

Equilibrium Beach Profile Model for Reef-Protected Beaches

J.J. Muñoz-Pérez[†], L. Tejedor[‡], and R. Medina[‡]

[†]Departamento de Física Aplicada,
Universidad de Cádiz CASEM
Polígono Río S. Pedro, s/n.
Puerto Real
Cádiz, Spain

[‡]Ocean & Coastal Research Group
Departamento de Ciencias y Técnicas del Agua y del Medio Ambiente
Universidad de Cantabria
39005 Santander, Spain

ABSTRACT

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A beach equilibrium model for reef-protected beaches is presented. The profiles analyzed in this work intersect a reef and, consequently, the entire profile is not sand rich. The model assumes uniform energy dissipation per unit volume and considers the wave decay due to the wave breaking over the submerged reef. The resulting beach profile form is similar to the one proposed by LARSON and KRAUS (1989); however, the profile shape parameter is not the same as the A value used in the usual DEAN (1977) equilibrium profile, due to the wave decay dependence. A simple relationship between the new shape parameter for reef-protected beaches and the A value for non-reef-protected beaches is determined using the ANDERSEN and FREDSOE (1983) wave decay model. The proposed relationship is validated using over 50 profiles measured on seven beaches. It is concluded that the shape parameter usually used in equilibrium profile models can not be represented in all cases by a simple function of the sediment grain size or fall velocity. It is also concluded that no equilibrium beach profile is possible within a distance of about 10 to 30 h_r from the edge of the reef, where h_r is the water depth over the reef.

ADDITIONAL INDEX WORDS: Beach profiles, equilibrium beach profile, reefs, reef-protected beaches, Spanish beaches.

INTRODUCTION

One of the most important concepts in the field of near-shore processes is that of the equilibrium profile of beaches. In a broad sense, the equilibrium beach profile is the result of the constructive and destructive forces acting in a beach profile. The hypothesis behind the equilibrium beach profile is that beaches respond to wave forcing by adjusting their form to an equilibrium or constant shape attributable to a given type of incident wave or sediment characteristic. The existence of this equilibrium profile has been a matter of great interest to numerous investigators and various expressions have been proposed over the years (KEULEGAN and KRUMBEIN, 1919; SUNAMURA and HORIKAWA, 1974; HAYDEN *et al.*, 1975; VELLINGA, 1983; BODGE, 1992). The most widely-used formulation, very simple and easy to apply, was proposed by BRUUN (1954) and DEAN (1977).

BRUUN (1954) assumed a beach profile shape given by:

$$h = Ax^{2/3} \quad (1)$$

where h is the total water depth, A is a dimensional shape parameter and x is the horizontal distance from the shoreline. This equation was found by fitting beach profiles from California and the Danish North Sea coast. DEAN (1977) adjusted the same expression to 504 beach profiles collected by HAYDEN *et al.* (1975), along the U.S. East Coast and the Gulf of Mexico. Fur-

thermore, DEAN (1977) showed that the assumption of constant wave energy dissipation per unit volume, due to wave breaking, is consistent with the previous equation (1).

Although the existence of a shoreface profile of equilibrium is generally accepted, several authors have questioned the validity of equation (1) for describing all shoreface profiles (*e.g.* WRIGHT *et al.*, 1991; PILKEY *et al.*, 1993; RIGGS *et al.*, 1995). PILKEY *et al.* (1993) stated that the most fundamental problems with the equation are the assumptions that: (1) only wave orbitals move sediment, (2) underlying shoreface geology is unimportant, and (3) differences in profile shape from place to place are only due to variations in grain size, in other words, that the A parameter is only a function of the sediment grain size.

In this paper the importance of the underlying geology is addressed analyzing the particular case of beaches in which the entire profile is not sand rich and areas of hard bottom or mud are encountered (*e.g.* coral reefs, perched barriers). Many characteristics and informative details about these kinds of beaches, which will be denoted as reef-protected beaches, have been previously studied: water level fluctuations (KARUNARATHANA and TANIMOTO, 1995), bore-like surf beat (NAKAZA and HINO, 1990), sediment flux (ROBERTS, 1980), wave set-up and cross-reef currents (SYMONDS *et al.*, 1995). In a special way, wave breaking and wave attenuation over submerged horizontal shelves have been considered (HORIKAWA and KUO, 1966; GERRITSEN, 1980; SEELIG, 1983; GOURLAY, 1994; NELSON, 1994; YU *et al.*, 1995; HARDY and YOUNG, 1996).

In the analysis presented, assumption (1) is accepted and the influence of the reef on assumption (3) is analyzed. In the paper several reef-protected beaches along the Spanish coast are studied and their profiles are examined.

MODIFIED EQUILIBRIUM BEACH PROFILE

Assuming wave energy dissipation per unit water volume to be the dominant destructive force, DEAN (1977) proposed that a sediment of specific size is stable in the presence of a particular level of wave energy per unit water volume, D^* , leading to the following equation for equilibrium beach profiles:

$$\frac{1}{h} \frac{\partial}{\partial x} (EC_g) = D^* \tag{2}$$

in which, E is the local wave energy density, and C_g is the local wave group velocity. Assuming shallow water linear wave theory and constant breaker-to-depth ratio, equation (2) can be integrated to yield equation (1).

A more general derivation of the equilibrium profile form with a sloping beach-face has been given by LARSON and KRAUS (1989). In this work, the profile shape is again assumed to result from uniform wave energy dissipation; however, unlike DEAN'S (1977) derivation, wave breaking is not restricted to spilling breakers with a constant breaker-to-depth ratio. Instead, wave energy dissipation per unit volume is assumed to be given by the dissipation model of DALLY *et al.* (1985). This dissipation model is solved for a beach in equilibrium, to find the breaker height at any depth. The resulting form of the equilibrium beach profile is:

$$x = \frac{h}{m} + \left(\frac{h}{A}\right)^{3/2} \tag{3}$$

where m is the beach face slope. In shallow water, the first term in eq. (3) dominates, simplifying to:

$$h = mx. \tag{4}$$

In deeper water, the second term in eq. (3) dominates with the following simplification:

$$h = Ax^{2/3}. \tag{5}$$

For practical applications, KRIEBEL *et al.* (1991), suggested that the limit between eqs. (4) and (5) may be given by:

$$h = \frac{4A^3}{9m^2}. \tag{6}$$

Consequently, for sandy beaches, eq. (5) dominates for $h > 0.3-0.5$ m. Spilling-wave breaking assumption with a constant wave height to water depth ratio, γ , is not adequate for waves breaking on a shelf. HORIKAWA and KUO (1966), computed theoretical curves that have a consistent agreement with experimental data in the case of wave transformation on a horizontal bottom. The ratio between the local wave height and the mean water depth decreases from 0.8, at the initial wave breaking point, to become almost constant, about 0.5, in the inner zone.

Several wave-decay expressions have been proposed (*e.g.* DALLY *et al.*, 1985; ANDERSEN and FREDSOE, 1983). FREDSOE and DEIGAARD (1992), for example, gave the following exponential decay:

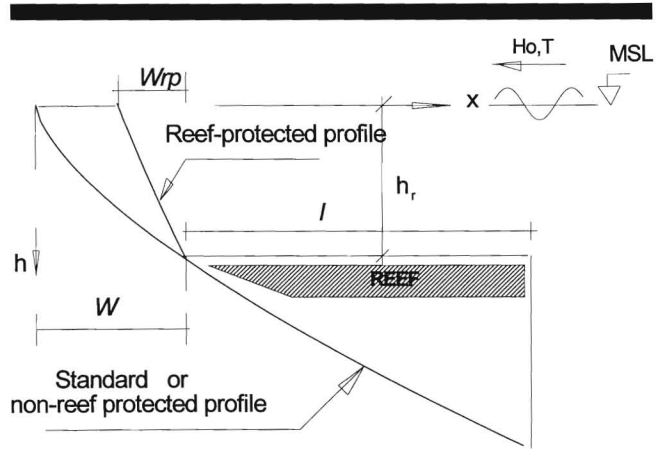


Figure 1. Sketch of protected and non-protected profiles.

$$\frac{H}{h_r} = 0.5 + 0.3 \exp\left(-0.11 \frac{l}{h_r}\right) \tag{7}$$

where h_r is the water depth over the reef, H is the wave height and l is inshore distance from the edge of the shelf (see Figure 1).

From eq. (7) it can be concluded that the wave height that reaches the sandy beach toe, which is located at the depth h_r , is less than the wave height that would reach that particular depth in a beach without the hard shelf. Consequently, the total amount of energy that has to be dissipated by the sandy profile is minor.

The beach profile form of a reef-protected beach can be determined by means of LARSON and KRAUS'S (1989) derivation of the equilibrium profile, taking into account the available wave energy at the toe of the beach and assuming the dissipation model of DALLY *et al.* (1985). The resulting beach profile form will be given by an expression similar to eq. (3). However, for the same grain size, the profile shape parameter for a reef-protected beach will not be the same as the A value used in the usual Dean equilibrium profile in eq. (1) or eq. (5) due to the shelf wave-decay dependence.

A simple relationship between the shape parameter for reef-protected beaches, hereafter denoted as A_{rp} , and non-reef-protected beaches can be obtained considering that the energy flux, EC_g , at h_r must be dissipated along the beach profile in both cases.

$$(EC_g)_{h_r} = \int D^* h \, dx. \tag{8}$$

Assuming linear wave theory and that eq. (5) is valid along the entire profile, it yields:

$$\left(\frac{H_{rp}}{H}\right)_{h_r}^2 = \left(\frac{W_{rp}}{W}\right) \tag{9}$$

where H is wave height, W is the total length of the profile and the subscript $()_{rp}$ indicates the reef-protected beach (see Figure 1).

Since H_{rp} at h_r is less than H at the same depth, the total

length of the profile for the reef-protected beach will also be less than the non-reef-protected beach and, consequently, the beach profile slope will be steeper.

Equation (9) can also be written in terms of the breaker-to-depth ratio as:

$$W_{rp} = W\left(\frac{\Gamma}{\gamma}\right)^2 \quad (10)$$

where Γ is the breaker-to-depth ratio for a reef-protected beach (e.g. equation (7)) and γ is the breaker-to-depth ratio in a non-reef-protected beach. For a wide shelf ($l \sim \infty$), typical values of Γ range between 0.55 to 0.35 (NELSON, 1994). Values of γ depend on beach slope and wave steepness, and have a wider range of variability. KAMINSKY and KRAUS (1993) compiled a large database of wave breaking parameters and showed that for typical field beach slopes (1/30 to 1/80) most of γ values are encountered in the range 0.65 to 1.1 with an average value of 0.79.

Introducing equation (5) in equation (10), a relationship between the shape parameters can be found as:

$$\frac{A_{rp}}{A} = \left(\frac{\gamma}{\Gamma}\right)^{4/3} \quad (11)$$

where A_{rp} is the shape parameter for the reef-protected beach and A is the non-reef-protected beach shape parameter.

FIELD DATA

Using the set of field data compiled by GOMEZ-PINA (1995), beach profile data from reef-protected beaches along the Spanish coast have been collected to verify the proposed model. Over 50 profiles from seven beaches have been analyzed (see Figure 2 for their locations). The main characteristics of these beach profile data are shown in Table I. It is noted that the values of A_{rp} listed in Table I have been determined by best fitting and the values of A by means of MOORE's (1982) relationship.

Figure 3 graphically compares the "actual", the "best-fit" and "MOORE's" (1982) beach profiles for each of the beaches analyzed. It is clearly shown in Figure 3 that the beach slope predicted by MOORE's (1982) relationship is much milder than the actual slope. It can also be observed that, for engineering purposes, a simple mathematical expression like equation (1) can properly describe the actual profile if an adequate shape parameter is provided.

In order to compare the quality of the fit for each profile, a parameter ϵ is determined by:

$$\epsilon = \frac{\sum (h_i - h_{pi})^2}{\sum h_i^2} \cdot 100\% \quad (12)$$

where h is the actual depth and h_p is the depth predicted by either MOORE's (1982) profile and the present model. The subscript $()_i$ refers to each of the points used to describe the profile. Values of ϵ are shown in TABLE I. The value of $\epsilon = 0$ corresponds to a perfect fit, and increasing values of ϵ refers to increasingly poorer fit.

The predicted values of A_{rp} using equation (11) and the best-fitted values listed in Table I are compared in Figure 4. The predicted values are computed using FREDSOE and DEIGAARD'S (1992) model for Γ . It is seen in Figure 4 that equa-

tion (11) provides a good representation of the beach shape parameter A_{rp} . The asymptotic best fit for a wide shelf ($l/h > 60$) is $A_{rp} = 1.48A$ which corresponds to a value of $W_{rp} = 0.56 W$. Regrettably, no data for dimensionless shelf width less than 35 meters are available. A possible explanation for this lack of data will be discussed later.

DISCUSSION

The theoretical model of equilibrium beach profiles presented by DEAN (1977) is based on the hypothesis of uniform dissipation of wave energy per unit volume in the surf zone and the assumptions of shallow water linear wave theory and constant breaker-to-depth ratio.

The latest assumption is used in equation (2) to relate water depth, h , with wave height, H . However, this relationship is not adequate in many cases. LARSON and KRAUS (1989) modified this assumption so that wave breaking is not restricted to spilling breakers with constant breaker-to-depth ratio. The relationship between water depth and wave height is determined in LARSON and KRAUS'S (1989) approach by means of DALLY'S *et al.* (1985) dissipation model:

$$D = \frac{K}{h^2}(F - F_s) \quad (13)$$

where K is related to the length scale over which the wave energy flux is reduced from its value during breaking, F , to a stable value F_s .

From equation (13) it can be concluded that if the energy flux, F , is modified during breaking due to external processes, the breaker-to-depth ratio will change as will the form of the profile. This conclusion has been formulated previously by GONZALEZ *et al.* (1997) when analyzing beach profiles in which wave propagation phenomena during breaking are important (e.g. diffraction behind breakwaters, refraction due to shoals, *etc.*). In reef-protected beaches the energy flux dissipation over the reef reduces the total energy flux that has to be dissipated by the beach resulting in a steeper profile. From the engineering point of view the profile form is, however, adequately represented by an expression similar to equations (1) or (5) with a different shape parameter.

An important conclusion is that the shape parameter cannot be represented, in all cases, by a simple function of the sediment grain size or fall velocity as proposed by MOORE (1982) or DEAN (1987). The underlying geology and the wave propagation characteristics along the profile play an important role in the profile shape. It is remarkable that the hypothesis of uniform dissipation of wave energy, which leads to a 2/3-power profile, is still consistent with the profile fitting results for reef-protected beaches.

The application of equation (7) is restricted to waves that break on a shelf. GOURLAY (1994) studied the wave transformation of waves approaching a fringing reef with a steep face and outer reef-top slope gently decreasing in the landward direction. A nonlinear parameter, F_{c0} ,

$$F_{c0} = \frac{g^{1.25} H_0^{0.5} \Gamma^{2.5}}{h_r^{1.75}} \quad (14)$$

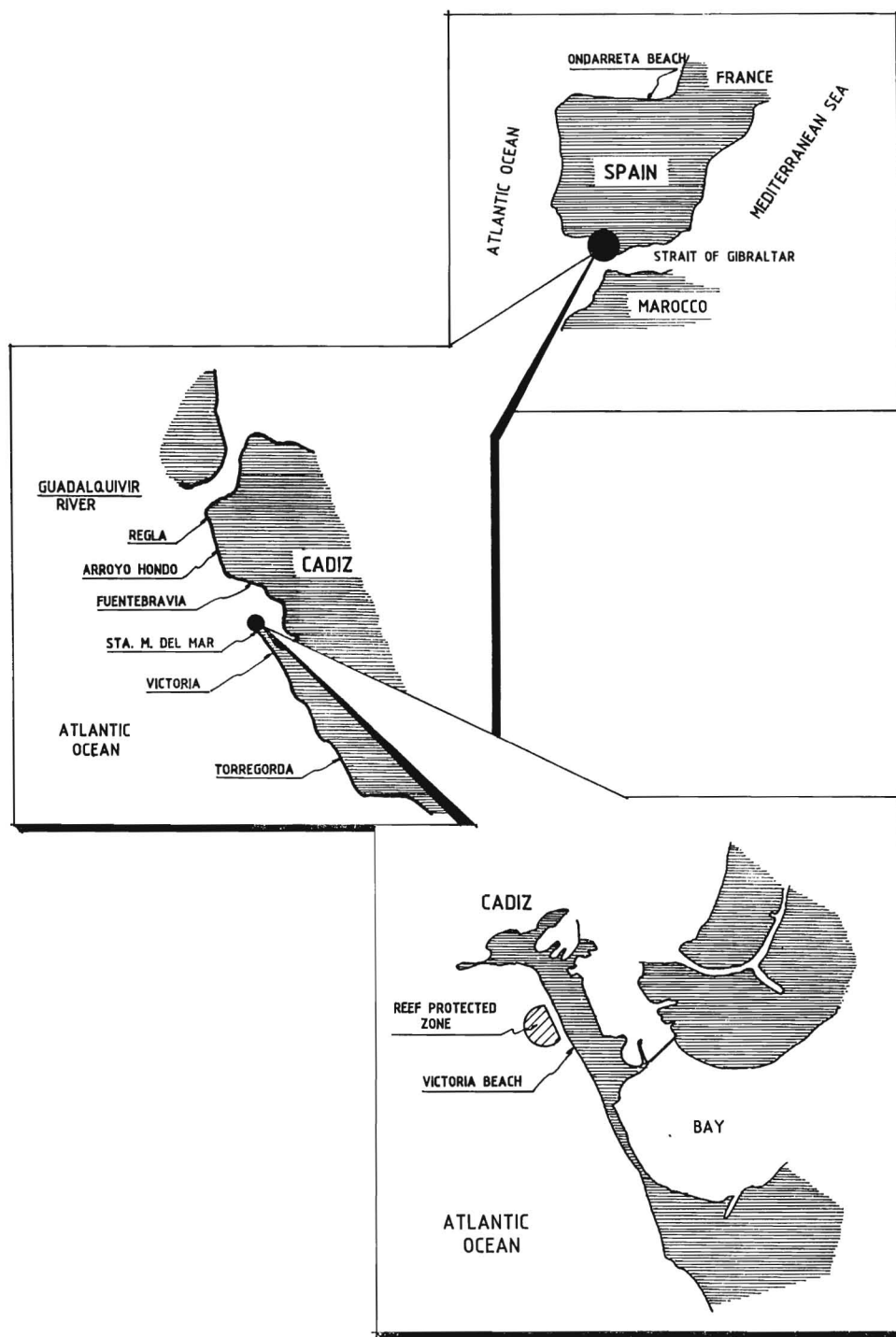


Figure 2. Location of beaches in which the proposed model has been validated.

based upon one proposed by SWART and LOUBSER (1979), was suggested as a suitable parameter for classifying wave transformation regimes on the reef.

In particular, when $F_{c0} > 150$, waves plunge on the reef edge and the amount of wave energy reaching the shore is

small. However, for $150 > F_{c0} > 100$ the waves increase in height as they cross the reef edge and then break by spilling on the reef-top. The wave height on the reef-top can be as much as 1.2 times the incoming wave height and the wave energy reaching the shore is maximum.

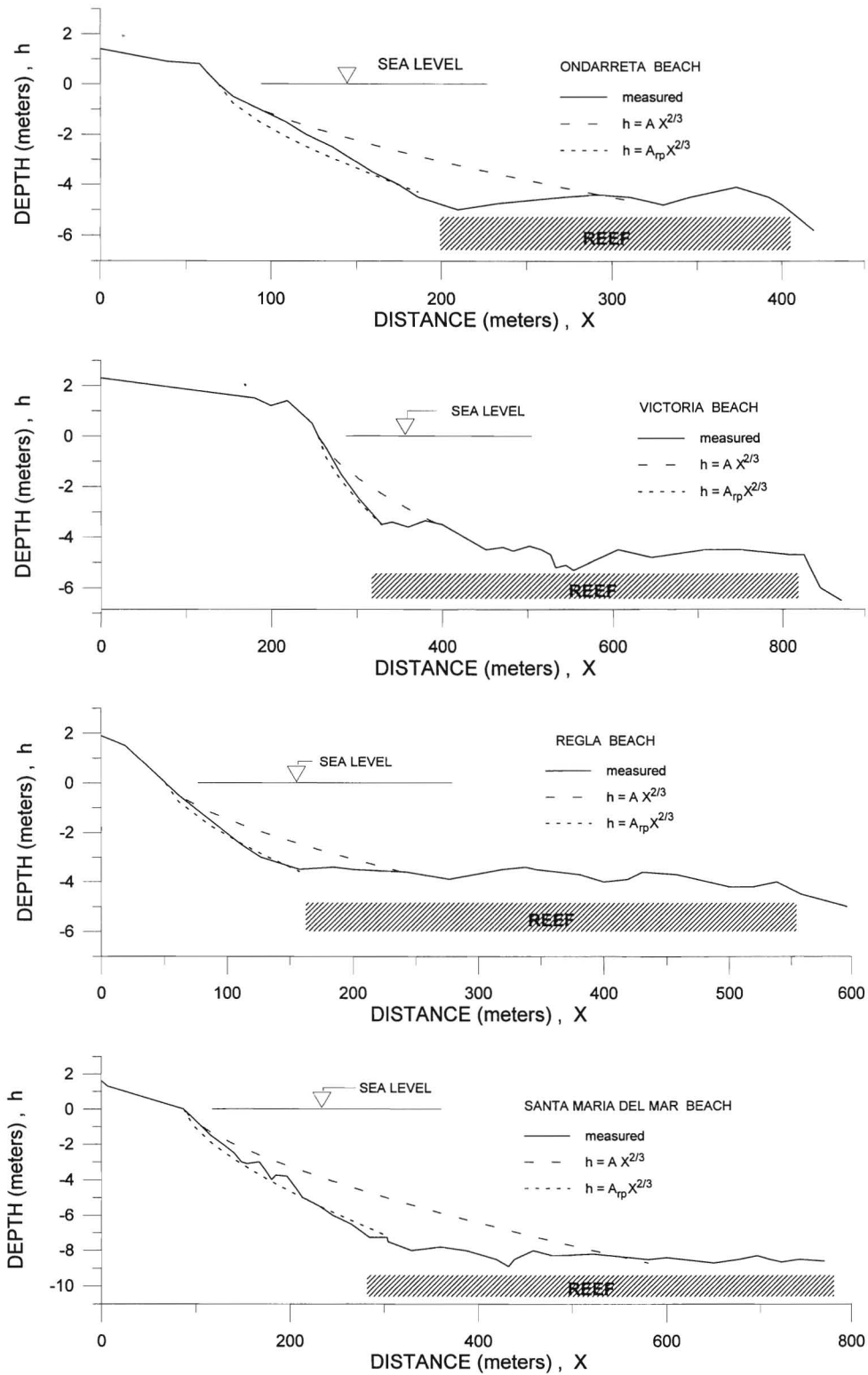


Figure 3. Comparison between measured profiles and the expressions $h = Ax^{2/3}$ and $h = A_{rp}x^{2/3}$.

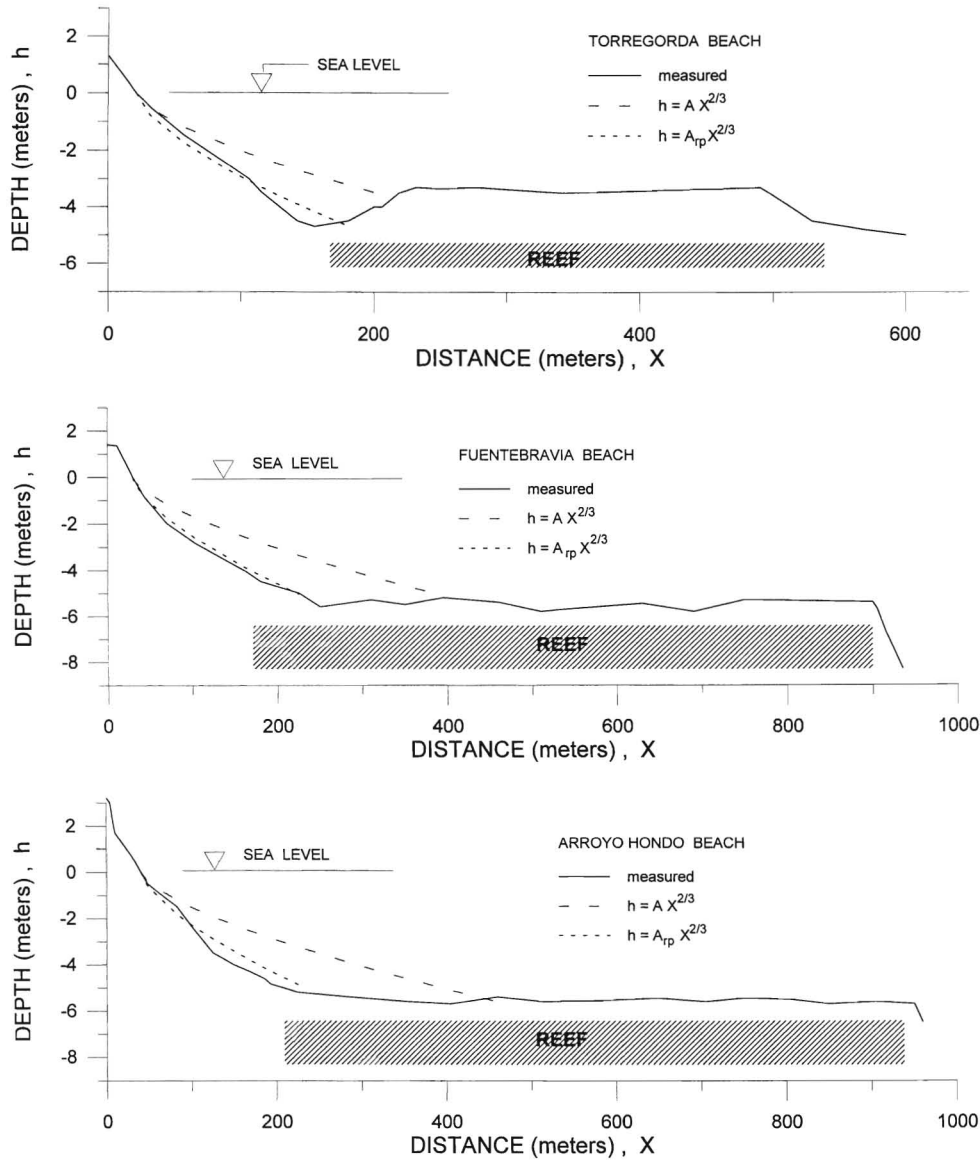


Figure 3 Continued.

Table 1. Main characteristics of the beach profile data.

| Beach | D_{50} (mm) | l (m) | h_r (m) | A ($m^{1/3}$) | A_p ($m^{1/3}$) | ϵ ($Ax^{2/3}$) | ϵ ($A_{rp}x^{2/3}$) |
|--------------|---------------|---------|-----------|-------------------|---------------------|---------------------------|--------------------------------|
| Ondarreta | 0.33 | 200 | 4.50 | 0.12 | 0.18 | 15.4 | 1.7 |
| Sta. María | 0.42 | 500 | 8.50 | 0.14 | 0.21 | 18.8 | 2.9 |
| Torregorda | 0.25 | 330 | 3.50 | 0.11 | 0.16 | 49.4 | 5.4 |
| Victoria | 0.32 | 470 | 4.00 | 0.13 | 0.20 | 13.5 | 0.9 |
| Arroyo Hondo | 0.25 | 750 | 5.30 | 0.10 | 0.15 | 9.3 | 1.4 |
| Regla | 0.25 | 380 | 3.50 | 0.11 | 0.16 | 12.6 | 0.6 |
| Fuentebravía | 0.27 | 740 | 5.50 | 0.10 | 0.15 | 37.2 | 2.6 |

The location of the breaking point for this kind of waves was found to be of the order of one reef-top wavelength:

$$l_p \approx T\sqrt{gh_r} \tag{15}$$

which yields an approximate value of $l_p = 10 h_r$. The surf zone width, l_s , for these kinds of waves was found to be within the range of two/three wave lengths,

$$l_s \approx 2 \div 3T\sqrt{gh_r}.$$

From Gourlay's results it can be concluded that, at least, at a distance of $l = 10h_r$, the wave height can be greater than the incoming wave and the wave energy flux can exceed the stable value of wave energy flux given by the constant breaker-to-depth ratio $\gamma = 0.8$ for that particular depth, h_r . Fur-

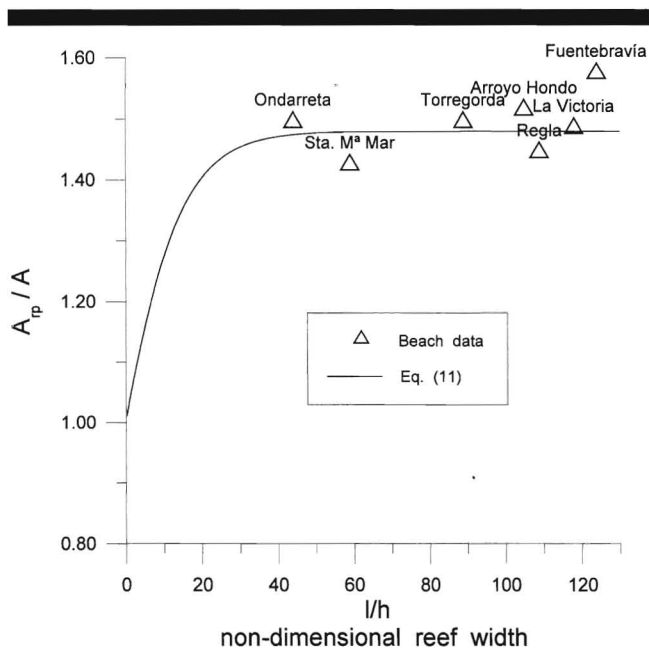


Figure 4. Comparison between best-fitted values of A_p values and predicted values of A_p using Equation 11.

thermore, the breaking process will take a distance (one or two wavelengths) to reduce this wave energy flux to a stable value.

KRIEBEL (1982), KRIEBEL and DEAN (1985) and ZHENG and DEAN (1997) have considered profiles out of equilibrium by hypothesizing that the offshore transport is proportional to the difference between the actual and the equilibrium wave energy dissipation per unit volume, *i.e.*:

$$Q = K(D - D^*) \quad (16)$$

If the actual wave energy flux, D , is greater than the equilibrium one, sand will be carried from onshore to offshore. Since that is not possible due to the hard shelf, it is concluded that no equilibrium beach profile is possible within a distance less than 10 to 30 h_r from the edge of the reef.

SUMMARY

Despite the advances in the physical understanding of the hydrodynamic processes that occur over submerged reefs, the stability of beaches protected by these reefs has received much less attention. In this paper a beach equilibrium model for reef-protected beaches has been developed. The model takes into account the wave breaking over the submerged reef and the wave energy flux reduction.

Although the resulting beach profile form is similar to the one proposed by LARSON and KRAUS (1989), the shape parameter is not the same as the A value used in the usual DEAN (1977) equilibrium profile due to wave decay dependence. From this dependence it is concluded that the A parameter cannot be adequately represented by a simple function of the sediment grain size or fall velocity. The underlying geology and the wave propagation along the profile play an

important role in the profile shape. Consequently, the A parameter usually used to fit real profiles with a 2/3-power shape must take these factors into account.

A simple expression has been proposed for the shape parameter A_p for reef-protected beaches based on ANDERSEN and FREDSOE's (1983) wave decay model. The proposed expression has been validated using over 50 profiles measured on seven beaches.

Using GOURLAY's (1994) study of the wave transformation of waves approaching a fringing reef and KRIEBEL's (1982) offshore transport model for profiles out of equilibrium, it is concluded that no equilibrium beach profile is possible within a distance less than 10 to 30 h_r from the edge of the reef.

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LITERATURE CITED

- ANDERSEN, O.H. and FREDSOE, J., 1983. Transport of suspended sediment along the coast. *Progress Report No. 54 Inst. of Hydrodynamics and Hydraulic Engineering*. ISVA. Technical University of Denmark. pp. 33-46.
- BODGE, K., 1992. Representing equilibrium beach profiles with an exponential expression. *Journal of Coastal Research* 8(1), 47-55.
- BRUUN, P., 1954. Coast erosion and the development of beach profiles. Beach Erosion Board, *Technical Memorandum, No 44*.
- DALLY, W.; DEAN, R., and DALRYMPLE, R., 1985. Wave height variation across beaches of arbitrary profile *Journal Geophysical Research*, 90(C6).
- DEAN, R.G., 1977. Equilibrium beach profiles: U.S. Atlantic and Gulf coasts. Department of Civil Engineering, *Ocean Engineering Report No. 12*, University of Delaware, Newark, Delaware.
- FREDSOE, J. and DEIGAARD, R., 1992. Mechanics of coastal sediment transport. *Advanced Series on Ocean Eng.* Vol. 3. World Scientific, 369p.
- GERRITSEN, F., 1980. Wave attenuation and wave set-up on a coastal reef. In: *Proceedings 17th Coastal Engineering Conference* (Sydney. ASCE, New York), Vol. 1, pp. 444-461.
- GÓMEZ-PINA, G., 1995. Análisis de perfiles de playa en las fachadas cantábrica y atlántica de la costa española y su aplicación a proyectos de regeneración. Universidad de Cantabria E.T.S.I.C.C.P. Dpto. de Ciencias y Técnicas del Agua y Medio Ambiente.
- GONZÁLEZ, M.; MEDINA, R., and LOSADA, M.A., Equilibrium beach profiles: effect of refraction and diffraction. *Proceedings Coastal Dynamics'97*, In press.
- GOURLAY, M.R., 1994. Wave transformation on a coral reef. *Coastal Engineering*, 23: 17-42.
- HARDY, T. and YOUNG, I., 1996. Field study of wave attenuation of an offshore coral reef. *Journal of Geophysical Research* Vol. 101, No. C6 pp. 14311-14326.
- HAYDEN, B.; FELDER, W.; FISHER, J.; RESIO, D.; VINCENT, L., and DOLAN, R., 1975. Systematic variations in inshore bathymetry. Department of Environmental Sciences, *Technical Report No. 10*, University of Virginia, Charlottesville, Virginia.
- HORIKAWA, K. and KUO, C.T., 1966. A study on wave transformation inside surf zone. *Proceedings 10th Coastal Engineering Conference*, (Tokyo), pp. 217-233.
- KAMINSKY, G.M. and KRAUS, N.C., 1993. Evaluation of depth-limited wave breaking criteria. *Proc. Ocean Wave Measurement and Analysis, Waves 93* (ASCE), pp. 180-193.
- KARUNARATHNA, H. and TANIMOTO, K., 1995. Numerical experi-

- ments on low-frequency fluctuations on a submerged coast reef. *Coastal Engineering* 26, 271–289.
- KEULEGAN, G.H. and KRUMBEIN, W.C., 1919. Stable configuration of bottom slope in a shallow sea and its bearing on geological processes. *Transactions American Geophysical Union*, 30(6), 855–861.
- KRIEBEL, D., 1982. Beach and Dune response to Hurricanes. M.S. Thesis, Department of Civil Engineering, University of Delaware, Newark, Delaware.
- KRIEBEL, D. and DEAN, R.G., 1985. Numerical simulation of time-dependent beach and dune erosion. *Coastal Engineering*, 9.
- KRIEBEL, D.; KRAUS, N.C., and LARSON, M., 1991. Engineering methods for predicting beach profile response. *Proceedings Coastal Sediments'91* (ASCE), pp. 557–571.
- LARSON, M. and KRAUS, N.C., 1989. SBEACH: Numerical model to simulate storm-induced beach change. Army Corps of Engr., Waterway Experiment Station, *Technical Report CERC-89-9*.
- MOORE, B., 1982. Beach Profile Evolution in Response to Changes in Water Level and Wave Height. M.S. Thesis, University of Delaware.
- NAKAZA, E. and HINO, M., 1990. Reef-zone disaster caused by bore-like surf beat. In: *Flows and waves*, Tokyo, Institute Technology Report No. 5, pp. 85–97.
- NELSON, R.C., 1994. Depth limited design wave heights in very flat regions. *Coastal Engineering*, 23, 43–59.
- PILKEY, O.; YOUNG, R.; RIGGS, S.R.; SAM SMITH, A.W.; WU, H., and PILKEY, W.D., 1993. The concept of shoreface profile of equilibrium: A critical review. *Journal of Coastal Research*, 9(1), 255–278.
- RIGGS, S.; CLEARY, W., and SNYDER, S., 1995. Influence of inherited geologic framework on barrier shoreface morphology and dynamics. *Marine Geol.*, 126, 213–234.
- ROBERTS, M.H., 1980. Physical processes & sediment flux through reef-lagoon systems. *Proceedings 17th International Conference Coastal Engineering*. (ASCE), Vol. 1, pp. 946–962.
- SEELIG, W.N., 1983. Laboratory study of reef-lagoon system hydraulics. *J. Waterway, Port Coastal Ocean Engineering* (ASCE), 109, 380–391.
- SUNAMURA, T. and HORIKAWA, K., 1974. Two-dimensional beach transformation due to waves. *Proceedings, Fourteenth International Conference on Coastal Engineering* (ASCE), pp. 920–938.
- SWART, D.H. and LOUBSER, C.C., 1979. *Vocoidal Water Wave Theory*. Vol 2: Verification CSIR South Africa. National Research Institute Oceanology, Coastal Engineering and Hydraulics Division CSIR Research Report 360.
- SYMONDS, G.; BLACK K.P., and YOUNG, I.R., 1995. Wave-driven flow over shallow reefs. *Journal of Geophysical Research*, 100(C2), 2639–2648.
- VELLINGA, P., 1983. Predictive computational model for beach and dune erosion during storm surges. *Proceedings* (ASCE), *Specialty Conference on Coastal Structures'83*, pp. 806–819.
- WRIGHT, L.D.; BOON, J.D.; KIM, S.C., and LIST, J.H., 1991. Modes of cross-shore sediment transport on the shoreface of the middle Atlantic Bight. *Marine Geology*, 96, 19–51.
- YU, X.; ISOBE, M., and WATANABE, A., 1995. Wave breaking over submerged horizontal plate. *Journal of Waterway, Port, Coastal and Ocean Engineering*, 121(2).
- ZHENG, J. and DEAN, R.G., 1977. Numerical models and intercomparisons of beach profile evolution. *Coastal Engineering*, 30, 169–201.