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Eigenfunction Analysis of Decadal Fluctuations in Sandbank Morphology at Gt Yarmouth

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ABSTRACT

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Empirical orthogonal function (EOF) analysis (or Principal component analysis) has become an established technique for investigating interannual beach level fluctuations. In this paper, EOF techniques are applied to a series of historical bathymetric surveys of a nearshore sandbank system in order to investigate their long term morphological behaviour. The sandbanks cover an area approximately 30km alongshore and 10km cross-shore and lie ~2 kilometres offshore from Gt. Yarmouth which is situated on the East Coast of the UK. Geographical Information Systems (GIS) techniques were used to create digital models of the seabed from historic survey data. The models were analysed using EOF and GIS methods to identify any trends or cyclic behaviour in the morphology of the sandbanks. Evidence of changes in gross sandbank configuration, long term trends and potentially recurrent behaviour have been identified, and their significance for current coastal management practice are discussed.

ADDITIONAL INDEX WORDS: *Sandbanks, empirical orthogonal functions, GIS, coastal morphology, strategic management.*

INTRODUCTION

The recognition of coastal areas as a resource and the concept of resource management have gained widespread exposure in recent years. The approach to coastal planning and management varies from country to country and has led to a large number of types of coastal management plans, (see *e.g.*, Kay and Alder, 1999). This has, in many respects, emphasised the dichotomy faced by those charged with responsibility for coastal management between the scientific and engineering questions on one hand and the administrative and policy issues on the other. Effective planning implicitly relies on an accurate understanding and ability to predict both short- and long-term natural coastal processes. However, while significant advances have been made in short-term predictions (*e.g.*, beach response over the course of an individual storm), the same is not true of predictions of morphological changes over periods of years or decades, which is a primary requirement of strategic plans. Nevertheless, administrative and political initiatives are creating pressure to adopt a strategic approach with a specified planning horizon. In situations where the requisite understanding of the key coastal processes is not complete this may lead to management decisions being taken now which may, in the light of further physical understanding, be seen to be unsustainable over the timescale of natural variations that may show no regard for administrative boundaries or timescales.

The planning of coastal defences of the UK has undergone rapid changes recently. Following the issuance of guidelines by the Ministry of Agriculture, Fisheries and Food, (MAFF), in 1995, Shoreline Management Plans (SMPs) are in place for England and Wales. Emphasis within the SMP programme was on the derivation of sustainable coastal defence policies that reflected the many demands on the coastline in an integrated fashion. Indeed, it was recognised that knowledge of the processes and their interaction along the coast is crucial for constructing the SMP. This theme was reiterated by Burgess and Frew (1996) who argued that all management decisions ought to be linked primarily to the coastal processes and that management strategies need to be informed by the regional coastal dynamics.

In this paper we have analysed the long-term changes in a system of sandbanks which lie a few kilometres off the East Coast of the UK. The sandbanks have environmental, commercial, touristic and physical importance to the neighbouring coastline. Coastal development within this area is covered by the corresponding SMP that has a ‘planning horizon’ of approximately 70 years. Major changes in the sandbank configuration would undoubtedly have an impact on the nearby shoreline and the strategy for management of the coast. Figure 1 shows a map of the area together with the bathymetry for 1992.

The sandbanks off Great Yarmouth are believed to constitute a significant volume of material, estimated to be in excess of $10^9 m^3$, Clayton *et al.* (1983), and are a potential source of marine aggregates. These offshore banks provide protection to the coast from wave action, behaving in a manner similar to a large offshore reef. Robinson (1960, 1966) suggested that the banks exhibit recurrent configurations

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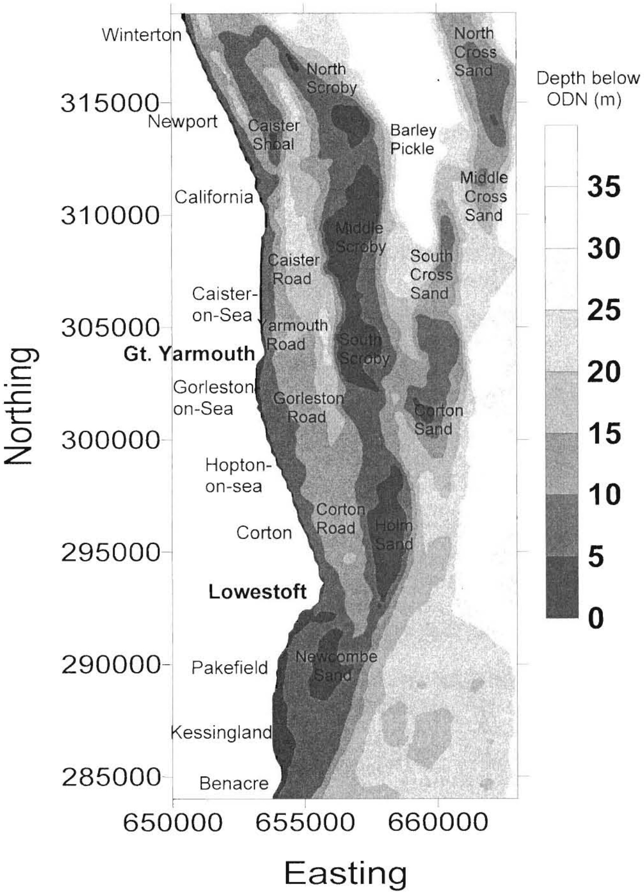

Figure 1. Map of study area showing the contoured digital terrain model for the 1992 survey data, (metres below Ordnance Datum Newlyn).

over a period of decades, on the basis of analysing historical bathymetry charts. In addition, he also noted correlations between changes in the sandbanks and beach movements at the nearby shore. The ability to predict changes in sandbank configuration and hence the corresponding impact on neighbouring beaches is clearly of importance to the strategic management of this shoreline. A conceptual model of the long-term sediment movement in the banks was proposed by Halcrow (1991), and discussed further by Reeve and McCue (1997). The conceptual model comprises a nearshore circulatory 'cell' within the Yarmouth banks, with material arriving from the north at Winterton Ness and from the south of Benacre Ness. It also included northward transport along the seaward edges of Corton, Cross and Scroby Sands, a southward transport along Barley Pickle, Caister and Yarmouth Roads, and a shoreward transport between North Scroby and Caister Shoal.

Figure 2 is an aerial photograph of Scroby Sands which shows that the sandbanks were sufficiently high to pierce the sea surface for at least part of the tidal cycle and also that they provide valuable habitat for seals which is largely secluded from human intrusion.

There are a number of quantitative methods that can be employed to investigate the movements of the sandbanks. These may be classified into the three broad categories: 'standard' statistics, volumetric analysis and advanced statistical methods. Each one has its own advantages and disadvantages. Used in isolation each technique provides an incomplete picture, but used together a more complex and, perhaps arguably, a more complete appreciation of the variability of coastal morphology can be obtained. For example, standard statistics (such as calculating the time mean or variance at each point) are straightforward to interpret but may not extract information about the spatial or temporal links within the data that can be revealed using volumetric or Empirical Orthogonal Function, (EOF), analysis.

The EOF method was introduced by meteorologists to investigate the patterns in time series of observations at fixed observing stations, Lorenz (1956). The method was subsequently used to describe seasonal changes in beach profiles by Winant *et al.* (1975). This method was extended to two spatial dimensions by Uda and Hashimoto (1982) and Hsu *et al.* (1994) who analysed a series of long-shore and cross-shore beach transects with the aim of developing a predictive scheme based on EOFs. Latterly, the temporal and longshore variations in a record of beach profiles covering a period of several decades have been studied using EOF methods, Wijnberg and Terwindt (1995).

In this study, EOF methods are applied not to beach profile data but to the time series of offshore sandbank surveys. This analysis significantly extends the preliminary studies of Reeve and Townend (1991) which were based on a coarser spatial resolution and did not include the 1992 data set. The temporal sampling interval is approximately one decade and so the temporal scale of any oscillations that one might expect to detect will be of the order of several decades. The analysis presented here does not address the question of detailed sediment transport patterns but rather, large scale changes in the morphology of the seabed.

BATHYMETRIC SURVEY DATA

Historical data were abstracted from sixteen bathymetric surveys for dates between 1846 and 1992, for an area that extends from Winterton in the north to Covehithe in the south. The dates of the charts used were 1846, 1864, 1875, 1886, 1896, 1905, 1916, 1922, 1934, 1946, 1954, 1962, 1974, 1982, 1987 and 1992. Maps and charts earlier than 1800 are not usually reliable, in that they are rarely based on an adequate ground-triangulation control, and often were simply sketch surveys, Carr (1962). The data on which the charts were drawn had a mean density of 30 points per km^2, corresponding to a mean resolution of approximately 180m. The surveys were taken relative to local datums and all measurements from charts had to be converted to a single reference datum prior to any analysis. It is difficult to obtain an absolute measure of accuracy for the chart measurements. A figure of between 25–50cms is suggested as being reasonable. Of potentially more concern is the scope for correlation in errors either in space or time. An assumption is made here that any such correlation is negligible. Chart surveys were

Figure 2. Aerial photograph of Scroby Sands in 1977.

traditionally performed with a plumb line and positioning by triangulation against visible coastal landmarks. Around the 1940's depth measurements began to be made with echo sounders and position-fixing by radar and latterly satellite global positioning systems (GPS). Both plumb line and echo sounders are susceptible to errors. The former through vessel movement and the plumb sinking into the sediment, the latter through multiple echoes and inclination of the seabed. When a change of technology is introduced one would expect to see any impacts as a 'one-off', systematic step change. Neither any such changes nor documented cases of the same area being surveyed using two different techniques simultaneously, (which would allow a direct comparison of the methods), were found for this dataset.

The bathymetry charts were digitised, reduced to a common projection, (OSGB38 Ordnance Survey National Grid), and common datum, Ordnance Datum Newlyn, (ODN). They were then imported into a Geographical Information System (GIS). The UK National Grid co-ordinates of the Southwest and Northeast corners of the study area are 650000E, 284000N and 663000E, 319000N respectively. The reduced data were then used to create a Triangular Irregular Network, (TIN), model for each survey date. The TIN models were then used to interpolate to a set of digital terrain models, (DTMs), covering the area of the sandbanks at a resolution of 100m. Figure 1 shows a contour map of the DTM for the 1992 survey. Note that depths below ODN are positive. Key features of the sand bank system are:

- Holm Sand, almost 3km due east of Lowestoft;
- Corton Sand, 5km due east of Gorleston-on-Sea;
- Middle and North Scroby Sands lying 2km offshore and running almost parallel to the mainland between Gt Yarmouth and Newport;
- South, Middle and North Cross Sands running almost parallel to the Scroby Sands but approximately another 3km further offshore;

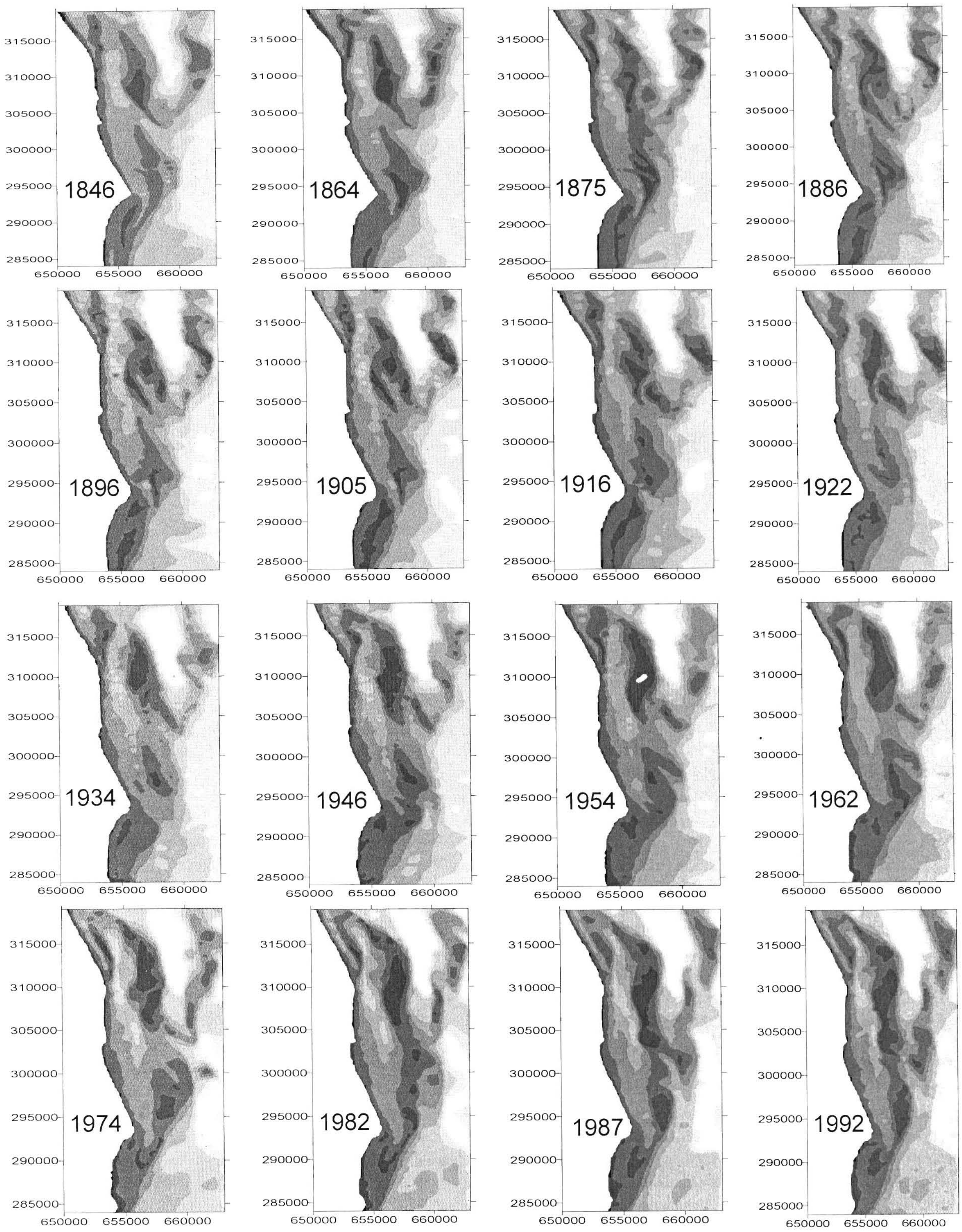

Figure 3. 'Thumbnail' plots of the offshore sandbanks between 1846–1992, (same contouring convention as Figure 1).

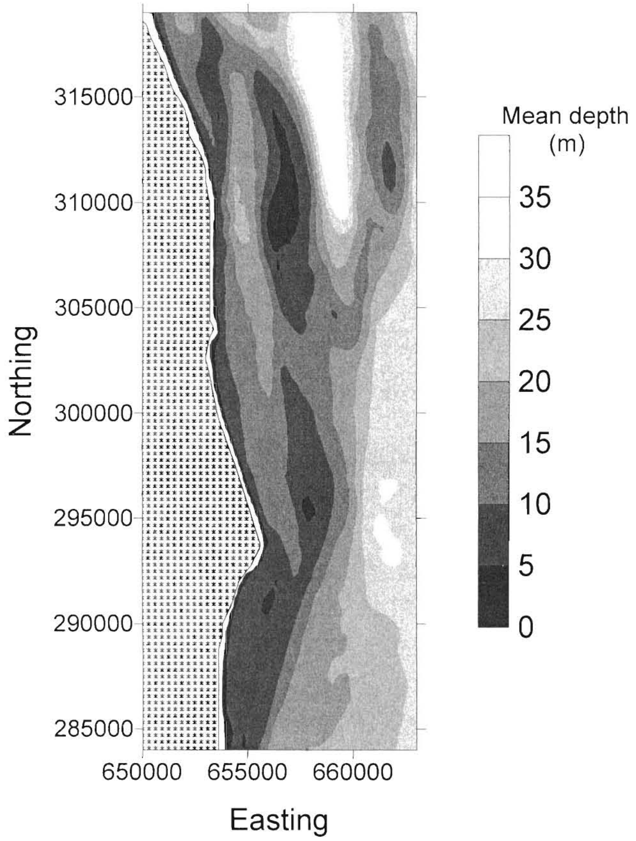

Figure 4. The mean bathymetry calculated from the sixteen DTMs, (metres below ODN).

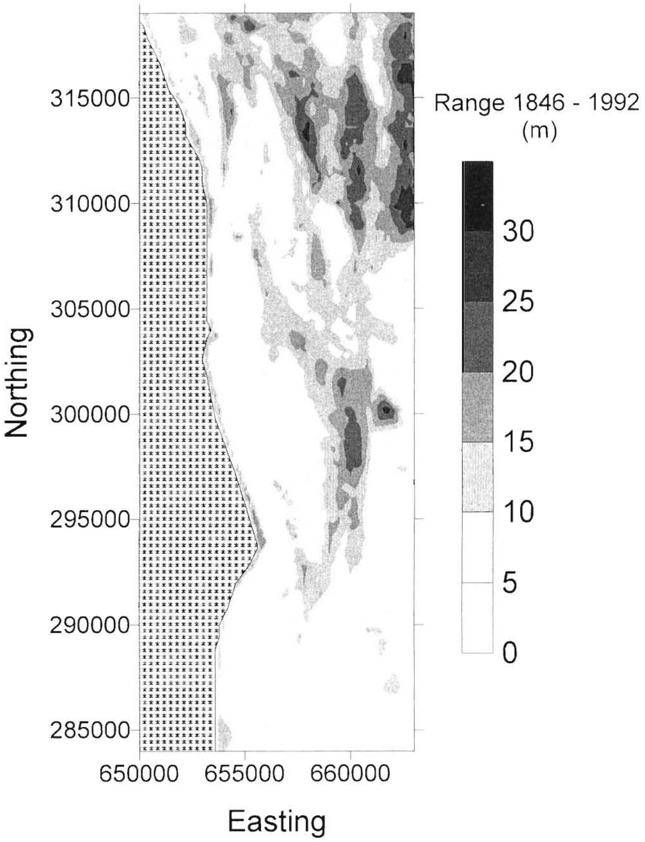

Figure 5. The range in seabed depths over the period 1846–1992, (metres).

- Caister Shoal which runs parallel to the mainland between California and Winterton, approximately 1km offshore;
- Holm Channel; Barley Pickle (a deep channel) and Caister Road (also a channel).

To evaluate the general spatial movement of the sandbanks over time, a visual comparison of the DTM's was conducted using the GIS. A full set of DTMs is shown in Figure 3.

There was a distinct period of erosion between 1846 and 1875, which resulted in an average increase of approximately 1m in the sea bed depth over the study area. Changes were particularly evident at Holm Sand, where there was a reduction in the height and width of the sandbank, and between Holm Sand and Middle Scroby Sands, where Holm Channel decreased in depth. In the addition, the configuration of the outer Cross sandbanks (in the north-east of the study area) changed significantly during this period.

Between 1875 and 1922, there was continued development of the outer sandbanks, notably at Middle Cross. In addition, there was overall shortening and broadening of the main body of Scroby Sands. Further, there was some erosion of the bar off Lowestoft and the formation of a channel across the southern flank of Holm Sand.

From 1922 to 1974, there was accretion across much of the study area resulting in a decrease of 1.4m in mean seabed depth over the study area. The pattern of accretion is clearly visible after 1934, where there was growth of many of the major sandbanks (*e.g.* Newcombe Sand, Holm Sand and the Scroby Sands) and disappearance of the channel off Lowestoft. Although the accretion is dominant, there was erosion of the outer South, Middle and North Cross Sands. The reduction in volume of these features coincides with growth of the inner sandbanks.

Between 1974 and 1982, Holm Channel (between the main inner sandbanks) was closed through increased deposition, creating a continuous bar aligned parallel to the coastline. This sandbank configuration, which is approximately 5km from the shoreline, remains largely evident today. In addition to the infilling of the channel, there was enlargement of the Middle Scroby Sands and Holm Sand; and enlargement and separation of Corton Sand from the main sandbank system. Over the same period the outer Middle and North Cross Sands expanded primarily on their north-eastern flanks.

THEORETICAL CONSIDERATIONS

Background

The seabed surface is a function of horizontal position and time. Let the seabed surface be denoted by

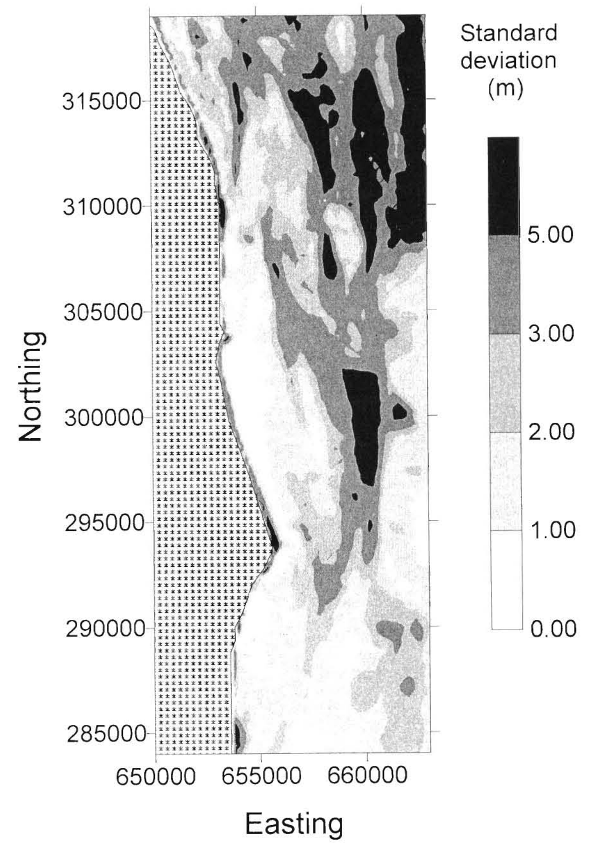

Figure 6. The standard deviation of seabed level changes over the period 1846–1992, (metres).

$$h \equiv h(x, y, t)$$

where x and y are distances along the axes in a Cartesian coordinate system, t is time and h is depth below datum. It is customary to align the coordinate axes so they correspond approximately to the cross-shore and alongshore directions of the beach data. In this case we choose x and y as Easting and Northing respectively, coinciding with the primarily north-south elongation of the sandbanks. The function h can be used to describe changes in seabed level along a fixed line in the (x, y)-plane or over an area. Some useful insights into the long-term morphological behaviour of the sandbanks can be gained by calculating straightforward statistics such as the variance of the seabed at each point or the change in mean depth over the study area with time.

Discretisation

The data are defined on a regular spatial grid of points at 100m intervals over the domain

$$x_{min} \leq x_i \leq x_{max}$$
$$y_{min} \leq y_j \leq y_{max}$$
$$t_{min} \leq t_k \leq t_{max} \qquad (2)$$

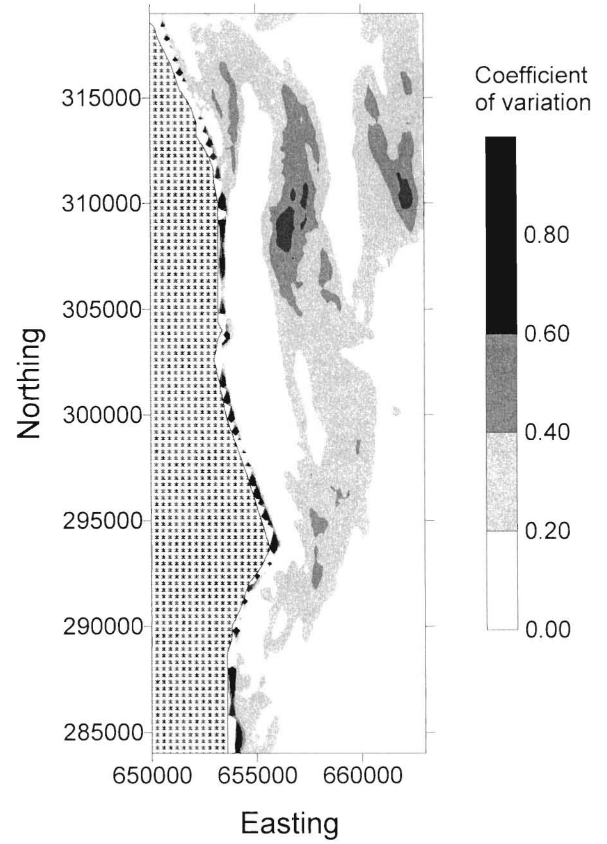

Figure 7. The coefficient of variation of the changes in seabed level over the period 1846–1992.

where $x_{min} = 650000$m, $x_{max} = 663000$m, $y_{min} = 284000$m, $y_{max} = 319000$m, $t_{min} = 1846$ A.D. and $t_{max} = 1992$ A.D. The index k runs from 1 to 16 and the spatial increments are defined by

$$x_i = x_{min} + (i - 1)\cdot\Delta x \quad \text{for } i = 1, 2, \ldots, I \quad \text{and}$$
$$\Delta x = \frac{x_{max} - x_{min}}{I - 1} \quad \text{and} \qquad (3)$$
$$y_j = y_{min} + (j - 1)\cdot\Delta y \quad \text{for } j = 1, 2, \ldots, J \quad \text{and}$$
$$\Delta y = \frac{y_{max} - y_{min}}{J - 1} \qquad (4)$$

General Statistics

The movement and variability of the seabed can be characterised using standard statistical measures. It is usual to calculate these in a pointwise manner. That is, treating the series of seabed levels at a particular point in space in turn. Repeating the calculations for each grid point in the DTMs provides a set of data values that can be contoured over the DTM grid. Thus maps of the mean, range, standard deviation and coefficient of variance maps were produced for the sixteen digital terrain models described earlier. The rationale for selecting these statistical parameters is described below.

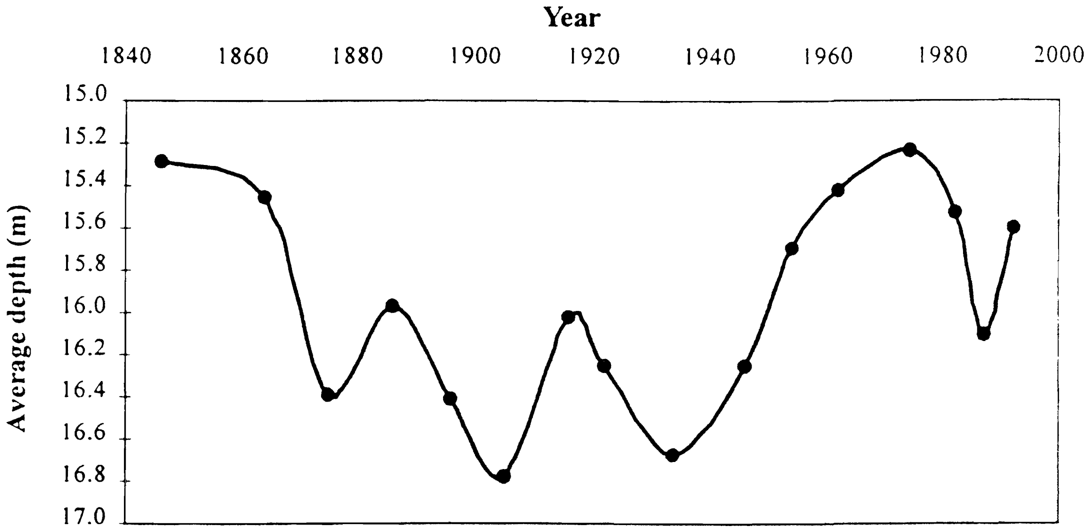

Figure 8. Average depth of seabed in study area between 1846–1992.

The mean bathymetry gives an impression of what the sandbanks would look like if all temporal variation were removed. Although this is important, it does not provide any information concerning the magnitude or temporal pattern of movement over time. The range, standard deviation and coefficient of variation can be used to provide a measure of these changes.

The range of bathymetry (difference between maximum-minimum) highlights the absolute changes in bathymetry. This allows regions that have large or small variation over the period to be readily identified. However, the range tells us little about the spread of variation over time. This is an important issue, as a large range could be associated with change between two consecutive charts (*e.g.*, large storm event) and is then followed by very stable behaviour.

The standard deviation and coefficient of variation (standard deviation divided by the mean) provide a measure of the spread of variation with respect to the mean value over time. The standard deviation is the spread of bathymetry values about the mean, whereas the coefficient of variation is a dimensionless measure of variability relative to the mean. Thus for two locations with the same variance in bathymetry levels but different mean depths, the coefficient of variation will be smaller for the location with the deeper mean bathymetry. The coefficient of variation tends to emphasise variability of shallow water such as the sandbanks and beaches.

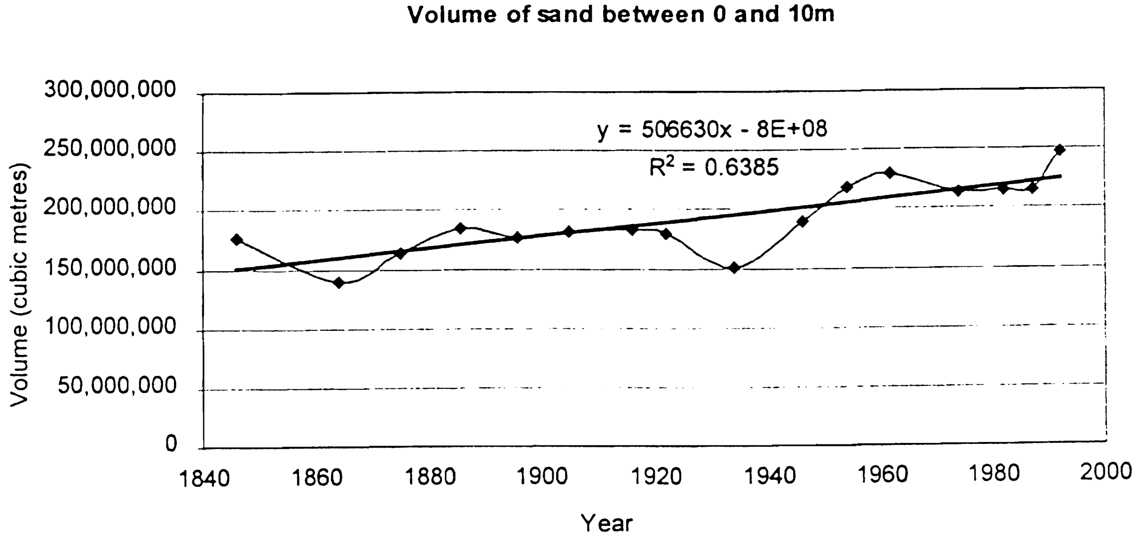

Figure 9. Volume of sand between 0 and 10m below OS datum.

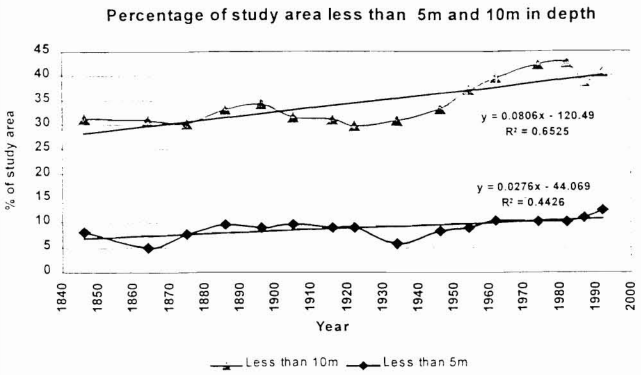

Figure 10. Percentage of the study area, which is between 0–5m and 0–10m below ODN.

In addition, to these statistical measures, further GIS analysis has concentrated on assessing the volume of material within the sandbanks. This analysis was conducted to provide a more detailed comparison against previous estimates of the volume of the sandbank system, CLAYTON *et al.* (1983). To evaluate the volumetric changes over time, the TIN terrain models stored in the GIS were used to assess the volume above (cut) and below (fill) 0, 5, 10, 15, 20, 25 and 30m below OS datum. Some results from this volumetric analysis are presented later.

One-dimensional EOF

In this case the problem is to determine the eigenfunctions of the variations in seabed level in time along a fixed cross-shore or longshore transects. Denote the discrete data by $g(\xi_l, t_k)$, where $1 \le l \le L$ and $1 \le k \le K$. The idea of EOF analysis is to express the data as

$$g(\xi_l, t_k) = \sum_{p=1}^{L} c_p(t_k)\cdot e_p(\xi_l) \qquad (6)$$

where e_p are the eigenfunctions of the square $L \times L$ correlation matrix of the data and c_p are the coefficients describing the temporal variation of the p'th eigenfunction. In practice, many fewer than L eigenfunctions may be required to capture a large proportion of the variation in the data, and (6) can provide an efficient means of identifying standing wave behaviour in the data. The correlation matrix, A, is calculated directly from the data and has elements

$$a_{mn} = \frac{1}{L\cdot K}\sum_{k=1}^{K} g(\xi_m, t_k)\cdot g(\xi_n, t_k) \qquad (7)$$

A is real and symmetric and has L real eigenvalues, λ_p, with $1 \le p \le L$. The L corresponding eigenfunctions, $e_p(\xi_l)$, satisfy the matrix equation

$$Ae_p = \lambda_p e_p \qquad (8)$$

The eigenfunctions of a real $L \times L$ symmetric matrix are mutually orthogonal. It is common practice to normalise the eigenvectors so they have unit length and thus

$$\sum_{l=1}^{L} e_p(\xi_l)\cdot e_q(\xi_l) = \delta_{pq} \qquad (9)$$

Table 1. *EOF results*

Eigen-function Number	Normalised Eigenvalue	% Mean Square (cumulative)	% Variance	% Variance (cumulative)
1	0.9735	97.4	—	
2	0.0085	98.2	32.0	32.0
3	0.0033	98.5	12.5	44.5
4	0.0029	98.8	10.9	55.4
5	0.0023	99.1	8.6	64.0
6	0.0022	99.3	8.1	72.1

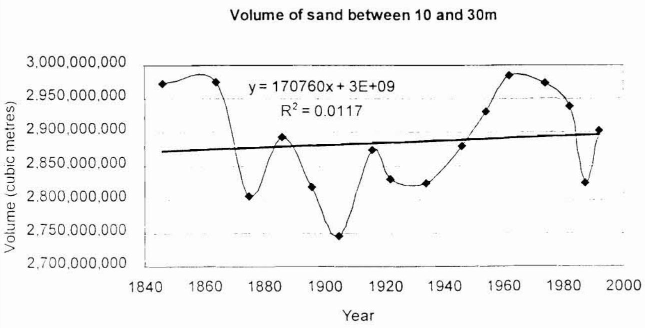

Figure 11. Changes in the volume of sediment between 10 and 30m below ODN, (cubic metres).

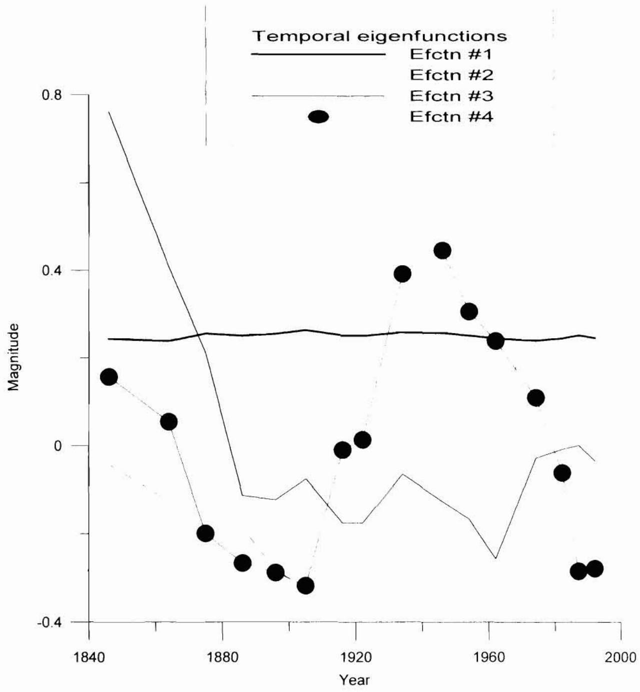

Figure 12. First four temporal eigenfunctions.

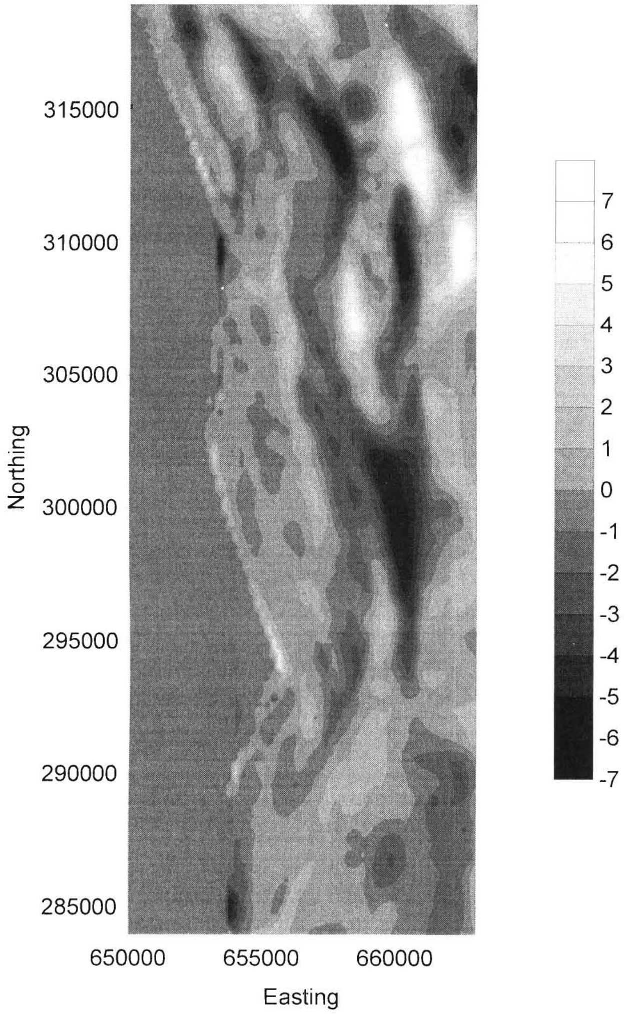

Figure 13. Contour plot of the second spatial eigenfunction.

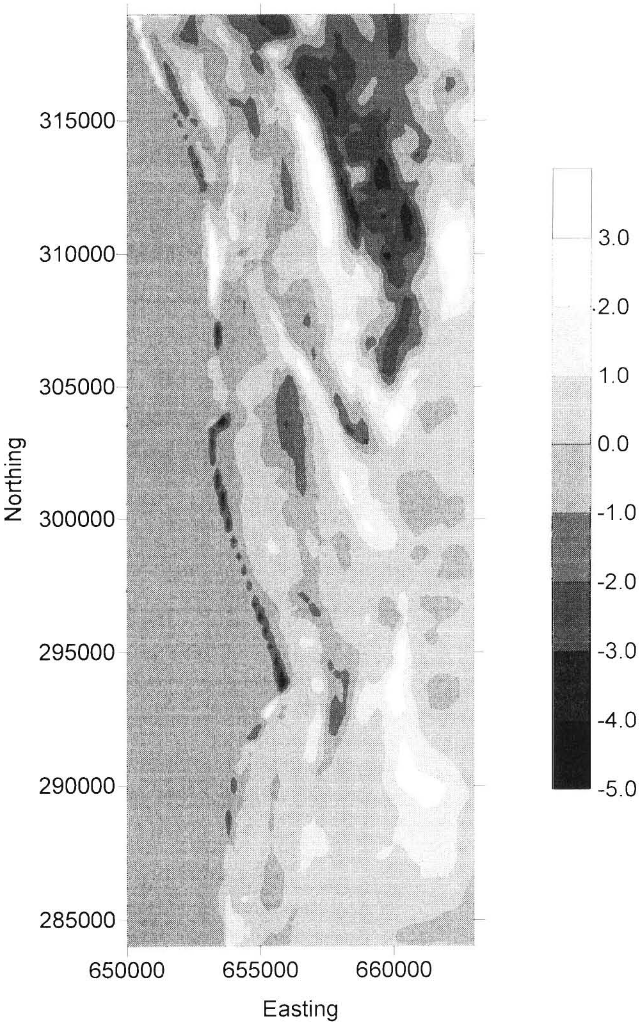

Figure 14. Contour plot of the third spatial eigenfunction.

where δ_{pq} is the Kronecker delta. From equations (6) and (9) the coefficients c_p may be calculated from

$$c_p(t_k) = \sum_{l=1}^{L} g(\xi_l, t_k)\cdot e_p(\xi_l) \qquad (10)$$

Two-dimensional EOF

In this case, the problem is to determine the eigenfunctions describing the variations in seabed level in time over an area in the (x, y)-plane. Denote the discrete data by $h(x_i, y_j, t_k)$, where $I \leq I \leq I$, $I \leq j \leq J$ and $I \leq k \leq K$. In contrast to the 1-D case there are several ways in which the data may be expanded in terms of eigenfunctions. The first is a natural extension to the 1-D case and was described by UDA and HASHIMOTO (1982):

$$h(x_i, y_j, t_k) = \sum_p e_p(x_i, t_k)\cdot f_p(y_j, t_k) \qquad (11)$$

where e_p are the cross-shore eigenfunctions and f_p are the longshore eigenfunctions. The eigenfunctions e_p and f_p can themselves be expanded in a fashion exactly analogous to that described in the previous section. An alternative expansion has been proposed by HSU *et al.* (1994):

$$h(x_i, y_j, t_k) \approx \sum_m e_m(x_i)\cdot f_m(y_j)\cdot c_m(t_k) \qquad (12)$$

In the analysis of the sandbank data, which exhibit strong variability in their joint cross-shore and longshore dependence, this assumption of separability is not appropriate. Here, the two-dimensional grid of points is treated as one larger one-dimensional data set, and we use Equation (6) with ξ_l covering all combinations of (x_i, y_j). Thus, we construct the correlation matrix, A, whose elements a_{mn} are the temporal correlation coefficients between seabed levels at any selected grid point (denoted by the m subscript) and those at every other point on the grid (denoted by the n subscript). The eigenfunction calculations then proceed as in the 1-D case. The spatial structure of each spatial eigenfunction can

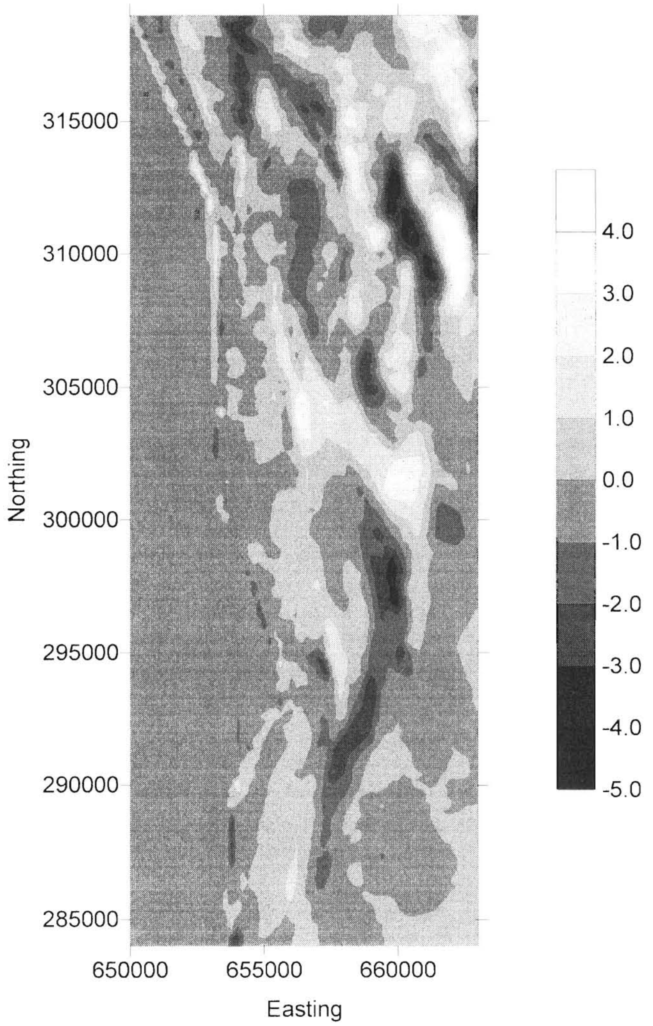

Figure 15. Contour plot of the fourth spatial eigenfunction.

be retrieved by noting the correspondence between ξ_l and (x_i, y_j), and plotting the l^{th} value of the eigenfunction at position (x_i, y_j). This procedure is more straightforward than the other 2-D methods and the results easier to interpret. Any similarity in behaviour at different points in the domain arises naturally from the analysis and will be evident in the results. This procedure makes no *a priori* assumption of separability of cross-shore and longshore dependence.

RESULTS

The results of the analyses are presented in three parts: the basic statistics; the volumetric calculations and finally the EOF analyses.

General Statistics

As shown by the mean bathymetry map (see Figure 4), the features of greatest persistence have been in the main central Scroby Sands, Holm Sands and Newcombe Sands. In addition, it is interesting to note that the pattern shown in Figure 4, mirrors very closely the most recent configuration of sandbanks, see Figure 1. In contrast, the only outer bank that appears to have maintained its general form over time is Middle Cross Sand. The variability of the other outer sandbanks tends to suggest that they are more much sensitive to changing patterns of sediment movement.

In the study area, the locations of greatest bathymetric range are situated offshore of Newport and California at Cross Sands and offshore of Hopton at Corton Sand, see Figure 5. In these locations, there has been over 20m change in the period studied. The largest of these changes has occurred on the north-east shelf of Scroby Sands before the drop into Barley Pickle, on the north west flank of Barley Pickle before the rise onto Cross Sands and on the east shelf of Middle and North Cross Sands.

The standard deviation and coefficient of variation maps, Figures 6 and 7 respectively, show that the immediate nearshore area and south of Lowestoft has been the most stable and slowly changing, with the main inner sandbanks (*e.g.*, Scroby Sands and Holm Sands) being relatively stable. In contrast, there has been greater variability at Caister Shoal, North Scroby and Middle Cross Sand in the north-east of the study area, and across Corton Sand offshore of Hopton-on-the-Sea, see Figures 6 and 7. This pattern re-emphasises that the regions of highest variability are oriented north-south, being predominantly shore parallel. Also, the complexity of the pattern increases as one moves northward from Lowestoft to Winterton.

'Volumetric' Analysis

As described earlier, GIS volume analysis techniques were used to assess the temporal changes in the average depth of the seabed and the volume of material within the sandbank system, see Figures 8 to 11.

As shown in Figure 8, there was a distinct period of erosion between 1846 and 1875 associated with an average increase of approximately 1m in the sea bed depth. After this phase, there were two smaller cycles of erosion and accretion between 1875 and 1934, followed by a period of accretion to 1974. After this lengthy period of accretion, there was a phase of erosion between 1974 and 1987, and a return to accretion between 1987–1992.

The GIS was also used to assess the changes in the pattern of sediment accumulation at different depths. This analysis revealed there has been a steady and gradual increase in amount of sediment, which has accumulated between 0 and 10m below ODN, see Figures 9 and 10. The square of the correlation coefficient, R^2, may be interpreted as the proportion of the variance in the data that is accounted for by the linear regression. The closer the value is to 1 the better the linear regression models the data. As shown in Figure 10, the percentage of the study area, 0 to 10m below ODN increased from 31% to 41% between 1846 and 1992. The positive trendline highlights this gradual accumulation of sediment in the shallow water zone over time. In contrast, there has been no long-term linear trend to the movement of sediment at lower depths (*e.g.*, the channels between the sandbanks), see Figure

11. This suggests that the most important mode of sediment accumulation (and transport) is the shallow water zone between 0 and 10–15m below ODN.

As a final observation, the total volume of sediment within the sandbanks in 1992 has been estimated at between 1.1 and 3.1×10^9 m^3. The lower limit is the volume between 0 and 20m below ODN, while the upper limit is the total volume between 0 and 30m below ODN. These figures provide a more definite estimate of the volume than the previous estimate of more than 1×10^9 m^3, CLAYTON *et al.* (1983). The calculation of the volume of the sandbanks clearly has important considerations for the future management of the offshore area (*e.g.*, for navigation and dredging).

EOF Analysis

Table 1 summarises the results of the EOF analysis. It demonstrates that over 97% of the mean square of the data is contained in the first function and that more than 99% of the mean square of the data is captured by only 5 functions. The mean square of the data is the average of the square of all the seabed levels in the whole dataset. The first eigenfunction corresponds to the mean bathymetry over the period, with the subsequent eigenfunctions representing the variation about the mean. In this case the 2nd through 6th eigenfunctions together account for over 72% of the variance about the mean. This is a large proportion of the variance for analyses of this kind and suggests that the EOFs can describe the variability in the sandbank data extremely efficiently.

The EOF expansion expresses the data as a sum of products of functions depending on, individually, space and time. It thus provides a means of separating variations in space and time, which have distinct frequencies and spatial scales. Figure 12 shows the graphs of the first four temporal eigenfunctions, which describe the variation in time of the corresponding spatial functions. There are several features of note:

- The first function is almost constant, as expected because it corresponds to the time mean;
- The second function shows a slow variation from negative to positive values over the period and is, arguably, indicative of a recurrent variation with a period of the order of 200 years;
- The third function shows a sharp drop from positive to negative values from 1840 to 1880 and smaller variations subsequently. This is indicative of an isolated switch in state followed by relatively stable behaviour;
- The fourth function shows a variation in time which is strongly suggestive of repetitive behaviour with a period of 100–120 years.

Figures 13, 14 and 15 show contour plots of the second, third and fourth spatial eigenfunctions respectively. One striking feature that they have in common is their strong spatial structure. Note that as the shape of the eigenfunctions is determined solely from the structure of the data there is no reason a priori that the individual eigenfunctions correspond directly with particular physical mechanisms.

For the second eigenfunction, minima are found on the seaward flanks North Scroby Sand, South Cross Sand and North Cross Sand, and around Holm Channel. Maxima occur in the southern end of Barley Pickle, and the landward flank of North Cross Sand. The second eigenfunction appears largely to capture the variability of the fringes of the sandbanks. The edges of the sandbanks were identified in the earlier sections as being the regions of largest variation, a point that was also noted by ROBINSON (1960). This eigenfunction also captures the link in behaviour between Holm Channel and Corton Sand. That is, when Holm Channel is well defined, Corton Sand is usually eroded or present in a very diffuse state. Conversely, when Corton Sand is well-defined Holm Channel is usually blocked with material. Where the eigenfunction is of the same sign then some similarity in morphological behaviour can be anticipated. Thus, the structure of the second eigenfunction suggests that Corton Sand, the landward flank of Middle Cross Sand and the seaward flank of North Scroby Sand will wax and wane together.

The third eigenfunction has a rather more diffuse structure. It attains large magnitudes in Barley Pickle, Yarmouth Road, Corton Road, Middle Cross Sand, the seaward flank of N. Scroby Sand and an offshore area centred on 661000 E, 290000 N.

The structure of the fourth eigenfunction includes a number of 'cross-shore' features such as the 'ridge' linking Corton Sand to Yarmouth Road and N. Scroby Sand to Caister Shoal. In addition, it has large magnitude in the vicinity of South and Middle Cross Sands, Holm Sand and the seaward flank of Newcombe Sand. The morphology of this eigenfunction suggests that the opening/closing of Holm Channel and the northern end of Caister Road, evident in Figure 3, are projected strongly onto this function. The associated temporal eigenfunction shows a quasi-sinusoidal variation. It is interesting that the sign of the eigenfunction also suggests that the behaviour of the two channels is out of phase. That is, it is negative in the north of Caister Road and positive in Holm Channel. The structure of the eigenfunction over Cross Sands is reminiscent of a localised wave pattern with a NNW-SSE tilt and a wavelength of approximately 3 kilometres.

DISCUSSION AND CONCLUSIONS

One finding of this study is a gradual long-term upward trend in the volume of the upper portion of the sandbank system. This, coupled with the observation that there has been no clear pattern of change in sediment volume at lower depths, suggests that the sandbanks have been growing taller over the period of many decades. Assuming that the sandbank system was in some form of dynamic equilibrium at the start of the study period, it might be argued that this growth was merely a morphological response to sea level rise to maintain this equilibrium. The results of some simple calculations are instructive in this regard. First we consider the shape of the upper section of the sandbanks to be well approximated by trapezoidal prisms, with parallel top and base with lengths T and b respectively. If the height of the prism is h and the total length of the prisms is l then the volume, V, is given by $l.h. (b + T)/2$. Choosing representative values as $l = 15000$m, $b = 2000$m, $T = 500$m, $h = 8$m gives $V = 1.5 \times 10^8$m^3, corresponding approximately to the volume of sand between 0 and 10m for 1846 given in Figure 9. Assuming that the increase in volume arises solely from an increase in h over the same period then the volume rate of increase 5 $\times 10^5 m^3$/yr corresponds to a rate of increase in sandbank height of 27mm/yr. This is close to an order of magnitude larger than the mean sea level rise observed over the same period. This suggests that the trend in sandbank volume is not solely a response to sea level rise but due mainly to some other mechanism(s). The volumetric analysis also demonstrates that a significant amount of material is available and the sandbank system appears to be acting in some ways as a repository. Given the almost zero trend in volume changes at lower depths this implies that the sandbank growth is being fed either by material from the coastline via cross-shore links or by material from outside the area being transported across the open boundaries of the system which is subsequently retained.

The analyses have identified long-term trends in volume and areal extent, as well as evidence for oscillatory behaviour within the sandbank system. Evidence of variations, which had several distinct cycles within the period covered by the study, would increase confidence in the predictability of the variations. As noted by WALLACE and DICKINSON (1972), EOF analysis is designed to identify standing wave features in the data. The technique of complex EOF, (CEOF), could be used to investigate the presence of travelling morphological disturbances. While CEOF analysis of the sandbank data is beyond the scope of the current paper, further research is planned on this topic.

Nevertheless, the presence of trends and recurrent behaviour provides some scope for predicting the future development of the sandbanks. In this regard it could be noted that the period of 'oscillations' identified in the eigenfunctions is greater than the time horizon currently adopted in many coastal zone plans. This raises the possibility that long-term recurrent behaviour is treated as an underlying trend on the planning timescale. The consequences could be a planned situation that is unsustainable when it becomes apparent that the 'trend' is actually a long-term 'oscillation'.

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