The newly hypothesized interaction between currents and waves can also decrease wave heights. Here we derive the cross-shore rate of wave-height decrease, based on the mechanism described in the second section, using approximations to determine the wave energy lost to the generation of smallscale turbulence. We will assume the case of a wave crest offset horizontally by current refraction or a current-generated eddy with a vertical axis (Figure 1). Similar results could be derived for the cases of wave-crest terminations (from wave diffraction, for example) or offsets from eddies with horizontal axes. We are not attempting to analyze the hydrodynamics in detail, but to derive an estimate of the wave-dissipation rate.

The rate of production of turbulent energy per volume, PROD, is given generally by:

$$
PROD = \tau \partial u / \partial y, \tag{2}
$$

where  $\tau$  is the shear stress and  $\partial u/\partial y$  is a velocity gradient. In the case we are treating, the pertinent gradient is  $\partial u_o/\partial y$ , the gradient in orbital velocities where a wave crest is offset (Figure 1), where *y* is the alongshore direction. (We are considering spatially abrupt wave-crest offsets, in which the offset occurs over along-crest distances that are small compared to the wavelength. In this case, shear in orbital velocity will exist, and the most appropriate direction in which to consider



Figure 3. Definition sketches for the derivation of the newly hypothesized wave-current interaction. (a) Plan view of an offset wave crest (Figure 1), showing orbital velocities directed onshore behind the wave crest and offshore behind it. (b) Orbital velocity cross-shore profiles of the two segments of the wave crest offset from each other.

the gradient in orbital velocity is that parallel to the largescale trend of the wave.)

Using a mixing-length approach,  $\tau$  can be approximated by

$$
\tau = \rho L^2 |\partial u_o / \partial y| \partial u_o / \partial y,\tag{3}
$$

where  $L$  is the characteristic eddy size and  $\rho$  is density. We estimate *L* to be equal to the depth, *D*. We approximate  $\partial u_{\alpha}$ /  $\partial y$  as  $\Delta u_o/\Delta y$ , where  $\Delta u_o$  is the maximum difference in orbital velocities over the spatial scale of wave-crest offset,  $\Delta y$ , which is taken to be a constant on the order of 10 meters, commensurate with typical rip-current widths. (Figure  $3(a)$ ).

With these assumptions, calculating the rate of turbulentenergy production averaged over a wave period requires an expression for  $\langle \Delta u \rangle$ , where the angle brackets denote averaging over a wave period. Instantaneously,

$$
\Delta u_o = (\partial u_o / \partial x) \Delta C r, \tag{4}
$$

where *x* is the cross-shore coordinate,  $\partial u / \partial x$  is the local slope of the wave orbital velocity profile, and  $\Delta Cr$  is the offset, in the direction of wave propagation, between wave-crest segments (Figure 3(b)).  $\Delta Cr$  will depend on the structure of the current-generated turbulence, refraction, and diffraction, which will depend on the magnitude of the current velocity. In this model we take  $\Delta Cr$  as proportional to the current velocity,  $u_{\alpha}$ , with a constant of proportionality,  $\alpha$ , on the order of 10 s. This choice is consistent with the simplest (though not the only) way that a wave crest can be offset—slowing of one segment of the crest by the current (refraction): A wave spends on the order of 10 s interacting with a rip current in the field (with wave velocity approximately 5 m/s and crossshore rip-current extents on the order of 100 m). Then, for example, if the rip current has a velocity of 1 m/s, the wave

crest will be offset on the order of 10 m during its interaction with the current. Combining equations  $(2)-(4)$  and these approximations, we have:

$$
\langle \text{PROD} \rangle = (\rho D^2 \alpha^3 u_c^3 / \Delta y^3) \langle (\partial u_o / \partial x)^3 \rangle. \tag{5}
$$

Using the sawtooth wave orbital velocity profile shown in Figure 3(b), and assuming the maximum orbital velocities of linear waves in the shallow water limit,

$$
u_{o,max} = \lambda H/2TD,\tag{6}
$$

where  $\lambda$  is wavelength, and  $T$  is the period, integrating over a wavelength (or period) yields:

$$
\langle (\partial u_o/\partial x)^3 \rangle = (H^3/T^3D^3)[1/(1-\epsilon)^2 + 1/\epsilon^2],\tag{7}
$$

where  $\epsilon$  is the proportion of the wavelength occupied by the steeper face of the wave. Assuming  $\epsilon \ll 1$ , we approximate:

$$
\langle (\partial u_o / \partial x)^3 \rangle = H^3 / T^3 D^3 \epsilon^2. \tag{8}
$$

Combining equations (5) and (8) gives the rate of turbulentenergy production per volume. Then, multiplying by the volume (per unit width and length) of a wave,  $D$ , leads to an expression for the change of wave energy, *E:*

$$
\partial E/\partial x = 1/c(\partial E/\partial t) = (\rho \alpha^3/\sqrt{g} \Delta y^3 \epsilon^2 T^3) (H^3 u_c^3/\sqrt{D}) \quad (9)
$$

where wave celerity, c, is approximated by  $\sqrt{\overline{gD}}$ , where g is the acceleration of gravity. Finally, approximating wave energy as proportional to  $H^2$  leads to an expression for the rate of change in wave height caused by this mechanism:

$$
\partial H/\partial x = 1/(2B\rho gH)(\partial E/\partial x) = C_{int} H^2 u_c^3/2T^3 \sqrt{D}, \quad (10)
$$

$$
C_{int} = \alpha^3 / B g^{1.5} \epsilon^2 \Delta y^3, \qquad (11)
$$

where *B* and  $C_{int}$  are constants. The value of *B* depends on wave shape (SVENDSEN, 1984b), and is 1/12 for sawtooth waves.

Because of the crude approximations involved in its derivation, equation (10) is almost certainly not correct in detail; Altering the assumptions alters the exact values of the exponents and C*int.* Experiments using different values of the exponents have shown that the qualitative behavior of the model does not depend on the exact values used. The strongly non-linear dependence on  $u_{\alpha}$ , however, does play a key role in the model, as we will discuss in a section on the results of the basic model. We treat  $C_{int}$ , which depends partly on poorly constrained constants of proportionality and spatial scales, as a free parameter, as we discuss in a section on the results of the embellished model. Using  $\epsilon = 0.1$  and the order of magnitude estimates for the other unknown quantities in equation (11), and assuming  $T = 10$  s,  $H = 1$  m, and  $u_c = 1$  m/s, equation (11) gives a cross-shore rate of wave-height loss on the order of  $10^{-2}$ . This rate is commensurate with that caused by breaking in a 100 m wide surf zone.

The rate of dissipation from the proposed mechanism depends sensitively on the magnitude of the shear in orbital velocities in an offset (or terminated) wave crest. (Combining equations (2) and (3) shows that the velocity gradient is cubed in the expression for turbulent energy production.) The large dissipation rate estimated for waves in surf-zone rip currents derives from treating waves that are shoaled (with large orbital velocities and a highly skewed shape,  $\epsilon \ll 1$ ), and from the small characteristic value of the along-crest length scale of the velocity gradient,  $\Delta y$ . Calculations for linear (sinusoidal) waves yield estimated dissipation rates approximately two orders of magnitude smaller. In a similar calculation for the dissipation this mechanism would cause in the case of wave terminations (as in a directionally spread wave field, for example),  $\Delta y$  would be commensurate with the length of wave crests, and  $\Delta u/\Delta y$  would be approximately  $u/\Delta y$ . The dissipation rate in this case, assuming shallow water waves and  $\Delta y = 10$  m (a generously small value), is an order of magnitude smaller than that estimated for waves interacting with rip currents. This result for deep-water waves and a larger value for  $\Delta y$  (e.g. the case of directionally spread waves propagating far from shore) would typically be several orders of magnitude smaller yet.

(In the deeper water well outside the surf zone, the choice for the length scale of eddies that mix the momentum across the velocity gradient in the mixing-length approximation of equation (3) is not obvious. In fact, in that case the existence of such eddies is questionable, because in an Eulerian frame of reference, as a wave passes, the shear in orbital velocities only exists for on the order of seconds-not long enough to accelerate large eddies. In the rip-current context, currentor breaking-wave-generated turbulent fluctuations would serve the purpose of transporting momentum across a shear in the flow, much as is envisioned in the derivation of the mixing length approximation.)

#### **Mass Transport**

Wave mass transport is approximated as the drift from sawtooth waves, with an additional contribution for a breaking wave from a roller, a mass of water rolling down the wave face and moving at the wave-propagation speed (SVENDSEN, 1984a):

$$
Q_{\mu\nu} = BH^2 \sqrt{(g/D)} + (\text{if breaking})0.9H^2/T,
$$
 (12)

where  $Q_{wo}$  is the wave water flux per unit alongshore distance. The sawtooth profile may be the most accurate simple description of surf-zone waves using only *H* as a parameter (and possibly T) (MADSEN, 1997; GUZA, R.T., *personal communication,* 1998). In addition, this treatment gives a water flux within the range of values indicated by measurements (BAlLARD, 1987; SVENDSEN, 1984a; SVENDSEN, 1987b), using measurements of undertow as a proxy for wave flux (which would be exactly true in a steady-state, alongshore uniform situation). The surf zone is defined each iteration as the domain onshore of the farthest offshore occurrence of breaking. The water-surface elevation in each cell in the surf zone in which wave height decreases is adjusted according to the difference between the wave flux into and out of the cell during a wave period (Figure 4(a)). This water mass is conserved on a time scale longer than an iteration; all the flux entering the surf zone eventually leaves it as cross-shore current flux. The domain outside the surf zone is treated as an infinite source/sink; water surface elevations there remain constant.



**cross shore cross-section**

Figure 4. Schematic illustration of some of the surf-zone processes occurring in the model during a wave period. (a) The cross-shore rate at which a wave height, H, decreases defines 1) convergence in wave transport, and 2) a surface slope that would balance the radiation-stress gradient. (b) In each cell, a flux equal to the amount of water above the local stress-balance surface divided by the wave period is added to the offshore current in that cell and all the cells between it and the break point. On the large scale, if the slope is greater than that which would balance the radiation stress, the offshore current will accelerate by the addition of a flux that would decrease the slope to that balancing the radiation-stress gradient.

#### **Cross-Shore Currents**

In the next step in an iteration, cross-shore current accelerations are determined by rules that are based on conservation of mass and momentum.

## **Surface Slope Versus Radiation-Stress Gradient**

In each surf-zone cell, the cross-shore surface slope, S, that would produce a bed shear stress that would balance the local radiation-stress (wave momentum flux) gradient is calculated:

$$
\rho gDS = -\partial/\partial x (3B\rho gH^2/2 + (if breaking)\rho 0.9H^2c/T). \quad (13)
$$

The first term on the right represents the radiation stress for sawtooth waves, and the second term on the right represents a contribution to the breaking-wave momentum flux from rollers (SVENDSEN, 1984b). The balance of stresses represented in equation (13) is not assumed to hold in general. Rather, the slopes that would balance the radiation-stress gradient, defined by equation (13), are compared to the actual surface slopes to determine the acceleration or deceleration of the current, in the following way: The stress-balance slopes define a cross-shore profile (Figure 4(a)). In each cell, if the actual water surface is above this profile, the offshore current exiting that cell and passing through the cells between it and the outer limit of the surf zone is increased. The increase in flux is equal to the volume of water in that cell above the stress-balance profile divided by a wave period. This rule im-

plements the principle that the cross-shore current will be accelerated until the stress from the surface-slope balances the radiation-stress gradient (Figure  $4(b)$ ). If the actual water surface is below the stress-balance profile, the offshore current will be decelerated in an analogous manner. Then the water surface elevations are adjusted according to the difference between the offshore current flux into and out of each cell.

Although accelerations caused by the surface slope between neighboring cells cannot be directly included on these time and spatial scales because of the resulting instability, the increase in the current in a cell during an iteration (the local acceleration) can be affected by the water level in the neighboring cells with this algorithm. For example, if the surface in a cell is above the stress-balance profile, but the water level in the neighboring cell in the shoreward direction is below the stress-balance profile, some or all of the water from any cells shoreward of that will in effect stay in the shoreward neighbor, thus reducing the acceleration in the cell in question.

Equation (13) is strictly applicable in the case of zero crossshore flow, which is not generally the case in the model. A bed-friction term, which would typically be an order of magnitude smaller than the radiation-stress-gradient terms, is neglected in equation (13). (In the embellished version of the model, we include the slight current deceleration caused by bed friction when calculating changes in current velocities, as we discuss in a subsequent section.) In addition, although equation (13) is used as part of the algorithm that determines temporal acceleration, we omitted a convective acceleration term in equation (13), which would also typically be small compared to the radiation-stress-gradient terms. We make these omissions in the interest of simplicity, and of including as few processes as possible.

## **Current Dispersion**

Dispersion of rip-current momentum in the alongshore direction occurs in the model. Organized cross-shore motions in the surf zone are believed to affect the distribution of velocity of alongshore currents generated by breaking waves approaching shore at an oblique angle (LONGUET-HIGGINS, 1970; SVENDSEN, 1994). Similarly, turbulent eddies generated by a current redistribute current momentum in a direction lateral to the current (PARKER, 1978). In LONGUET-HIG-GINS' (1970) treatment of alongshore currents, the crossshore momentum flux per unit cross-shore distance,  $M_x$ , is:

$$
M_x = -\rho CD \partial V/\partial x \qquad (14)
$$

where *V* is alongshore current velocity and *C* is a momentumexchange coefficient. Alongshore mixing of rip-current momentum by eddies assumed to be generated by the rip current is treated in the model in an analogous manner:

$$
M_{\nu} = -\rho C_{M_0 E x} D \partial u_c / \partial y,\tag{15}
$$

where  $M_{y}$  is the alongshore flux of cross-shore momentum, and  $C_{MoEx}$  is a momentum-exchange coefficient, which represents in part the characteristic size of large-scale eddies, and is poorly constrained. If *D* is replaced by the depth averaged in the alongshore direction, then the alongshore flux of crossshore momentum (or velocity) is proportional to the alongshore gradient in cross-shore momentum (or velocity). Flux of a conserved variable proportional to its gradient leads to the diffusion equation. The alongshore dispersion of crossshore velocity is treated as diffusion, in a manner analogous to that described in the next section.

## **Alongshore Currents**

The fluxes of water onshore in the waves and offshore in cross-shore currents in general vary in the alongshore direction, leading to alongshore differences in water-surface elevation. For example, in the vicinity of a rip current, surface levels will be lower because of the large offshore flux. In the model, alongshore flow driven by alongshore surface slopes is calculated after the adjustments of surface elevations caused by offshore flow during the wave period (10 s), Experiments performed with an alternation between offshore and crossshore flow every 1 s rather than every 10 s show that the model results are not qualitatively affected by varying this time step by an order of magnitude.

We determine the alongshore flow by balancing the pressure gradients resulting from the surface slopes against the bed resistance. This treatment stems from the approximation that, on time scales on the order of 10 s and spatial scales on the order of 100 m, alongshore flow will be in a quasi-steady state with respect to the surface slopes driving it; that flow responds to changes in the surface slopes on a time scale shorter than 10 seconds. (We neglect forces resulting from spatial gradients in the flow, which will typically be small in the alongshore direction. These forces would be largest in the vicinity of rip currents, and would likely tend to strengthen rip currents. However, we are attempting to determine the minimal set of processes sufficient to produce the main characteristics of rip currents on planar beaches, and as discussed in subsequent sections, strong, narrow rip currents result without this effect.) The treatment is also based on the approximation that, above the wave boundary layer that exists near the bed, the eddy viscosity,  $v_t$ , which parameterizes the flow-smoothing effects of turbulence, results predominantly from breaking-wave-generated turbulence, and that currentgenerated turbulence can be neglected. Eddy viscosity can be related to the density of turbulent kinetic energy, *k:*

$$
v_t = l \sqrt{k}, \tag{16}
$$

where *l* is a characteristic eddy size, which is assumed to be a small proportion of the depth. We assume that  $\sqrt{k}$  varies linearly with depth, approximating laboratory observations (SVENDSEN, 1987a), and has a magnitude determined by wave height, period and depth, using the analogy between a breaking wave and a hydraulic jump (FREDSOE, 1992, p. 113). This yields:

$$
v_t = \beta Hz/(TD)^{1/3},\tag{17}
$$

where  $\beta = 0.25$  (m/s<sup>2)13</sup>, and *z* is the elevation above the bed. This treatment is consistent with observations made by OKAYASU and *et al.* (1988), which show  $\nu$ , varying linearly with z under breaking waves.

We do not treat the bottom boundary layer in detail, but assume that the turbulence parameterized by eddy viscosity arises from wave breaking, and wave oscillatory motion near the bed. Assuming a balance between shear stresses caused by the surface slope, S, and vertical velocity gradients at each level (as is commonly assumed for steady-state, open-channel flow).

$$
\tau = \rho g(D - z)S = \rho v_t(\partial u_c/\partial z), \qquad (18a,b)
$$

the vertical velocity profile results from integration of equation (18b). Using equations (1) and (17), integrating the velocity profile leads to an expression for steady-state water flux, Q, with the following characteristics:

$$
Q = (gD^2/\kappa U_{fb})
$$
  
× {ln(D/δ) – 3/2 + [U\_{fb}/(U\_{fb} + U\_{fw})]ln(δ/k\_n/30)}S, (19)

In equation (19),  $\kappa$  is von Karman's constant, 0.4,  $k_n$  is the roughness height (a grain diameter, 0.001 m, assuming a lack of bedforms),  $\delta$  is the thickness of the bottom boundary layer, for which we use

$$
\delta = 0.09k_n[\sqrt{(g/D)}TH/4\pi k_n]^{0.82},\tag{20}
$$

(FREDSOE, 1992, p. 25), and  $U_{fb}$  and  $U_{fw}$  are the friction velocities associated with the turbulence from wave breaking and oscillatory motion near the bed, respectively. We use

$$
U_{fb} = 0.375D^{2/3}T^{1/3},\tag{21}
$$

$$
U_{fw} = 0.05D^{1/2},\t\t(22)
$$

where equation  $(22)$  is simplified from FREDSOE and DEI-GAARD (1992, p. 61). The third term in the brackets in equation (19) arises from the velocity at the top of the bottom boundary layer. We use equation (19) because it expresses the basic dependencies of quasi-steady-state flux responding to a surface slope, when turbulence is generated chiefly by wave breaking. We have performed experiments using different forms for equation (19), including that resulting from the inclusion of turbulence generated by the interaction of wave oscillatory motion and cross-shore currents, as treated by FREDSOE and DEIGAARD (1992, p. 59). These experiments show that the model results do not depend sensitively on the assumptions used in deriving equation (19).

Approximating the local depth and boundary layer thickness as those averaged in the alongshore direction leads to flux proportional to slope in equation (19). Flux proportional to slope leads to the diffusion equation when combined with conservation of mass. (In the open channel flow case,  $v_t$  is dependent on S, because increasing the slope increases the flow, which increases the eddy viscosity. In that case, flux is proportional to  $S^{1/2}$ . It is the assumption that eddy viscosity is independent of the surface slope driving the flow that leads to the diffusion equation for flow.) The linear nature of diffusion allows a treatment of the diffusion of each part of a diffusing system separately-in this case the water in each cell-and superposition of the results. Treating each cell as analogous to a dirac function, diffusion will give a gaussian distribution of water from each cell, the width of which is determined by the diffusion coefficient and the length of time passed, and the height of which is determined by the initial



Figure 5. Schematic illustration of the alongshore-current algorithm. (a) Diffusion acting on the water in one cell will give a gaussian distribution after a finite time step (treating the water in each cell as a dirac function). For simplicity, we show a case with water initially in only two cells. (b) Superposing the results from each cell gives the water-surface configuration. Formally, this configuration consists of a running, weighted average of the original elevations, using a gaussian weighting function.

water-surface elevation in the cell (Figure 5 (a)). Summing the results for each cell gives in effect a running, weighted average of surface elevations, with a gaussian weighting function that has a width related to the time step (Figure 5 (b)).

#### **BASIC-MODEL RESULTS**

The first robust result is that isolated, narrow rip currents do develop from the interactions simulated in the basic model. The model output in Figure 6 shows several such currents. These currents result from the nonlinear nature of the wavecurrent interaction (equation (10)). As an area of offshoredirected flow accelerates because of the feedback described in the second section, wave dissipation rapidly increases with increasing current velocity. In this case, more of the wave dissipation in the surf zone occurs in relatively deep water than would occur from breaking alone. A lower surface slope is required to balance a given amount of dissipation in deeper water (equation (13)). Thus the current-related dissipation lowers the stress-balance slopes (equation (13)) in individual cells, and consequently the cross-shore profile defined by those slopes. This lowered stress-balance profile is then likely



Figure 6. Results of a basic-model simulation using planar bathymetry with a slope of 0.02, with random elevation perturbations with an amplitude on the order of centimeters, and an average incident-wave height of 1 m. Incident wave heights include random, spatially and temporally uncorrelated perturbations, with an amplitude of 0.05 m. Time 0 represents a time after the transient approach to the statistically steady state shown. The alongshore boundary conditions are periodic. Current velocities are measured at the break point, which is defined each wave as the cross-shore location farthest from shore at which breaking occurs. Velocities that are plotted as a shade of blue can be interpreted as undertow, and velocities shown as warm colors can be interpreted as rip currents. In this simulation, cell width was 4 m, wave period was 10 s,  $C_{int}$  (equation (10)) = 35, and  $C_{MoEx}$  (equation (15)) = 0.25.

to be lower than the actual water-surface elevation in most or all surf-zone cells in that cross-shore column, causing acceleration of the current. Even if the actual water-surface elevations are low locally, the low stress-balance profile makes deceleration of the current less likely.

The narrowness of the rip currents in the model results largely from a selection process. A strong offshore flow removes water from the surf zone, lowering the actual watersurface elevations, at a rate dependent on its average velocity and its width. A wide and strong flow removes water from the surf zone faster than the alongshore flows that result from the lowered water-surface elevations can replenish the water in that region. In this case, the actual water-surface elevations decrease rapidly, quickly bringing them below the stress-balance profiles locally. This causes a wide, strong offshore flow to decelerate rapidly enough that the waves quickly become larger, increasing the radiation stress gradients locally and thus decelerating the flow further. This series of interactions causes wide, strong flows to disappear in a small number of wave periods A narrow (on the order of 10's of m wide) flow can reach relatively high velocities (approximately 0.4 m/s in Figure 6, but higher in the embellished model results) without removing water from its region of the surf zone faster than the resulting alongshore flows can replenish it, becoming a sustained rip current. Flows that start out somewhat more than a few tens of meters wide can become narrower and evolve into sustained rip currents.

In the basic model, which has temporally (and spatially) uniform incident waves, narrow rip currents will reach a quasi-steady state. The local actual water-surface elevation will decrease until it is nearly coincident with the stress-balance profile. In a steady state, the actual water-surface elevation must be just greater than the stress-balance profile, to accelerate the current during each iteration just enough to balance the local decrease in rip-current velocity that results from the alongshore dispersion of offshore-directed momentum. Velocities in developed rip currents remain approximately constant, with a configuration such as that shown in Figure 6.

The alongshore spacing of rip currents arises through essentially the same mechanism that leads to narrow rip currents. If two rip currents form too close to each other, their combined flow rapidly lowers the actual water-surface in the region, which causes the cessation of both rip currents. With temporally uniform incident wave heights, a consistent alongshore spacing evolves (Figure 6). In this simulation, as in all that we discuss in this paper, the bed is approximately planar, with small amplitude, uncorrelated, random elevation perturbations in each cell; the rip-current spacing shown in Figure 6 arises not from the bathymetry, but from the interaction between cross-shore and alongshore currents.

## **MODEL EMBELLISHMENTS**

In this section we describe some additional processes that affect both the dynamic behavior of rip currents in the model, and the strength of the feedback leading to those currents.

#### **Outside the Surf Zone**

In nature, rip currents extend outside of the surf zone. The embellished model we add the hypothesized wave-current interaction in this area, which reduces the heights of waves reaching the surf zone. This addition requires a treatment of waves and currents outside the surf zone.

#### **Cross-shore currents outside the surf zone**

We need to determine how the rip-current velocity varies with cross-shore distance outside of the surf zone. Pressure gradients from water-surface slopes are not likely to be an important force for the currents here, as they are in the surf zone. Where a rip current exists, the hypothesized wave-current interaction would tend to reduce wave heights outside the surf zone, relative to the wave-height changes shoaling alone would cause. This component of wave-height change would cause a relative elevation of the water surface, as occurs in the surf zone. However, we assume that in the relatively deep water outside the surf zone, alongshore flow would prevent this isolated elevation from becoming significant. (Wave shoaling generally causes a slight lowering of the water surface outside the surf zone-the set down-but shoaling and the associated radiation-stress gradients are continuous in the alongshore direction.) Given this assumption, we use the self-consistent approach of balancing the component of the radiation-stress gradient caused by the wave-current interaction against the spatial deceleration of the current (Figure 7):

$$
\partial/\partial x (3B \rho g H^2/2) = -\partial/\partial x (\rho D u_c^2), \text{ or } (23)
$$

 $\partial u_c/\partial x = -(3Bg/2)(H/u_cD)\partial H/\partial x$ 

$$
-(u_c/2D)\partial D/\partial x \tag{24}
$$

where  $\partial H/\partial x$  refers only to the component of wave-height



Figure 7. Schematic illustration of the spatial deceleration of currents outside the break point in the model. Shades of gray show different crossshore current velocities. The upper view zooms in on a model cell in which the hypothesized interaction between waves and currents causes the wave to decrease in height relative to what shoaling alone would have caused (which is implicitly assumed to be balanced by a small set-down slope). The resultant relative decrease of wave momentum flux is balanced by a change in current momentum flux.

change from the wave-current interaction (excluding shoaling). Using equation (10) for this change in wave height yields:

$$
\partial u_c / \partial x = -(3C_{int} Bg/4)(H^3 u_c^2 / T^3 D^{1.5}) - (u_c / 2D) \partial D / \partial x.
$$
 (25)

Starting with the first cell outside the surf zone in a crossshore column and moving offshore, the current flux is decreased according to equation (25), until the velocity falls be-Iowa small threshold.

## **Shoaling**

In the embellished model, shoaling tends to cause nonbreaking-wave heights to increase. As waves move into shallow water, conservation of energy and linear wave theory in the shallow water limit give:

$$
H_2^2 = \sqrt{(D_1/D_2)H_1^2},\tag{26}
$$

where  $H_1$  and  $H_2$  are the wave heights in the cells with depths  $D_1$  and  $D_2$ , respectively.

## **Non-Uniform Waves**

In embellished-model runs, incident wave heights consisted of an average height with added coherent perturbations, roughly simulating wave groups. These perturbations were changed at constant intervals, typically every five waves. They had gaussian-distributed heights that were uniform in the alongshore direction within each perturbation, and alongshore lengths that were constant during a simulation. One perturbation was placed at a random alongshore location, which changed every time the perturbations were changed, and the rest were lined up end to end, with small transition regions between them. We have tried other simple schemes for time-varying waves, but the results discussed in the next section do not depend sensitively on the particular algorithm.

## **Cross-Shore-Current Bed Friction**

Bottom friction, with a quadratic dependence on current velocity, decelerates cross-shore currents in the embellished version of the model. Bottom shear stress,  $\tau_h$ , is commonly related to velocity by using an empirical friction coefficient,  $C_f$ :

$$
\tau_b = \rho C_f u^2 / 2. \tag{27}
$$

If bottom friction is considered in isolation from other forces, the resulting deceleration,  $[\partial u / \partial t]_r$ , is then given by:

$$
\rho D[\partial u/\partial t]_f = \rho C_f u^2 / 2. \tag{28}
$$

In the model, each iteration the cross-shore velocities are adjusted by:

$$
\Delta u_c = TC_f u_c^2 / 2D. \tag{29}
$$

#### **EMBELLISHED-MODEL RESULTS**

Inclusion of the wave-current interaction outside of the surf zone alters the quantitative behavior of the model. In this case, if the velocity reaches a critical range (around 0.4 m/s), the waves reaching the surf zone have lost a significant portion (several percent) of their height. This significantly reduces the radiation-stress gradients locally in the surf zone, causing lower stress-balance slopes than in the case without the interaction outside the surf zone. This leads to a greater difference between the actual water-surface elevations and the profile defined by the stress-balance slopes, leading to stronger rip currents (Figure 8).

Figure 8(a) shows the self-organization of the rip current spacing, as described in the section on basic-model results, starting from an initial condition of a horizontal water surface and no currents, in the case of an incident wave height of 1.5 m. Figure 8(b) shows the spacing that evolves with an incident wave height of 1 m. (Time 0 in Figure 8(b) represents a time after the transient period of organization.) With smaller waves, the rip currents have a considerably lower maximum velocity (note the different velocity scales), and approximately steady rip currents can coexist in closer proximity.

In the model, non-uniform incident wave heights are required to produce long-term behavior with rip currents that have finite durations. With temporally varying wave heights,



Figure 8. Results of embellished-model simulations using planar bathymetry with a slope of 0.02. In (a) the incident-wave heights had an average of 1.5 m, plus random, spatially and temporally uncorrelated perturbations with an amplitude of 0.2 m. In (b) the incident-wave heights had an average of 1.0 m, plus random, spatially and temporally uncorrelated perturbations with an amplitude of 0.2 m, and time 0 represents a time after the transient approach to the statistically steady state shown. In (c) the incident-wave heights had an average of 1.0 m, plus perturbations that had a gaussiandistributed amplitude with a standard deviation of 0.085 m and crest lengths of 500 m. In (c) wave perturbations were changed every five waves, roughly simulating wave groups, and time 0 represents a time after the transient approach to the statistically steady state shown. The alongshore boundary conditions are periodic. Current velocities are measured at the break point, which is defined each wave as the cross-shore location farthest from shore at which breaking occurs. Velocities that are plotted as a shade of blue can be interpreted as undertow, and velocities shown as warm colors can be interpreted as rip currents. In these simulations, cell width was 4 m, wave period was 10 s,  $C_{\text{int}}$  (equation (10)) = 35, and  $C_{\text{MoEx}}$  (equation (15)) = 0.65.

rip current evolution is not directly controlled by changes in wave heights; the pattern of initiations and cessations of rip currents does not directly reflect the pattern of incident waves. For example, in the run shown in Figure 8(c), incident wave heights changed every five waves (50 s), yet rip currents generally last for minutes to tens of minutes. Rather, the perturbations caused by varying wave heights interact with rip currents. When a rip current has operated long enough to reduce the actual water-surface elevations locally, it becomes susceptible to an increase in local incident wave heights, which will increase the radiation-stress gradients, raising the stress-balance profile. If the stress-balance profile rises above the actual water-surface elevation, at least at most crossshore locations in the surf zone, the negative feedback that causes rip currents to cease can be initiated.

While a clearly preferred alongshore spacing between rip currents is not obvious in model results when incident wave heights vary temporally, a rough minimum spacing does arise (Figure 8(c)), through the interaction described in the basicmodel-results section.

Model experiments have shown that the inclusion or exclusion of wave shoaling or friction on cross-shore currents does not change the results significantly, merely altering the rip current velocities slightly.

The embellished version of the model can match semiquantitative observations. Based on Doppler sonar observations of velocities just outside the surf zone, SMITH and LAR-GIER (1995) reported that with incident wave heights averaging 1 m, rip currents next to the Scripps pier were typically around 15 m wide, had peak velocities of approximately 0.5 to near  $0.7$  m/s, and lasted approximately 10 min. Figure  $8(c)$ shows model current velocities measured at the break point in a run conducted with a beach slope and average wave height consistent with the conditions reported by SMITH and LARGIER (1995). The rip currents in Figure 8(c) exhibit typical characteristics that roughly match the observations.

However, we achieved this match between model and natural data by adjusting two poorly constrained parameters, as well as the variability of the incident wave heights. Changing the coefficient,  $C_{\text{int}}$  in the wave-current interaction (equation) (10)), affects the quantitative output of the model, as is shown by Figure 9. The coefficient determining the strength of the alongshore diffusion of offshore-directed momentum,  $C_{M_0R_x}$  in equation (15), also affects the quantitative behavior of the model, as is shown by Figure 10. Changing the amplitude of the variations in incident wave heights affects mainly rip current durations; increasing the amplitude leads to shorterlived rip currents, and vice versa.

## DISCUSSION AND FUTURE WORK

#### Implications of results

The fact that the model can produce quantitatively accurate results, as is shown by Figure 8(c), is consistent with the model being valid, in the sense of capturing the key processes and interactions in the context of rip currents in the absence of alongshore bathymetric variations. The results shown in Figure 8(c) indicate that the model offers a plausible explanation of a wide range of rip-current phenomena.

The qualitative, robust model results provide perhaps a more reliable indication of model validity. More aspects of natural rip currents arise from the model interactions than were intentionally put in. In particular, while the feedback leading to strong offshore currents was hypothesized before construction of the model, and may be seen as being "built in" to the interactions, the role of flow width in allowing or



equation (10). The simulation shown in (b) had the same parameters as those in the simulation shown in Figure S(c). In each run, the sequence of random numbers for the variations in incident waves is the same.

prohibiting the development of a sustained, jet-like feature strongly resembling rip currents on alongshore-uniform beaches was unforeseen. A previous hypothesis for why rip currents tend to be narrow (in the absence of alongshore bathymetric variations) involves a flow narrowing as it moves into deeper water, in the absence of a cross-shore acceleration or an increase in the cross-shore mass flux (ARTHUR, 1963). The model presented here predicts that rip currents are narrow all the way across the surf zone as well as outside of itor alternatively offers an explanation for that observation. The relatively wide spacing between sustained but dynamic



in equation (15). The simulation shown in (b) had the same parameters as those in the simulation shown in Figure S(c). In each run, the sequence of random numbers for the variations in incident waves is the same.

rip currents also falls out of the model, in the sense of arising from the interactions rather than being prescribed explicitly.

The results show that strong, narrow, widely spaced rip currents can result robustly from some relatively simple interactions between a small number of processes. Model experiments have shown that this qualitative result does not depend sensitively on model parameters, or the details of the treatments of the processes in the model. The basic version of the model, which includes only a small number of surf-zone processes, provides an example of such an experiment. The fact that this version of the model produces rip currents sug-

gests the robustness of the qualitative results. The instability that leads to these qualitative results does not depend on the proposed mechanism for wave-energy dissipation being correct; any mechanism that causes the disordered waves in a rip current to dissipate will cause the feedback described in the second section. In the model, rip currents arise from 1) a wave-current interaction that dissipates wave energy; 2) onshore mass transport by waves; 3) offshore flow responding to imbalances between radiation-stress gradients and set-up slope in the surf zone; 4) alongshore flow responding to alongshore surface slopes; and 5) alongshore dispersion of crossshore momentum. Time-varying incident wave heights are also necessary to produce rip currents with finite durations. Inclusion of other processes in the model, including wave shoaling and bed friction acting on cross-shore currents, do not affect the qualitative, robust results.

Rip currents are not universally present in natural surf zones. In this model, as is generally true in rip-current models, strong rip currents do not develop if the waves are small. And in this model, either 1) increasing the amplitude of temporal variations in incident wave heights or 2) increasing the bed slope inhibits rip-current development. The latter occurs because, to the first approximation, the height of the hill of water piled up against the beach in the surf zone is not dependent on the slope of the bed. Integrating across the surf zone any equation like  $(13)$ —in which local set up slope is a function of local wave height and depth—across the surf zone, when combined with the approximation that wave height is proportional to depth in a surf zone, leads to a maximum set up elevation that depends only on the height of the waves at the break point. Thus, for a given wave height, a surf zone with a steeper bed slope, which will be narrower, will have a smaller reservoir of the elevated water that drives rip currents in the model. For this reason, if other factors are held constant in the model, increasing the bed slope will decrease the prevalence of strong rip currents in the model. In addition, either factor can prevent rip-current development altogether if increased enough.

## **Model Testing and Future Work**

The need remains to compare model results to field data in a way designed to test whether the interactions in the model really are those that are important in nature. We are in the process of collecting field data to test model predictions that result robustly from the basic interactions in the model. Positive results of such tests could suggest the need for a program to more directly test the wave-current interaction, which is potentially of interest in applications outside the ripcurrent context.

In a physical model (HALLER *et al.,* 1997), and possibly in the field (SMITH, J.A., *personal communication,* 1998), rip currents associated with alongshore bathymetric variations have been observed to be non-steady-state. We are investigating whether this model makes predictions for rip-current behavior in the presence of alongshore bathymetric variations that are significantly different from the zones of offshore flow caused by rip channels or gaps in alongshore bars in some other surf-zone-circulation models (SANCHO, 1995; HAAS *et al.,* 1997; SORENSEN *et al., 1998).*

#### **ACKNOWLEDGEMENTS**

We thank L. Clarke, R. T. Guza, M. Kessler, M. Okihiro, L. Plug, anonymous reviewers, and especially B.T. Werner for very helpful conversations and suggestions. Thank you Michael LeBars and Cyril Guillon for help with the model and the manuscript. Supported by grants to B. T. Werner from the Office of Naval Research, Coastal Dynamics and Scholar Programs [N00014-96-1-0078 and N00014-97-1- 0154], and the Andrew W. Mellon Foundation.

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