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Beach Cusps: A Comparison of Data and Theories for Their Formation

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ABSTRACT

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In recent years, there has been considerable debate about whether beach cusp formation is associated with the presence of standing edge waves, results from self-organising feedback between changing topography and swash motion or is attributable to a number of other less popular mechanisms.

In this paper we utilise a large amount of data from laboratory experiments and field studies (from lakes and sea coasts, and from calm to storm conditions) published over the last 50 years to test the predictions of the two main cusp forming hypotheses. After a review of the relevant theories, comparison is made between measured cusp spacing and edge wave wavelength in order to test the edge wave theory. The self-organisation theory is examined by considering the variation of cusp spacing with some metric describing a swash length. These analyses, using more data than previous attempts, confirm that there is a possible link between cusp development and both edge waves and swash-sediment feedback, and that it is not possible to produce conclusive support for one theory above the other with the simple measurements that have been made previously.

Furthermore, we report evidence for a specific breaker type (plunging) being associated with cusp presence and suggest that if sub-harmonic standing edge waves are present on a beach (which does not seem to be likely for a large number of the measurements considered, and lead to cusp development) there will necessarily be a link between their spacing and swash length in the form predicted by Werner and Fink (1993).

ADDITIONAL INDEX WORDS: *Beach cusps, self-organisation, edge waves, swash length.*

INTRODUCTION

On many beaches, particularly steeper beaches with coarse-grained sediments, regular cusp-like features stretching for large distances along the shoreline are commonly observed. Despite the interest that has been shown in the development of beach cusps, there is still considerable debate on the fundamental process(es) underlying their formation. Several theories exist which attempt to explain or suggest why swash cusps form with particular along-shore spacings, including the formation of cusps by standing edge waves (*e.g.* KOMAR, 1973; GUZA and INMAN, 1975; INMAN and GUZA, 1982), feedback between topography and fluid motions (RUSSELL and MCINTIRE, 1965; DEAN and MAURMEYER, 1980; WERNER and FINK, 1993), instability of breaking waves (CLOUD, 1966), instability of littoral drift (SCHWARTZ, 1972), the presence of velocity salients (GORYCKI, 1973) and intersecting wave trains (DALRYMPLE and LANAN, 1976). All but the first two of these theories have generally been ignored on the basis of poor agreement with observed cusps in most situations, so that hypotheses of beach cusp formation centre on the question of linking cusps either to standing edge waves or to sediment-wave feedback. It should be noted here that given the occurrence of cusps in a wide range of conditions, it is possible that both mechanisms are viable or, indeed, may both occur together.

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Most studies of beach cusps have attempted to relate cusps to standing edge waves. Edge waves (ECKART, 1951; URSELL, 1952; BOWEN and INMAN, 1971) are free modes of near-shore water motion trapped against the shoreline by refraction, and have amplitudes that decay exponentially offshore and vary sinusoidally along-shore.

When standing edge waves are considered to act in superposition with the incident waves, the result is a regular longshore variation of the wave run-up on a beach. According to the edge wave theory for cusp formation, this variation is associated with the cusp spacing or wavelength (λ_c). For synchronous edge waves (edge wave time period T_e equal to the incoming wave time period T_i one obtains $\lambda_c = L_e$ (where L_e is the edge wavelength), while for sub-harmonic edge waves (edge wave time period T_e equal to twice the incoming wave time period T_i) $\lambda_c = L_e/2$. Thus for mode zero edge waves, the following simple relationships between cusp spacing and incident wave and beach characteristics exists:

$$\lambda_c = m\frac{g}{\pi}T_i^2 \sin \beta \tag{1}$$

with m = 1 and m = 0.5 for sub-harmonic and synchronous edge waves, respectively. The reader is referred to INMAN

Table 1. *Summary of parameters for field data (for notes see bottom of Table 2).*

Author	tan β		D (mm)		λ (m)	H (m)	T (sec)	S (m)
Krumbein (1947)[a]	0.179[b]		0.83[b]		66.6	1.84	9.3	
	0.188[b]		0.60[b]		54.6	1.85	9.8	
	0.099[b]		0.51[b]		52.5	1.71	8.8	
	0.124[b]		0.34[b]		52.2	1.58	9.9	
	0.179[b]		0.23[b]		52.5	1.37	8.7	
	0.198[b]		0.90[b]		44.7	1.88	12.5	
	0.146[b]		0.51[b]		44.1	1.88	10.7	
	0.204[b]		0.27[b]		45.9	1.40	11.1	
Longuet-Higgins & Parkin (1962)								
12-4-56	0.134		2.54		4.9		3.7	
23-4-56	0.157				2.8		2.85	
24-4-56	0.114				2.8		3.55	
9-6-56	0.2		0.254–2.54		4.3	0.46	6.2	1.82
16-6-56	0.16				5.5	0.76	6.1	5.79
21-6-56	0.14				4.3	0.30	5.4	3.66
22-6-56	0.14				3.7	0.23	6.2	1.98
23-6-56	0.14				4.0	0.08	6.85	1.37
6-7-56	0.1				10.0	1.07	5.0	13.72
8-7-56	0.09				6.4	0.46	6.3	6.10
9-7-56	0.09				6.4	0.38	6.3	5.18
10-6-56	0.1				8.8	0.76	6.2	10.67
King (1972)	0.045		horn	bay	14.8		9.5	
			0.346	0.277				
Williams (1973)								
2-10-69	0.105		0.1–2		5.6	0.15	3.5	3.5
13-1-70	0.094				4.4	0.15	5	4.5
19-1-70	0.096				7.0	0.10	6	4
21-1-70	0.096				7.3	0.15	6	4
22-1-70	0.144				6.0	0.15	4	5
2-2-70	0.135				14.2	0.20	5	6
3-2-70	0.156				10.0	0.10	5.5	4
14-4-70	0.1				11.8	0.10	3.5	5.5
25-6-70	0.137				3.8	0.25	4	2
25-7-70	0.144				4.5	0.15	6	2
7-9-70	0.094				6.0	0.15	9	4
2-12-70	0.149				5.6	0.15	7.5	4
19-1-71	0.138				6.4	0.10	10	5.5
1-6-71	0.124				4.8	0.25	4	2
Komar (1973)	horn	bay	horn	bay	0.304		1.0	
May-1971		0.087	2.19					
21-6-71	0.123	0.07			0.322		1.0	0.13
22-6-71	0.154	0.07	1.15	0.76	0.591		1.4	0.20
29-6-71		0.07	2.30	0.52	0.292		2.0	
8-7-71	0.092	0.087			0.229		1.0	0.18
15-6-72	0.128	0.07	0.49	0.23	0.11		1.3	0.21
17-6-72	0.056	0.056	1.60	1.21	0.21		0.9	0.22
15-5-73	0.105	0.105	1.41	0.91	0.275		1.0	0.15
17-5-73	0.114	0.114	1.20	0.87	0.207		0.77	0.15
Darbyshire (1977)[a]								
18-7-75	0.062[b]				20.0	1.3	8.5	
20-8-75	0.088[b]				22.85	1.5	12.5	
					11.42	0.5	8.3	
6-2-76	0.053[b]				29.0	1.5	19.5	
5-9-75	0.034[b]				6.0	0.4	11.75	
26-2-76–3-3-76	0.049[b]				12.0	1.5	13.33	
21-5-76	0.059[b]				13.7	5	8.0	
Dubois (1978) 10-8-76	horn	bay	0.31		35.0	0.6	10	
	0.146	0.1						
Dubois (1981) 5–28-6-79	horn	bay	0.33		32.5	1.3	7–11	
	0.096	0.078						

Table 1. *Continued.*

Author	tan β	D (mm)	λ (m)	H (m)	T (sec)	S (m)
Sallenger (1979)	0.07		12.3	0.15–0.20	6.5	
	0.07		10.9	same	6.5	
	0.06		8.6	same	6.5	
	0.111		5.4	0.06–0.08	3.9	
	0.091		5.4	same	3.9	
	0.099		3.6	same	3.9	
	0.16		0.7	few cms	2.3	
	0.099		0.7	few cms	2.3	
Huntley & Bowen (1978)	0.081	sand	12.7	0.12	6.9	7.8
Dean & Maurmeyer (1980)	0.04	0.2	23.2	0.335	15.4	12.5
Guza & Bowen (1981)	0.075		12.6		7.14	
Takeda & Sunamura (1983)[a]	0.120[b]	0.34	35	1.30	8.0	15
	0.097[b]	0.34	20	0.66	6.3	14
	0.083[b]	0.30	28	0.98	9.2	12.5
	0.106[b]	0.30	22	0.74	6.5	8.5
	0.097[b]	0.30	16	0.66	6.3	8
	0.053[b]	0.28	21	0.98	9.2	11
	0.109[b]	0.28	15	0.76	8.4	15
	0.126[b]	0.28	25	0.86	7.8	15
	0.060[b]	0.28	22	1.48	9.8	18
	0.091[b]	0.28	17	0.74	6.5	12
	0.162[b]	0.28	15	0.66	6.3	11
	0.092[b]	0.28	19	0.48	5.6	12.5
Orford & Carter (1984)	0.1–0.13	0.176–1.0	42.5 61.3	<10	9–11.5	
Seymour & Aubrey (1985)	0.05	0.23	40	0.60	16	
Miller et al. (1989)	0.07–0.105	sand	36.0	5 (max)	10–12	
Sherman et al. (1993)	0.169	0.15–0.25 20–250	20–25	0.41	6.5	
Allen et al. (1996)	0.119	0.47	27.5	0.48	16.7	17
Holland & Holman (1996)						
15-10-94	0.083	0.35	36	3.1	10.9	
16-10-94			29	3.5	11.2	
17-10-94			32	2.1	10.6	
18-10-94			36	1.5	11.5	
19-10-94			40	1.3	14.2	
20-10-94			37	1.2	13.5	
21-10-94			20	0.9	11.3	
Masselink & Pattiaratchi						
8-3-95	0.10	0.4	30	0.3	10	15
10-2-96	0.10	0.4	30	0.55	10	18
14-2-96	0.12	0.5	20	0.4	11	10

and GUZA (1982) for a more detailed explanation of standing edge wave theory in relation to beach cusp formation.

WERNER and FINK (1993) modelled swash motion as a series of slabs of sediment-laden water which run up the beachface with pre-defined (but essentially arbitrary) angle and initial velocity. Unlike the spectrum of waves found normally in the real world of field study, there is no interaction between one swash cycle and another. The subsequent trajectory of these swash particles is determined by the local slope of the beach, which directs the down-slope acceleration due to gravity. The sediment carrying capacity is considered to be dependent on the flow kinetic energy such that the accelerations and decelerations will result in erosion and deposition of sediment. Incipient topographic depressions are amplified by attracting and accelerating water flow, thereby enhancing local erosion, while topographic highs are regions that repel and decelerate the fluid leading to further deposition. Their model also included the influence of local pressure gradients arising from the convergence and divergence of swash particles during swash motion.

Simulations with non-uniform (random) initial water particle velocities resulted in the development of regular cusp systems in which the cusp spacing was observed to be proportional to the size of the cross-shore swash length S:

$$\lambda_c = fS \quad (2)$$

where f is a constant having a value between 1 and 3 depending upon the exact algorithm used in the simulations. They show that this relationship agrees with earlier findings obtained from a much simpler analytical model (DEAN and MAURMEYER, 1980) and from laboratory and field measurements (TAKEDA and SUNAMURA, 1983).

Following Werner and Fink's work, renewed interest in beach cusps led several authors to attempt to provide a comparison of the two theories for explaining observed measurements on a natural beach as the cusps were forming (ALLEN et al., 1996; HOLLAND and HOLMAN, 1996; MASSELINK and PATTIARATCHI, 1998). The results of these studies taken together have not proved conclusive. This is not surprising given Werner and Fink's own assertion that "remarkably, these two incompatible, physically distinct mechanisms lead to similar predictions for spacing and formation conditions of beach cusps over a wide range of scales" observed so far.

In this paper we draw together data from previously published studies of beach cusp development both in the field and in laboratory conditions, though a compatibility problem between the two sources of measurements arises. In fact it is clear that the model by WERNER and FINK (1993) should better fit laboratory conditions where the swash length is essentially constant while for field observations one is forced to refer to the statistical representation of the swash spectra given by each author. Measurements of cusp spacing from these studies are compared with the predictions of the main theories proposed for cusp development (equations 1 and 2) in an attempt to see whether the sum of our knowledge of cusp systems enables us to say if there is a "best" theory for cusp formation. On the basis of these comparisons and of some results following an analysis of the breaker type associated with cusps, we make some recommendations for the directions of future research in this area.

DATA ANALYSIS

In order to examine the applicability of the standing edge wave and self-organisation models for beach cusp development, comparisons are made between measured cusp spacings, and cusp spacings predicted by equations 1 (edge wave theory) and equation 2 (self-organisation). Similar comparisons for the standing edge wave theory but with less data have been made previously by INMAN and GUZA (1982), TAKEDA and SUNAMURA (1983), and KANEKO (1985). Of these studies, only TAKEDA and SUNAMURA (1983) also compared measured cusp spacings with swash length. Such comparisons require data giving the following measured parameters—cusp spacing, incident wave period, beach gradient and swash length.

After an extensive search through relevant literature, data from 26 previously published papers were selected as being suitable for use here. It is important to note that not all the data sets include measurements of swash length, restricting their use only to the examination of the standing edge wave theory. In addition, we rely heavily on the original author's interpretation of how to measure the parameters, with the beach gradient being the most problematic. In some cases only a single gradient figure is given and we have taken this to be representative of the overall beach slope, whereas other authors quote gradients for cusp horns and bays, which we have averaged to give an overall gradient value (note that throughout the analysis the approximation $\tan\beta \approx \sin\beta$ has been used). Another problem arising when trying to test the self-organisation theory is related to the statistic describing the swash length. ALLEN et al. (1996) clearly pointed out the difficulties related to the evaluation of a significant measure of the swash length as related to the process of beach cusp formation, but ended up using what they defined as a "typical" value. Other authors clearly reported the value of the mean swash length (LONGUET-HIGGINS and PARKIN, 1962; DEAN and MAURMEYER, 1980) while for the other cases it is only possible to hypothesise that "typical" values are reported rather than the maximum length. Finally, it should be mentioned that some of the data were obtained by reading values from graphs and so are potentially less accurate. Nevertheless, the full data set provides a fairly complete picture of the field (Table 1) and laboratory (Table 2) measurements of beach cusp spacing made over the past 50 years and usefully updates previous similar comparisons. Also listed in the tables are the incident wave height and the mean grain diameter of the sediment present though again it must be stressed that for most of the observations the location of the sediment samples is not clearly stated.

RESULTS

Standing Edge Wave Theory

For the standing edge wave theory to be valid, a graph of measured cusp spacing (λ_c) against the parameter given by (1) should be a straight line with zero intercept and gradient of 1 or 0.5 depending upon whether sub-harmonic or synchronous mode zero edge waves are associated with the cusps. This graph is shown in Figure 1, along with lines which show the expected relationship for cusps due to both sub-harmonic (solid) and synchronous (dashed) edge waves. The graph is plotted on log-log scales to allow easy visualisation of the 151 data points over three orders of magnitude and, although the use of log-log scales visually compresses much of the data variability, it reveals a relationship between cusp spacing and the edge wave parameter given by eq. 1 (a least square fit to quantify the behaviour of the edge wave parameter results in a regression coefficient equal to 0.72). There is a cluster of data points for which both edge wave relationships over-predict the cusp spacings, but the vast majority of the data lie close to one or other of the lines, and within a margin of error which is reasonable considering the inaccuracies inherent in estimating the various parameters (particularly beach gradient). It is less clear whether the data is better represented by the sub-harmonic or synchronous relationship, but close inspection of Figure 1 suggests that the sub-harmonic edge wave relationship provides the best description for the majority of the data.

Table 2. *Summary of parameters for laboratory data.*

Author	tan β	D (mm)	λ (m)	H (cm)	T (sec)	S (m)
Longuet-Higgins & Parkin (1962)	0.185	fine sand	0.533	1.7	1.5	0.254
Guza & Inman (1975)	0.105	fine sand	1.8		2.4	
	0.105	fine sand	1.0		2.7	
			0.025			
Dalrymple & Lanan (1976)	0.163	sand	1.14	1.25	1.04	
	0.146	sand	1.32	1.25	0.96	
			1.05		0.92	
			1.05		0.90	
Ann (1979)[a]	0.106[b]	0.2	0.20	0.030	0.85	
	0.100[b]	0.2	0.17	0.028	0.78	
	0.106[b]	0.2	0.24	0.041	0.86	
	0.100[b]	0.2	0.24	0.041	0.78	
Sunamura et al. (1977)[a]	0.102[b]	0.20	0.13	0.013	0.56	
	0.102[b]	0.20	0.12	0.015	0.56	
	0.102[b]	0.20	0.16	0.020	0.56	
	0.102[b]	0.20	0.15	0.022	0.56	
	0.102[b]	0.20	0.16	0.028	0.56	
Tamai (1980)[a]	0.106[b]	0.28	2.21	0.075	2.20	
	0.106[b]	0.28	1.73	0.126	2.19	
	0.101[b]	0.28	128	0.113	1.78	
	0.109[b]	0.28	1.40	0.128	1.80	
Guza & Bowen (1981)	0.105	sand	1.3		2.7	
Takeda & Sunamura (1983)	0.1	0.69	0.34	0.042	1.0	0.20
	0.1	0.69	0.20	0.020	1.0	0.10
Kaneko (1985)	0.081	0.0028	1.5	3.4	2.38	
	0.081	glass beads	0.75	3.1	1.72	
	0.081	of ρ = 2.43	0.50	2.7	1.71	
	0.081		0.75	3.3	1.58	
	0.081		0.50	4.0	1.27	
	0.081		0.38	3.5	1.15	
	0.081		0.30	3.5	0.85	
	0.107		0.30	3.5	1.11	
	0.107		0.38	3.2	1.06	
	0.107		0.25	3.3	1.01	
	0.107		0.30	3.4	0.98	
	0.107		0.30	3.0	0.94	
	0.107		0.25	3.0	0.88	
	0.107		0.21	3.5	0.76	
	0.118		0.75	3.6	1.38	
	0.141		0.75	2.6	1.38	
	0.141		0.38	4.2	0.98	

Notes: [a] Edge wave wavelength taken from graph or table; [b] values estimated with formulae; tanβ = beach slope; D = mean diameter; λ = cusp spacing; H = wave height; T = wave period; S = swash length.

Self-Organisation Theory

According to Werner and Fink's self-organisation theory, and earlier work by DEAN and MAURMEYER (1980), cusp spacings should be directly related to the observed swash length. Thus a graph of cusp spacing against some estimate of swash length should be a straight line with zero intercept and a gradient in the range 1–3 (according to Werner and Fink), and 1.5 (according to Dean and Maurmeyer). This graph is shown in Figure 2 (again log-log scales are used to draw together 51 data points over three orders of magnitude) where evidence of a strong relationship between observed cusp spacing and swash length is given:

$$\lambda_c = 1.63\ S \qquad (R^2 = 0.83) \qquad (3)$$

By considering Figure 1 and 2 it is also possible to analyse whether significant discrepancies exist between the laboratory and field data. As regards the applicability of the edge wave theory no significant discrepancy exists, even though the laboratory data set is clustered along the sub-harmonic prediction. For the case of swash length only three points refer to laboratory experiments so that no conclusion may be drawn (see Figure 2).

Breaker Type

The data also allow us to determine the breaker types associated with the presence of cusps. In order to classify and distinguish between different breaker types the following inshore parameter defined by GALVIN (1968) has been used:

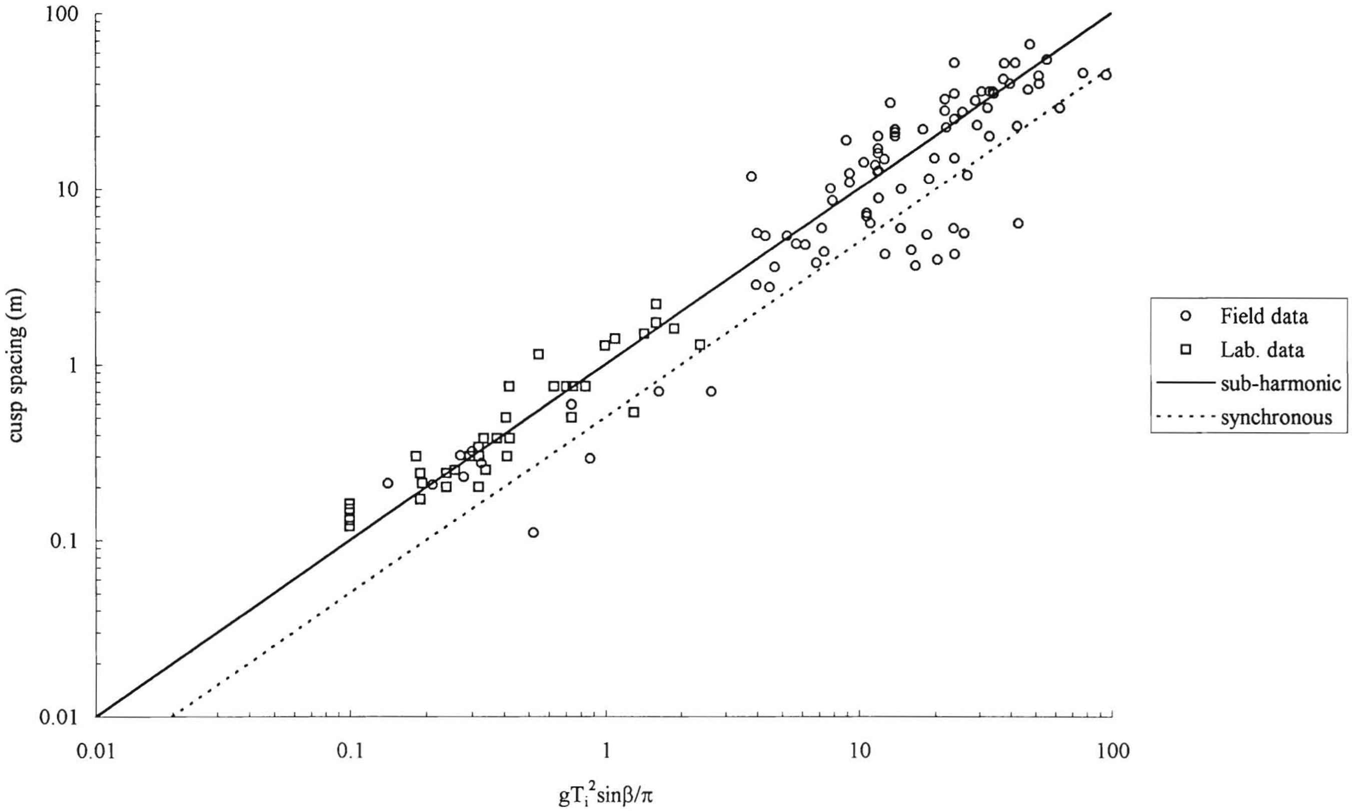

Figure 1. Comparison of measured cusp spacing with sub-harmonic and synchronous mode zero edge wave wavelength.

$$\frac{H_b}{gT_i^2 \sin \beta} \tag{4}$$

with the values of 0.003 indicating the transition from surging to plunging breaker and 0.068 the transition from plunging to spilling. Even before showing the results it must be again underlined how this analysis is potentially biased by the kind of data available as it is not always possible to assess whether the wave height reported is related to the breaking or to deep water measures or to the sensor position. The following analysis has been performed by considering the wave height measurements given by the different authors equal to breaking height. An estimation of the likely error has been made by considering exactly the opposite case, namely treating the measures given by the authors as referring to deep-water wave heights, and evaluating the breaker wave height through an expression given by TAKEDA and SUNAMURA (1983). Results indicate a potential error that, at worst, is around 75% (on average the error ranges between 40–60%) of the deep-water wave height. Such a variation does not substantially affect the results. Results shown in Figure 3 clearly identify plunging breakers as the predominant type associated with cusp presence. The measures well into the surging region all refer to WILLIAMS (1973).

DISCUSSION

Considering the complete data set it would appear that there are reasonably strong relationships between cusp spacing and the edge wave parameter (eq. 1) *and* swash length (eq. 2). Thus the data would appear to provide some support for both the standing edge wave theory and the self-organisation theory.

Taken on its own Figure 1 provides support for the standing edge wave theory. However, it should be noted here that edge waves characterised by a time period and mode number that, according to the theory, could have caused the measured cusp spacing were not explicitly observed in any of the studies from which the data are taken. Indeed, in one case (HOLLAND and HOLMAN, 1996), the presence of synchronous or sub-harmonic mode zero edge waves was convincingly ruled out. In the absence of direct evidence for edge wave motion, it is possible to assess the like-hood of sub-harmonic edge wave excitation by using the following parameter (GUZA and INMAN, 1975):

$$\epsilon = \frac{4\pi^2 a}{gT_i^2 \sin^2\beta} \tag{5}$$

with a equal to the breaking wave amplitude.

As a result of a series of laboratory experiments GUZA and

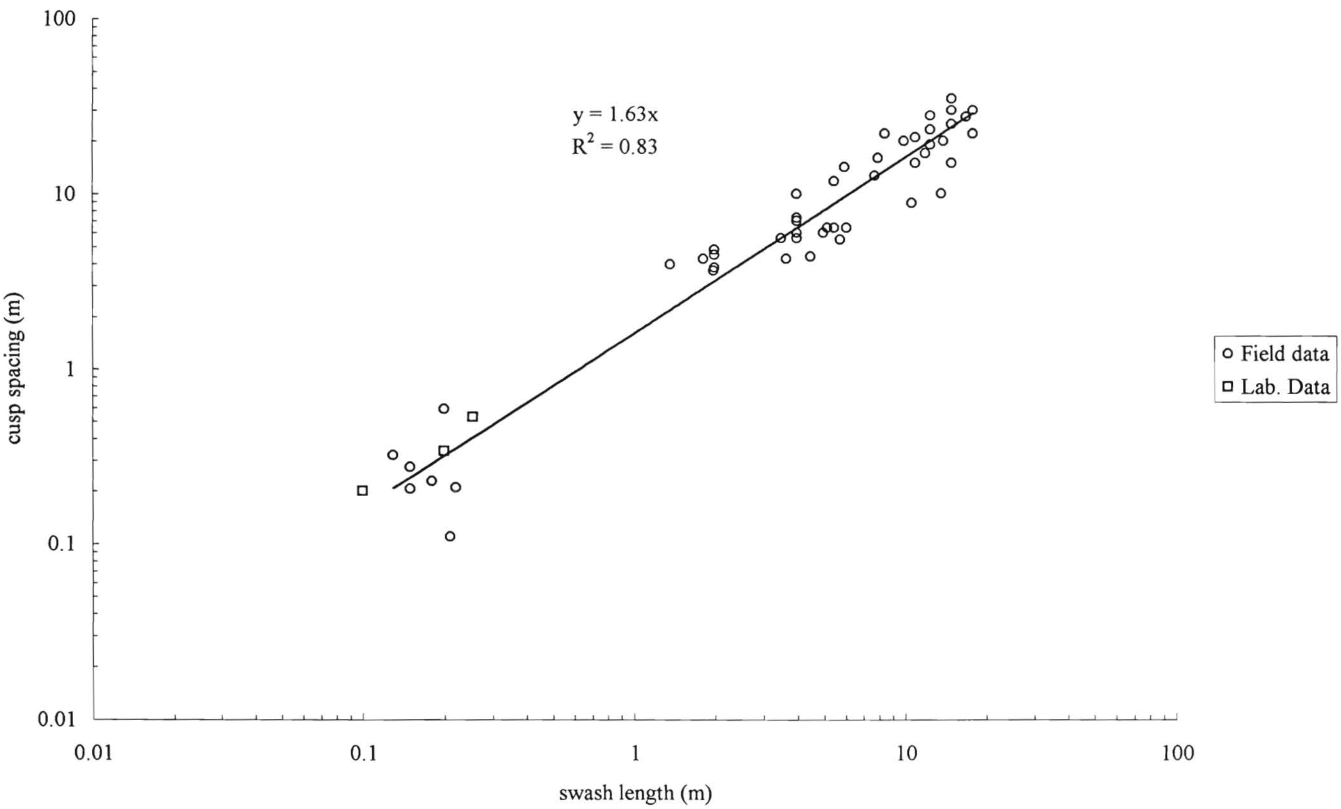

Figure 2. Variation of measured cusp spacing with swash distance.

INMAN (1975) suggested that if this parameter is bigger than 2, sub-harmonic edge wave excitation is weak or non-existent. This parameter has been plotted on Figure 3 (dotted line) and, although it must be considered as only a semi-quantitative criterion, it is evident that for a large number of the observed cusps, edge waves are not, on this basis, expected to exist. In fact ϵ would have to be as large as 18 in order to encompass all of the observations.

Figure 2, on the other hand, appears to provide support for the self-organisation theory. The question then arises as to why the cusp spacing appears to show a close correspondence to both parameters. In fact, WERNER and FINK (1993) suggested that if the swash is saturated when sub-harmonic edge waves occur, then the two mechanisms will be indistinguishable. If one considers an amplitude corresponding to the shoreline amplitude:

$$2a = S\sin\beta \qquad (6)$$

it follows that, for saturated swash, the value of ϵ (eq.5) ranges between $2 < \epsilon < 12$ (GUZA and BOWEN, 1976; VAN DORN, 1978; GUZA and THORNTON, 1982). Combining equation (1) with $m = 1$ (sub-harmonic case) with eq. 5 and 6, one obtains a simple relationship between the cusp spacing and swash length of the same form as equation (2), but with the constant, f, lying in the range $0.52 < f < 3.14$ depending on the exact value of ϵ chosen. This range has a mean value of 1.8 such that in conditions of saturated swash the standing sub-harmonic edge wave theory *would be expected* to give a relationship between cusp spacing and swash length in broad agreement with equation (3) as found here. In short, the agreement between variables shown in Figure 2 is consistent with that shown in Figure 1 if edge waves occur in saturated swash, and so a linear relationship between cusp spacing and swash length is not sufficient indication of the validity of the self-organisation theory. However, neither is the agreement shown in Figure 1 conclusive proof of the standing edge wave theory since in almost all cases it is not known whether edge waves were actually present. We believe that our analysis shows that it is simply not possible to find beach cusps in the field or generate them in the laboratory and make *simple* measurements which provide support for one theory above another.

Instead, beach cusp research is in desperate need of three-dimensional observations of *evolving* cusps along with detailed measurements of the surrounding fluid flows sufficient to establish what role, if any, edge waves play in determining the length scale of beach cusps, and, if none, what other factors, within the self-organisation mechanism, can create these length scales.

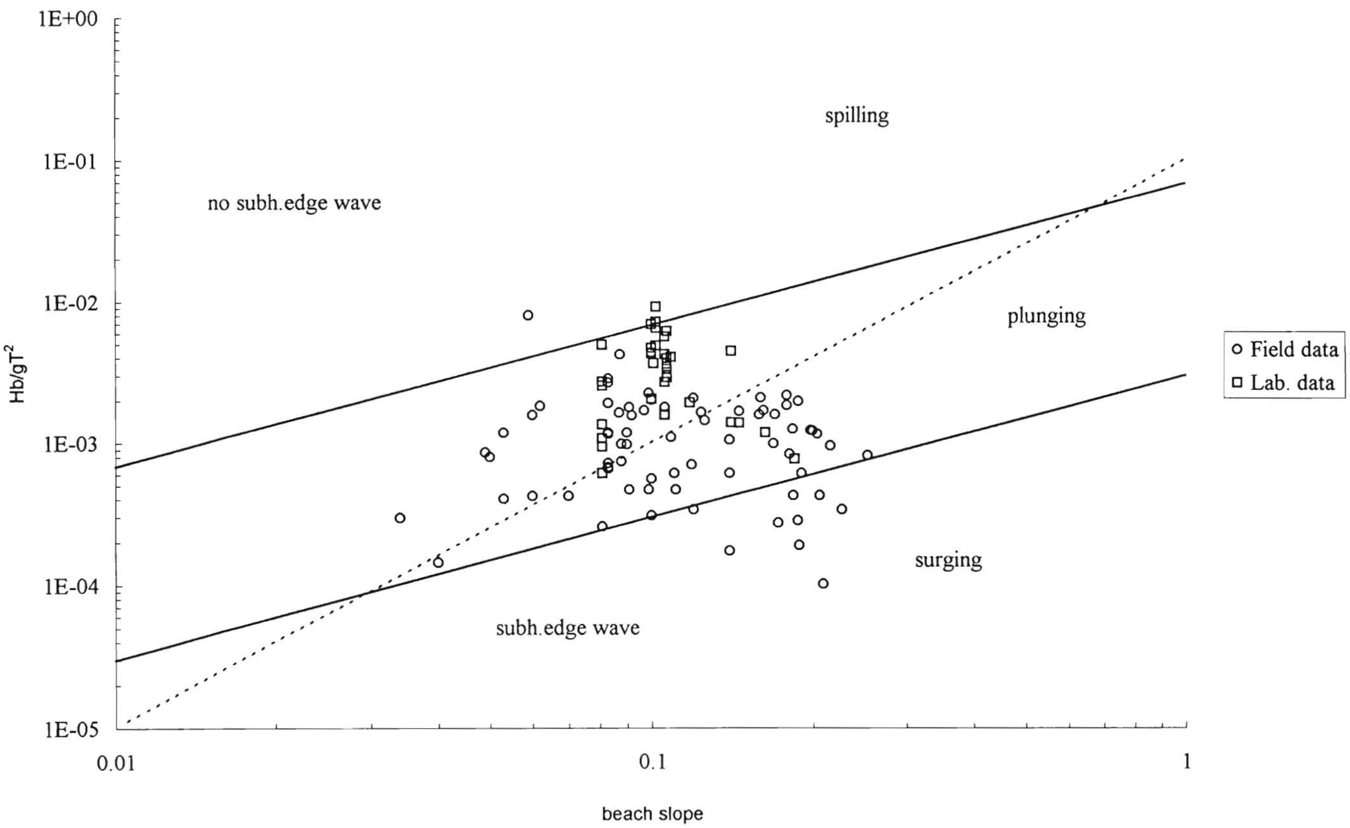

Figure 3. Breaker type and cusp existence.

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