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DISCUSSION

Discussion of: E. Robert Thieler, Orrin H. Pilkey, Jr., Robert S. Young, David M. Bush, and Fei Chai, 2000. The Use of Mathematical Models to Predict Beach Behavior for U.S. Coastal Engineering: A Critical Review. *Journal of Coastal Research*, 16(1), 48–70.

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THIELER *et al.* (2000) investigated a number of mathematical models used to predict beach behavior and concluded that such models should not be used for coastal engineering design because of invalid model assumptions. One of the models that was evaluated and judged to be incorrect was the Bruun Rule. They reported that "The Bruun Rule . . . provides the basis for most mathematical models used to predict the rate of shoreline retreat due to sea-level rise (*e.g.*, . . . DUBOIS, 1990; 1992; . . .)" (p. 58); . . . "geologists have embraced the Bruun Rule; the concept remains the basis for a number of quantitative models of coastal evolution (*e.g.*, . . . DUBOIS, 1976; 1990), and is used to predict coastal evolution in a variety of settings (*e.g.*, DUBOIS, 1975; 1976; 1992; . . .)" (p. 59). From 1990 to the present, THIELER *et al.* (2000) have misread the results of my work concerning the response of a sandy shore when subjected to a relative rise in sea-level. Since 1990 I stopped embracing the Bruun Rule in favor of a transgressive barrier model (DUBOIS, 1990; 1992; 1995; 1997). I am writing (a) to clarify the difference between the Bruun Rule and the transgressive barrier model, (b) to note that an element of the Bruun Rule may be applicable to the nearshore, and (c) to ask for clarification of comments made regarding mathematical models of beach behavior.

To preserve the shape of an equilibrium shoreface profile when subjected to a relative sea-level rise, BRUUN (1962) postulated that coastal processes erode beach and upper shoreface sediments and deposit these materials on the lower shoreface in order to elevate the offshore bottom in proportion to a sea-level rise (Figure 1). The volume of eroded sediments (V) is equal to the volume of deposited sediments (V′). V is given as

$$V = WY, \tag{1}$$

and V′ as

$$V' = XS, \tag{2}$$

where W is the rate of beach erosion, Y is the vertical height from the foredune crest to the limited water depth of sediment transport, which is about 16–18 m for a hundred year time frame (BRUUN, 1988), X is the horizontal length from the foredune crest to the limited water depth of sediment transport, and S is the rate of sea-level rise. Setting equation (1) to equal (2), W is solved as

$$W = XS/Y. \tag{3}$$

Note that a mathematical term representing a physical explanation of why beach erosion should occur in response to a relative sea-level rise is not included in the Bruun Rule.

Although the Bruun Rule requires the base of a shoreface profile to aggrade in proportion to a sea-level rise, geologists have noted that a beach-shoreface profile transgresses as sea-level rises during a relatively long time frame (HOYT, 1967; DILLON, 1970; KRAFT *et al.* 1973; SWIFT, 1975; KOCHEL and DOLAN, 1986). To accommodate the concept of a transgressing beach-shoreface profile, a new mathematical model was proposed by DUBOIS (1990). An assumption was incorporated into the transgressive barrier model to explain why a shoreface should erode in response to a relative sea-level rise, and from this assumption, an equation was formulated to predict the rate of beach erosion. The assumption and the equation are presented as follows.

The transgressive barrier model (Figure 2) assumes that as sea-level rises (S) the depth of initial shoreface forcing (D_1) located at the shoreface-ramp juncture—where the bottom slope begins to increase landward—is elevated by a distance equal to a sea-level rise and displaced horizontally landward from position X_1 to X_2. With D_1 now at X_2, the shoreface profile becomes shorter and steeper, and the rate of wave-energy dissipation increases along the bottom (BRUUN, 1988), causing wave action to erode the bottom until the length and shape of the shoreface profile are readjusted to the rate of wave-energy dissipation. The horizontal displacement of D_1 (ΔX_1) is equal the rate of beach erosion (ΔX). A shorerise profile—the shoreface segment located between the nearshore and ramp—is described by a power function (BRUUN, 1988) in the form of

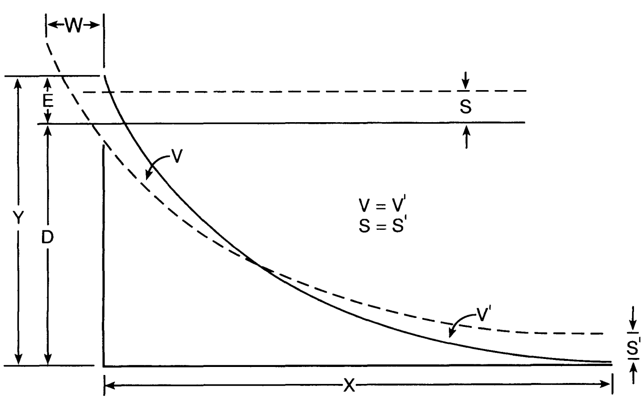

Figure 1. A beach-shoreface profile responding to a relative sea-level rise according the Bruun Rule (after DUBOIS, 1992). To preserve the shape of a transgressing shore profile, the volume of sediment eroded from a beach and upper shoreface is equal to that deposited on a lower shoreface.

$$D = AX_s^m, \quad (4)$$

where D is the water depth, X_s is the horizontal distance that begins at the shoreward edge of a shorerise and extends seaward to a referent depth, A and m are empirical coefficients (DUBOIS, 1999), although BRUUN (1988) theorized that m is equal to ⅔ for equilibrium profiles. Based on relatively large coefficients of determination, a good fit exists between equation (4) and observed shorerise profiles located along the Atlantic coast of Long Island, New York, and along the Gulf of Mexico coast of Mustang-Padre Island, Texas (DUBOIS, 1999). Given the empirical coefficients of A and m, and a rate of relative sea-level rise (S), which can be obtained from tidal stations, ΔX is estimated as

$$\Delta X = (D_1/A)^{1/m} - [(D_1 - S)/A]^{1/m}. \quad (5)$$

Equation (5) was applied to 43 profiles off of Long Island, New York (DUBOIS, 1995). The average ΔX for these profiles was 0.67 m/yr, which was within the reported range of 0.3 to 0.9 m/yr (LEATHERMAN and ALLEN, 1985) for this New York shoreline. However, one should not draw the conclusion that a relative sea-level rise is the sole driving force that governs observe retreat rates. The rate at which a shoreline changes its horizontal position is controlled by the sum effects of (a) the rate of sea-level change, (b) the change in the wave-climatic regime, and (c) the difference between the sediment discharge entering and leaving a referent shoreline.

The total sediment volume (V_t) eroded from a transgressing shore profile is given as

$$V_t = (\Delta X\ Y) - (XS), \quad (6)$$

where Y is now the vertical height between the foredune crest and the shoreface-ramp juncture, and X is the horizontal length between the foredune crest and the shoreface-ramp juncture (DUBOIS, 1995). For the equation that estimates the volume of washover deposits, see DUBOIS (1995; 1996).

The similarities between the Bruun Rule and the transgressive barrier model are that both models predict beach erosion occurs when an equilibrium shore is subjected to a relative sea-level rise, and both assume the shape of a cross-shore profile remains reasonably constant during transgression. The principle difference between both models is that the transgressive barrier model does not postulate where eroded beach-shoreface sediments will be deposited (Figures 1 and 2). In equation (6) V_t can be any number for the transgressive barrier model whereas for the Bruun Rule, V_t must be zero if ΔXY is to equal XS. However, if a shoreface slope is monotonic, then no deposition is required on any part of a profile in order to maintain its shape during transgression (Figure 2). Should overwash occur, then some sediments of V_t will come to rest as washover deposits (Figure 3). Some other sediments could be swept seaward by storm-driven downwelling currents and deposited on a ramp (Figure 3); but ramp aggradation is not mandatory to preserve the shape of a transgressing shoreface profile (Figure 2 and 3) (DUBOIS, 1995). In addition, some sediments should be transported downdrift by longshore currents and deposited at a shoreline terminus (DUBOIS, 1995; 1996). Finally, traversing shoreward from a shoreface-ramp juncture, the bottom slope gradually increases. In the nearshore, however, if the bottom slope decreases

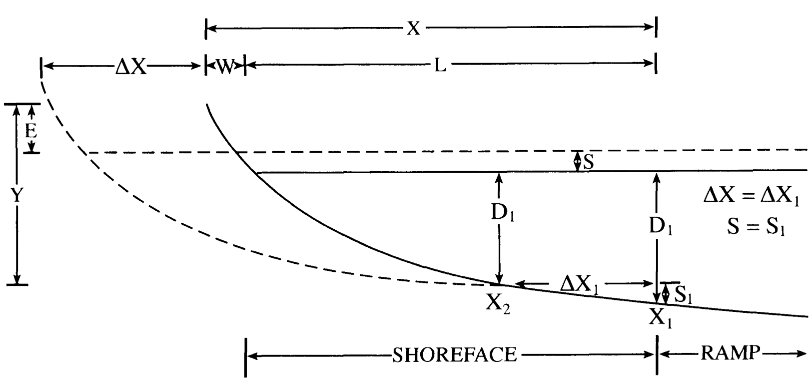

Figure 2. A beach-shoreface profile responding to a relative sea-level rise according to the transgressive barrier model (after DUBOIS, 1990). For this model with a monotonic curved profile, no deposition is required to elevate any part of a profile as a shore transgresses.

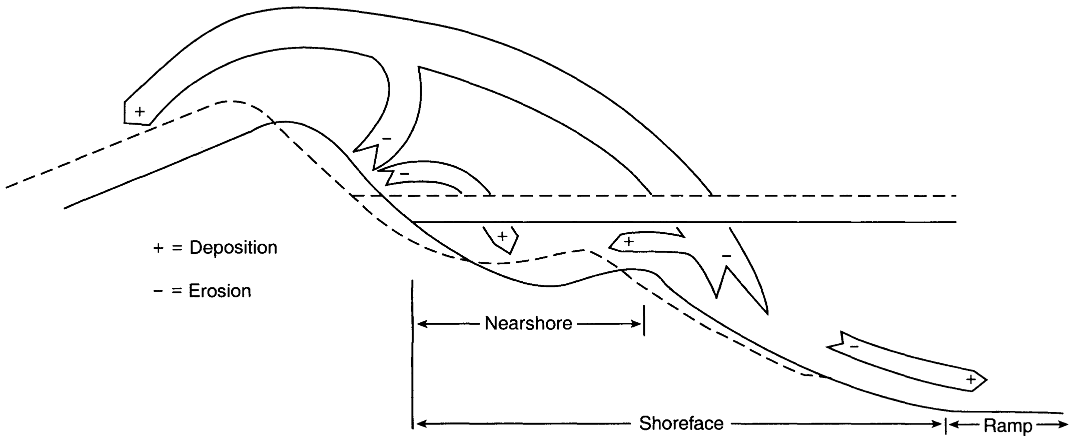

Figure 3. An extended two-dimensional view of the transgressive barrier model. Arrows show potential directions of sediment transport (after DUBOIS, 1992). Note that the shoreface slope increases as one traverses landward from the shoreface-ramp juncture to the seaward edge of the nearshore; upon reaching the nearshore, the slope decreases and reverses direction. Consequently, as sea-level rises, the nearshore aggrades while the lower shoreface transgresses. An element of the Bruun Rule may be applicable when confined to the nearshore zone (DUBOIS, 1997).

or reverses direction, then to preserve the shape of a nearshore bottom, some sediments will be deposited to elevate the bottom in proportion to a sea-level rise (Figure 3). The sediment volume deposited in the nearshore would be a small fraction of the total volume eroded from a transgressing beach and shoreface. In conclusion, there is a significant difference between the Bruun Rule and the transgressive barrier model because the models are based on different assumptions; therefore, each model employs its own equations to predict the rate of beach erosion and the volume of displaced beach-shoreface sediments. For more information about my assessment of the Bruun Rule consult DUBOIS (1992; 1995).

THIELER *et al.* (2000) are correct to note that equation (3) should not be used because the model assumptions are inaccurate. There is no need to aggrade the lower shoreface in order to preserve the shape of a transgressing shoreface profile, and the volumetric exchange of beach and shoreface sediments is not confined to just two cross-shore compartments (DUBOIS, 1995). On the other hand, contrary to what THIELER *et al.* (2000) have concluded about the Bruun Rule, an element of the Rule has value when applied to the nearshore where a bottom slope is gentle compared to the landward edge slope of a shorerise. As noted from results of an empirical study conducted at a shore segment along Lake Michigan (DUBOIS, 1975; 1976; 1977), a rising water level accompanied by varying wave energies induced beach erosion; the sediment volume eroded from a beach was about equal to the sediment volume deposited in a nearshore (DUBOIS, 1977). The cause of net beach erosion and net nearshore aggradation appears to be linked to the temporal cycle of storm and fair-weather waves. Following storm waves that erode a beach and aggrade a nearshore, swells erode the nearshore and rebuild a beach. However, during times of rising water levels, not all nearshore sediments are redeposited on a beach; a sediment layer equal to the rise in water level is left on the nearshore bottom (Figure 3) (DUBOIS, 1982).

Finally, I agree with some other conclusions presented by THIELER *et al.* (2000). If engineers wish to continue to employ predictive models, then they need to find a better way of calibrating and verifying their models, and they need to monitor projects so that they can objectively assess their predictions. There are, however, three statements written by THIELER *et al.* (2000) that I believe need further clarification; collectively, the statements seem unduly pessimistic about the future of mathematical models.

(1) In response to the question of what do we do if the mathematical models that predict beach behavior do not work, their reply is that "it is not incumbent upon us to offer any solutions at all" (p. 64). Is there a temporal condition set to this response? Does this statement mean that they will not now nor in the future work to develop better mathematical models that predict beach behavior?

(2) "The listing of assumptions in Tables 2 and 3 should not be construed as an appeal to make the models more complex by including more variables" (p. 65) if predictive models do not work. Why not add more variables to a model if additional variables will improve model accuracy? Why stop this line of research?

(3) "It is equally clear that there will be no universal model for coastal evolution. A local to regional approach is needed" (p. 64). How do they know the results of future coastal research? Is it not possible that future research could yield universal models with terms reflecting local or regional conditions?

Because—in my judgement—the paper by THIELER *et al.* (2000), as well as those by PILKEY *et al.* (1993a; 1993b), has been written with a pessimistic view of mathematical models that attempt to predict beach behavior, I can not help wondering if the authors have reached the conclusion that the human mind is incapable of formulating mathematical models that can reasonably replicate our complex coastal environment, and are, therefore, indirectly suggesting that we disengage from doing this kind of theoretical research and shift our attention to empirical studies. I hope my interpretation of what they have written is completely wrong. Mathematical model building is an important branch of study in any scientific discipline because when models are proven correct, they yield insights into how systems function, insights that can not be revealed by empirical studies alone. Mathematical model building, however, is a very challenging endeavor that normally requires a substantial investment of time. Thus, we need as many coastal scientists as possible to focus their efforts on creating better models that are verifiable by empirical studies and for coastal scientists in academia to train the next generation in the science (or art?) of mathematical model building. At this time, some of our models may not yield satisfactory predictions, but today's efforts will form a foundation upon which future generations can build reliable mathematical models and gain a greater understanding of how coastal systems function.

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