Journal of Coastal Research	16	3	782–789	West Palm Beach, Florida	Summer 2000

Characteristics of Waves off Goa, West Coast of India

V. Sanil Kumar, K. Ashok Kumar, and N.M. Anand

Ocean Engineering Division
National Institute of Oceanography
Goa 403 004, India

ABSTRACT

KUMAR, V.S.; KUMAR, K.A., and ANAND, N.M., 2000. Characteristics of waves off Goa, west coast of India. *Journal of Coastal Research,* 16(3), 782–789. West Palm Beach (Florida), ISSN 0749-0208.

Directional wave measurements were carried out using Datawell directional waverider buoy off Goa along west coast of India during the south west monsoon period in 1996 and the results are presented. Theoretical joint distribution of wave height and period was compared with that of observed. The maximum wave height was found to be 1.6 times the significant wave height with wave heights following the Rayleigh distribution. The highest waves were found to occur with intermediate periods. The measured significant wave heights show that the contribution from the swells were predominant than from the seas and the swell direction was from 230° to 270°. The unidirectional spectrum showed that it can be satisfactorily represented by Scott and Scott-Wiegel spectra for this location with an average correlation coefficient of 0.9 with better representation of peak by Scott-Wiegel spectra.

ADDITIONAL INDEX WORDS: *Wave spectra, joint distribution, Rayleigh distribution.*

INTRODUCTION

Proper understanding and evaluation of the wave climate at a given location is necessary and essential for adequate design and construction of offshore structures. The wave height is generally the most important parameter for designs. However, the wave is not specified until it is associated with a reasonable wave period. The knowledge of the short-term joint probability of wave heights and periods are important in analyzing more accurately a wide range of wave related problems. Theoretical aspects of the joint probability have been investigated in the past by various researchers (LONGUET-HIGGINS, 1975, CAVANIE *et al.* 1976, LINDGREN and RYCHLIK, 1982, MEMOS, 1994) and several types of different statistical and probability based models are proposed. Comparison of the distribution given by CAVANIE *et al* and LONGUET-HIGGINS with observation and with numerically derived data are given by SHUM and MELVILLE (1984) and by SROKOSZ and CHALLENOR (1987). They found good agreement in the region of high waves of a narrow spectrum. Since there is no known theoretical reason why any particular distribution function should fit the wave data best, the evaluation of the different distribution functions must be done on an empirical basis. The joint distribution function of the short term sea states for the shoaling region (*i.e.,* intermediate and shallow water depths) was applied by DOERING and DONELAN (1993) using a laboratory generated data. The applicability of joint distributions in the intermediate and shoaling regions for the measured wave data was done by GODA (1978), DATTATRI *et al.* (1979), HARISH and BABA (1986) and SHAHUL HAMEED and BABA (1995).

98221 received 28 September 1998; accepted in revision 18 October 1999.

Wave characteristics off the south west coast of India was studied by DATTATRI *et al.* (1979), HARISH and BABA (1986), BABA *et al.* (1989), SWAIN *et al.* (1993), SHAHUL HAMEED and BABA (1995) and MATHEW *et al.* (1995). There is no published work of the recorded wave data covering a full season off Goa coast. The data available along this region are the visual observations reported by ships (CHANDRAMOHAN *et al.,* 1991) and the short-term data covering a week at 80 m water depth (PRASADA RAO and BABA, 1996). To obtain a better understanding of the wave climate along the Goa coast waves were measured during the south west monsoon (June to September). Although severe storms normally do not occur regularly along this coast, high southwest monsoon waves attack the coast for a short period during June to September. Hence the wave data collection programme was scheduled for this season.

DATA MEASUREMENT AND ANALYSIS

Wave measurements were carried out using Datawell directional waverider buoy (STEPHEN and KOLLSTAD, 1991) off Goa along west coast of India where the water depth was 23 m during June to September 1996. The sampling interval was 0.78125 s and the data were recorded for 20 minutes duration at every 3 hr. interval. The data analysis is carried out by using the technique proposed by KUIK *et al.* (1988) wherein the characteristic parameters of directional spreading function at each frequency are obtained directly from Fourier coefficients a_0, a_1, b_1, a_2 and b_2 without any assumption of model. Fourier coefficients are estimated from auto, co- and quadrature spectra of the collected buoy signals. The significant wave height and the zero upcrossing period are obtained from the spectral analysis.

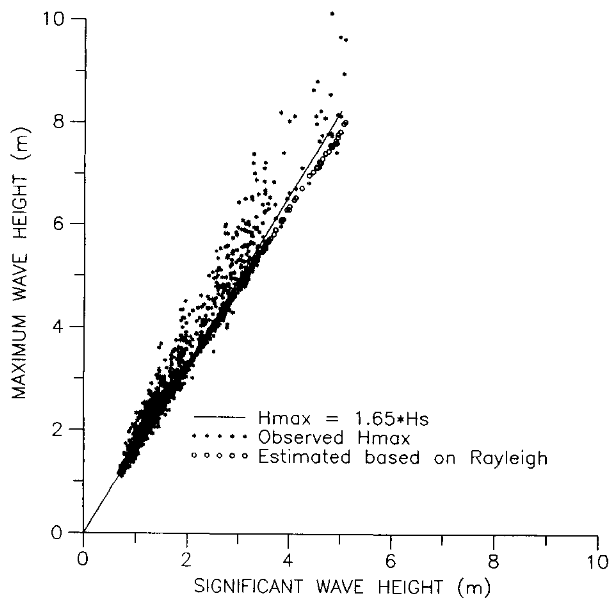

Figure 1. Correlation of maximum wave height and significant wave height.

Wave Parameters

The parameters obtained from the measured data are as under.

$$\text{mean wave direction,} \quad \theta_m = \arctan(b_1/a_1) \tag{1}$$

The sharpness or flatness in the shape of the uni-directional wave spectrum is judged through the spectral peakedness parameter, Qp (GODA 1970). Spectra with sharper peaks will have larger values of the peakedness parameter.

$$Q_p = \frac{2}{m_0^2}\int_0^\infty f[S(f)]^2\, df \tag{2}$$

where S(f) the spectral density corresponding to frequency f and m_0 is zeroth moment of energy spectrum about origin given below.

$$m_0 = \int_0^\infty f^0 S(f)\, df \tag{3}$$

The width of the spectra was defined by CARTWRIGHT and LONGUET-HIGGINS (1956) using the spectral width parameter, ϵ as follows,

$$\epsilon = \sqrt{\frac{m_0 m_4 - m_2^2}{m_0 m_4}} \tag{4}$$

where

$$m_n = \int_0^\infty f^n S(f)\, df$$

Another parameter defined by LONGUET-HIGGINS (1975) to estimate the width of the spectra is the spectral narrowness parameter, ν.

$$\nu = \sqrt{\frac{m_2 m_0 - m_1^2}{m_1^2}} \tag{5}$$

Small values of ϵ, ν represents narrow band spectra, while larger values represent broad band spectra.

Joint Distribution Models

The joint distribution models considered in the present study are given below.

LONGUET-HIGGINS (1975) provided a formulation for the joint distribution of wave amplitudes and periods. The distribution is applicable to a narrow spectrum of waves, ($\nu^2 \ll 1$, where ν is the spectral narrowness parameter).

However, this distribution has a symmetric wave period distribution that is not in keeping with observations (GODA, 1978). LONGUET-HIGGINS (1983) modified his distribution to include the asymmetry observed in wave period distribution as given below.

$$P_{LH}(H, T) = C_{LH}\left(\frac{H}{T}\right)^2 \exp\left\{-\frac{H^2}{8}[1 + \nu^{-2}(1 - T^{-1})^2]\right\} \tag{6}$$

where

$$C_{LH} = \frac{1}{8}(2\pi)^{-1/2}\nu^{-1}[1 + (1 + \nu^2)^{-1/2}]^{-1}$$

$$H = H/\sqrt{m_0} \quad \text{and} \quad T = T/(m_0/m_1)$$

CAVANIE *et al.* (1976) developed a model for the joint distribution of wave heights and period. This distribution is valid for narrow spectra and is uniquely defined by a spectral width parameter, ϵ, which is defined by CARTWRIGHT and LONGUET-HIGGINS (1956). This distribution is given below.

$$P_{CA}(H, T)$$

$$= C_{CA}\frac{H^2}{T^5}\exp\left\{-\frac{H^2}{8\epsilon^2 T^4}\left[\left(T^2 - \left(\frac{1 - \epsilon^2}{1 + \nu^2}\right)\right)^2 + a^2\left(\frac{1 - \epsilon^2}{1 + \nu^2}\right)\right]\right\} \tag{7}$$

where

$$C_{CA} = \frac{1}{4}(1 - \epsilon^2)(2\pi)^{-1/2}\epsilon^{-1}\alpha^{-1}(1 + \nu^2)^{-2}$$

$$\alpha = \frac{1}{2}[1 + (1 - \epsilon^2)^{1/2}] \qquad \alpha = \epsilon^2/(1 - \epsilon^2)$$

The joint probability density of wave heights and periods by the group of CNEXO (EZRATY *et al.*, 1977) has the following form.

$$P(H, T) = \frac{\beta^3 H^2}{4\sqrt{2\pi}\,\epsilon\alpha^2 T^5}\exp\left[-\frac{H^2 T^{-4}}{8\epsilon^2}\left\{(T^2 - \beta^2)^2 + \frac{\epsilon^2}{\alpha^2}\beta^4\right\}\right] \tag{8}$$

where

$$\beta = \frac{\sqrt{1 + \alpha^2}}{2}\alpha \quad \text{and} \quad \alpha = \sqrt{1 - \epsilon^2}$$

Since the spectral width parameter estimated based on spectral estimates is sensitive to the Nyquist frequency and

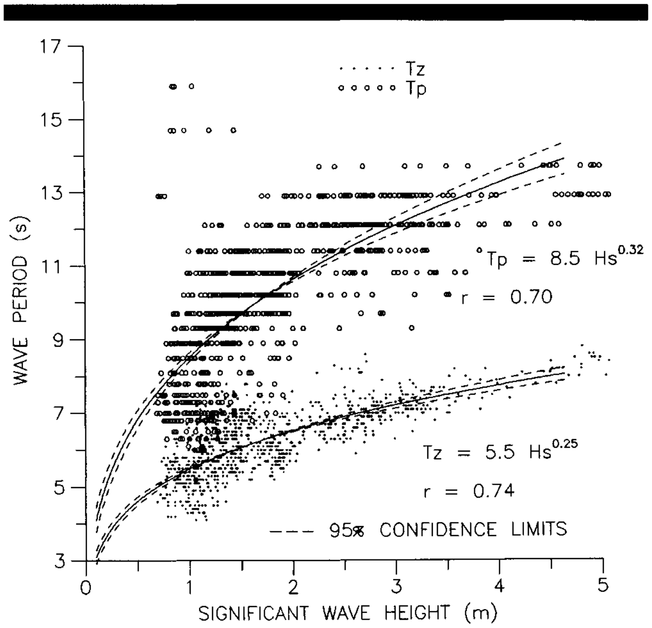

Figure 2. Variation of Tz and Tp with Hs.

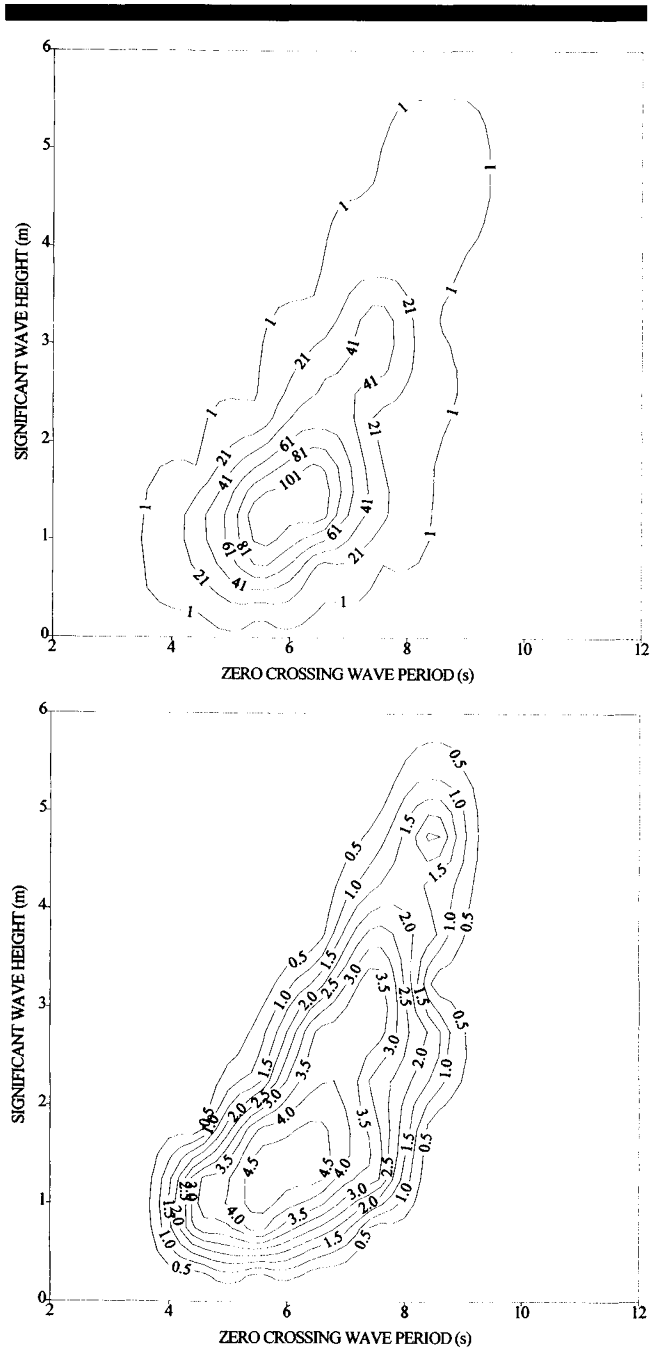

Figure 3. Joint distribution of Hs and Tz.

higher order moments, they recommend to use the same obtained from the zero crossing analysis. Hence the same has been used in the present case.

The distribution of LINDGREN and RYCHLIK (1982) was not considered for comparison for the reasons given by MEMOS (1994).

RESULTS AND DISCUSSION

Wave Height

The variation of significant wave height (H_s) with maximum wave height (H_{max}) during the observation period is shown in Figure 1. Hs varied from 0.8 to 5.1 m with a mean value of 1.81 m, H_{max} varied from 1 to 10.1 m with a mean value of 3.21 m. H_{max} observed from each 20 minutes record shows that it is approximately 1.65 times Hs with a correlation coefficient of 0.99.

The H_{max} estimated based on Rayleigh distribution also shows that this relation holds good (Figure 1). Which shows that the wave heights follow the Rayleigh distribution for this location. An earlier study by PRASADA RAO and BABA (1996) on the one week data collected off Goa in 80 m water depth had indicated the ratio between H_{max} and H_s as 1.75. DATTATRI *et al.* (1979) found that Rayleigh distribution predicts the wave height distribution very well to the data for Mangalore, west coast of India.

Hs-Tz Relationship

In general, normal short term sea states are characterized by the significant wave height and wave period and extreme sea states by significant wave height and peak period. From practical point of view it is therefore of some interest to replace Tp by Hs by a regression function. The variation of zero crossing wave period (Tz) and wave period corresponding to maximum spectral energy (Tp) with significant wave height (H_s) during the observation period is shown in Figure 2. It shows that correlation between Hs and Tz is good and Tz can be expressed in terms of Hs as given in Figure 2. Tz ranged from 4 to 9 s with a mean of 6.3 s while Tp changed from 4 to 16 s with a mean of 10.3 s. The wave period corresponding to maximum wave height varied from 4 to 16 s with a mean

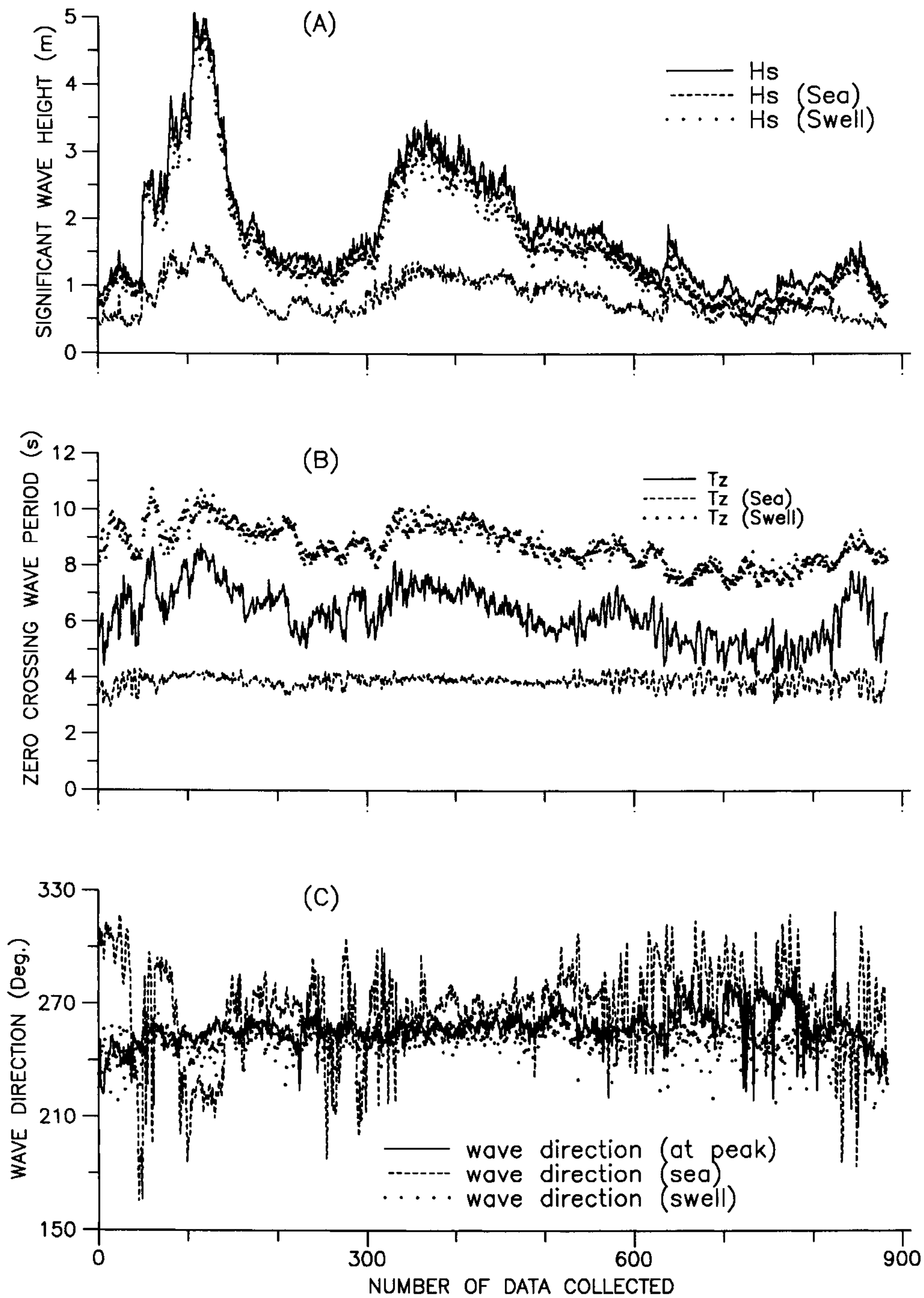

Figure 4. Variation of wave height, period and wave direction at sea and swell.

of 8.9 s. Hs seemed to be well correlated with Tz (which is an average value) with a correlation coefficient of 0.74, rather than Tp (which is a single unique quantity). By fitting non-linear regressions the relationships between Hs, Tz and Tp comes out as in Figure 2. SWAIN *et al.* (1993) also found very good correlation with a correlation coefficient of 0.79 between Hs and Tz for the shallow water waves off Cochin, south west coast of India.

Joint Distribution of Hs and Tz

A linear contour plot of the estimated significant wave height and zero crossing period is shown in Figure 3(A). The lowest contour is at the levels of one observation and the increment between the contour lines is 20. The logarithmic contour plot for the estimated significant wave height and zero crossing period is shown in Figure 3(B). The advantage of using a logarithmic contour plot over the linear ones are i) in linear plots the most contour lines are drawn where there are many values and few are drawn near the edge and ii) more emphasis can be placed on the rarer observations by using the logarithms. The results show that highest waves occur with intermediate periods. This is as per the theories of wave generation which indicate for shorter periods, larger waves break, where as for larger periods there is no enough fetch or duration to generate very high waves.

Sea and Swell

After estimating the one dimensional spectral density and the mean wave direction over the frequency band, the average directions of the sea and swell were estimated as follows. A cut frequency, 'fc' was determined based on the spectral density curve, mean direction and beam width. Then the wave parameters of both wave fields, sea and swell, were calculated for each record. The wave height, wave period and direction corresponding to sea and swell are presented in Figure 4. Figure 4A shows that the contribution from the swells were predominant in the recorded Hs values. The period of sea were predominantly between 3 and 4 s and that of swells were between 8 and 10 s (Figure 4B).

Mean Wave Direction

The mean wave direction varied from 170° to 320° with respect to north. The wave direction of the sea and swells are also shown in Figure 4C. This shows that the swells recorded are generated in the deep water under the south west monsoon winds which vary from 230° to 270°.

Wave Steepness

The steepness of the measured waves were determined using the wave period corresponding to maximum spectral energy. It was also obtained from the average zero crossing wave period (The underlying wave theory used was linear). When these two quantities were compared with each other (Figure 5), it was found that the former value was almost half of the later. This can be expected since the spectral peak period (that goes into the denominator when we calculate the wave steepness) is much higher than the average zero crossing period. The wave length estimated based on the spectral peak period varied from 25 to 225 m with a mean of 128 m. The wave length corresponding to zero crossing wave period varied from 20 to 110 m with a mean of 61 m.

Spectral Width Parameter

The variation of spectral narrowness parameter with spectral width parameter is shown in Figure 6 and it shows that there is a good correlation between these parameter with a regression coefficient of 0.71. The spectral width parameter varied from 0.7 to 0.9 with a mean value of 0.82, showing that the spectra recorded are broad band spectra. The spectral narrowness parameter varied from 0.35 to 0.75 with a mean value of 0.5. LONGUET-HIGGINS (1975) had shown that ν is apparently 0.5 ϵ for a narrow band spectra. DATTATRI *et al.* (1979) found that this equation holds good for the data collected off Mangalore, but in their work the spectral width parameter was estimated directly from the record by the number of crests and zero crossings than from the spectral analysis. For the present data it is found that $\nu = 0.73\epsilon^{1.75}$.

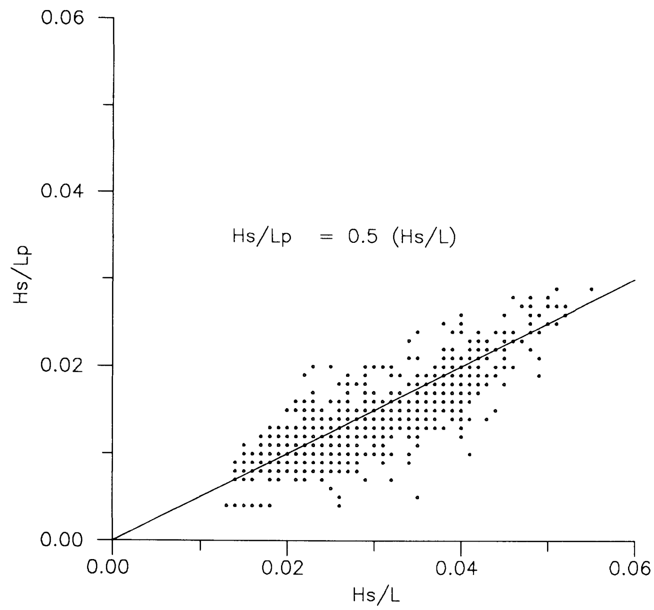

Figure 5. Variation of steepness corresponding to peak wave period and zero crossing wave period.

Figure 6 also shows the variation of peakedness parameter with spectral width parameter. The correlation between these two parameters is poor with a regression coefficient of 0.17. The peakedness parameter varied from 1 to 3.8 with a mean value of 1.96.

Wave Spectrum

The spectrum computed from measured data shows that it can be satisfactorily represented by Scott (SCOTT, 1965) and Scott-Wiegel (WIEGEL, 1980) spectrum for this region. The probable reason might be the fact that validation of Scott spectrum was carried out using considerable swell dominated data. Similar situation usually prevails along many sites along the west coast of India (DATTATRI *et al.,* 1977, NARASIMHAN and DEO, 1979, BABA and HARISH, 1986, BABA *et al.,* 1989). The maximum spectral density computed from the measured data and the same obtained from Scott and Scott-Wiegel spectra are presented in Figure 7. This shows that the better representation of peak is by the Scott-Wiegel spectra. BABA *et al.,* 1989 also found the same true for the data collected off Cochin, west coast of India at 15 m water depth. The variation of maximum spectral density (E_{max}) with significant wave height (H_s) shows that the following empirical relation can be used for this location.

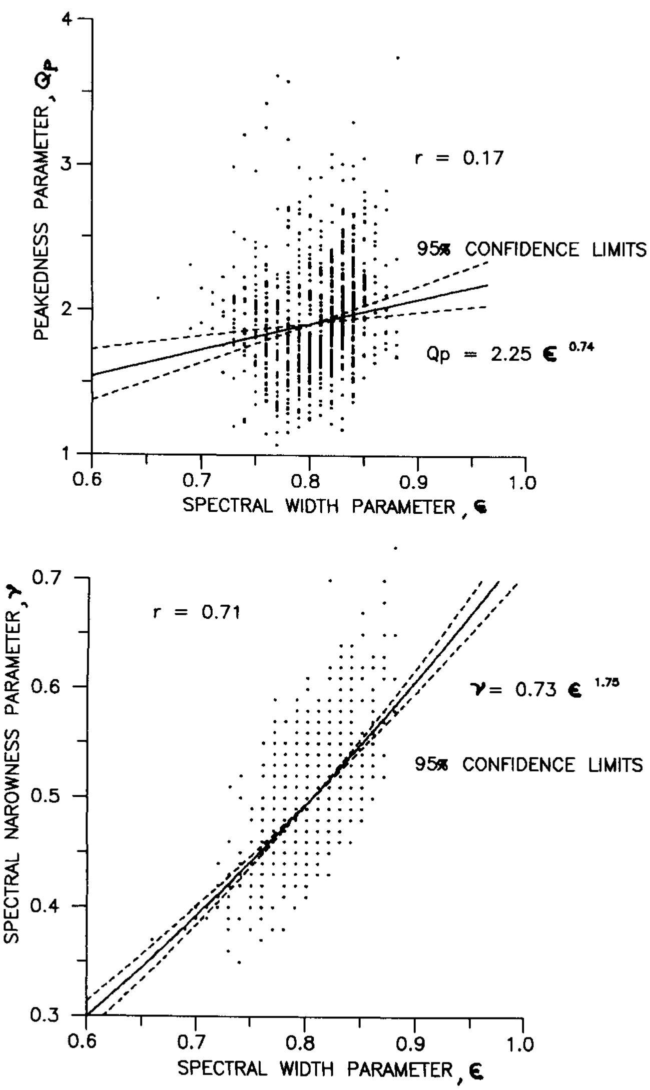

Figure 6. Variation of spectral narrowness parameter and peakedness. parameter with spectral width parameter.

$$E_{max} = 1.55H_s^2 \quad (9)$$

Figure 8 shows the correlation coefficient between the measured and theoretical spectral estimates. It shows that when the wave heights are higher, the correlation coefficient is more than 0.9.

Joint Distribution of H and T

For estimating the joint height and period distribution for zero crossing waves, first the wave heights and corresponding wave periods were extracted from the data. The wave heights (H) were normalised by $\sqrt{m_0}$ and the wave periods (T) by (m_0/m_1). The contour plots of the observed distribution (measured) of H and T in solid lines and the LONGUET-HIGGINS distribution in dashed line are shown in Figures 9A, 9C, 10A and 10C for different values of spectral width parameter. The

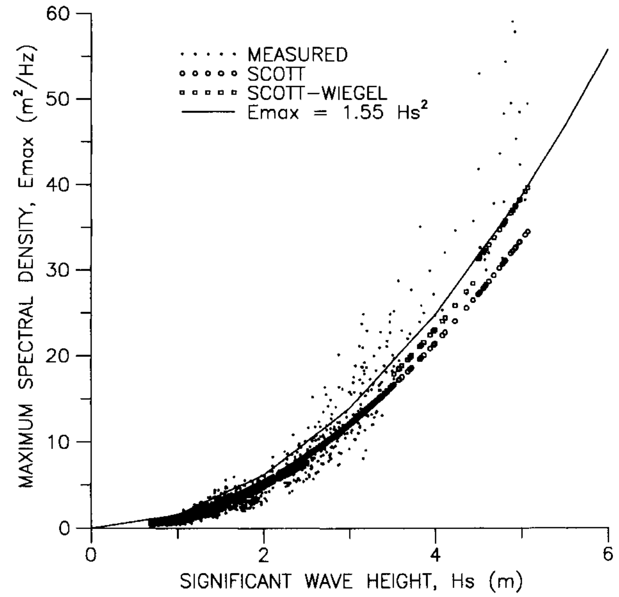

Figure 7. Variation of spectral density measured and estimated based on Scott and Scott-Wiegel with significant wave height.

CAVANIE distribution in solid line and CNEXO distribution in dashed line are shown in Figures 9B, 9D, 10B and 10D. Figures show that neither of the theoretical distributions considered represent the measured data perfectly. The reason is due to the fact that theoretical distribution are for narrow band approximations and the measured data are mainly broad banded. Also it can be seen that LONGUET-HIGGINS

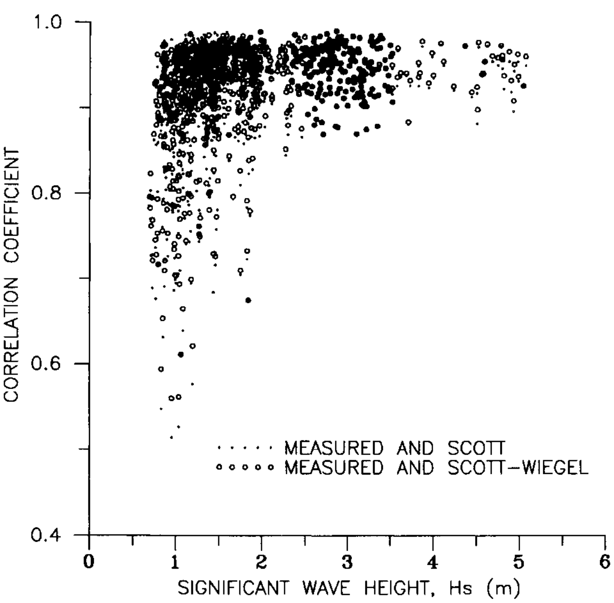

Figure 8. Variation of correlation coefficient between measured and theoretical spectra with significant wave height.

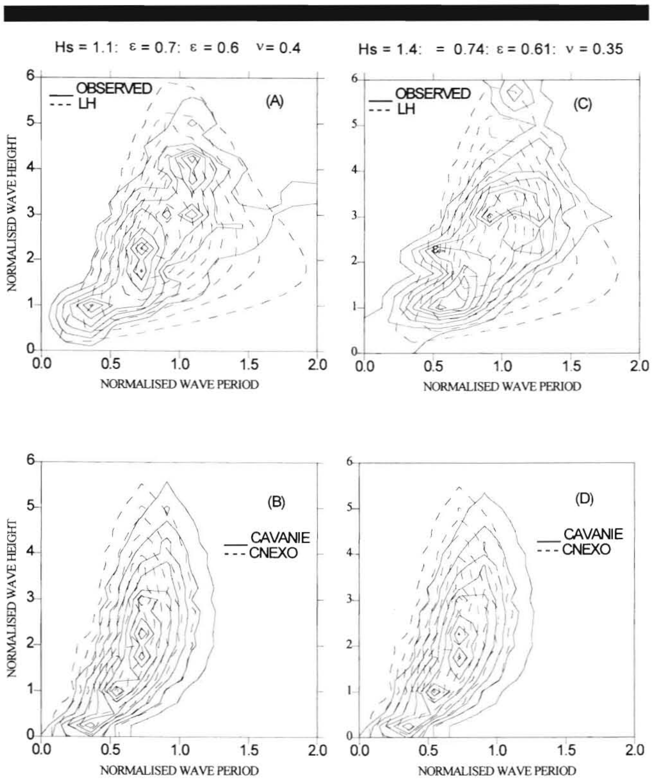

Figure 9. Joint distribution of H and T of measured data with LONGUET-HIGGINS, CAVANIE *et al.*'s and CNEXO distributions. Outermost contour for 0.1 subsequent contour for 0.1 increment.

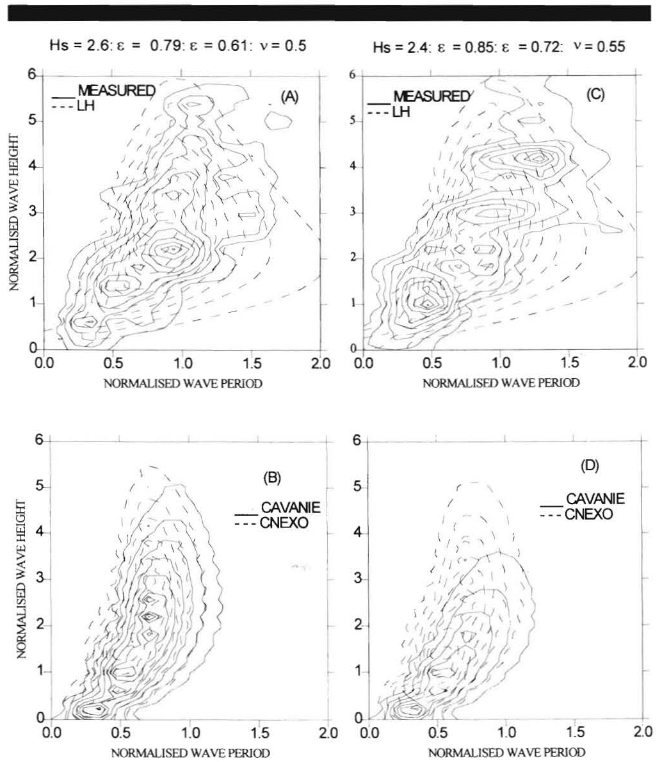

Figure 10. Joint distribution of H and T of measured data with LONGUET-HIGGINS, CAVANIE *et al.*'s and CNEXO distributions. Outermost contour for 0.1 subsequent contour for 0.1 increment.

distribution is close to the measured data than the other distributions considered. This shows that the south west monsoon wave data collected off Goa is not adequately represented by commonly followed theoretical distributions which is similar to the conclusion drawn by MYRHANG and KVAALSVOLD (1995). DATTATRI *et al.* (1979) found that the theoretical distribution given by LONGUET-HIGGINS (1975) was not applicable to the data for Mangalore, west coast of India. Based on the data collected at 4 shallow water locations along the Kerala coast, south west coast of India, HARISH and BABA (1986) found that joint distribution of non-dimensional heights and periods show good qualitative agreement with the probability density function proposed by CNEXO.

CONCLUSIONS

(1) During the measurement period significant wave height varied from 0.8 to 5.1 m with a mean value of 1.81 m.
(2) The maximum wave height was 1.65 times the significant wave height and the wave heights follow the Rayleigh distribution.
(3) Significant wave height was well correlated with zero crossing wave period rather than peak wave period.
(4) The waves recorded were predominantly swells. The period of sea were between 3 and 4 s and that of swells were between 8 and 10 s.
(5) The mean wave direction varied from 170° to 320° with respect to north with the swell direction varying from 230° to 270°.
(6) Steepness of the significant wave calculated based on the peak energy period was almost half of the same obtained through the use of average zero crossing period.
(7) The joint distribution of wave height and period was not adequately represented by the commonly followed theoretical distributions. This could be relegated to the broad banded nature of the observed wave spectra.
(8) The unidirectional spectrum showed that it can satisfactorily represented by Scott and Scott-Wiegel spectra for this location with a better representation of peak by Scott-Wiegel spectra.

ACKNOWLEDGEMENTS

The authors thank M/s Mormugao Maritima Limited, Goa for funding the wave measurement programme. They also thank their colleagues Mr. P. Pednekar and Mr. Manohar Mochemadkar for the help in data collection and the Director of the institute for providing facilities. N.I.O. Contribution Number 3542.

LITERATURE CITED

BABA, M.; DATTATRI, J., and ABRAHAM, S., 1989. Ocean wave spectra off Cochin, west coast of India. *Indian Journal of Marine Science,* 18, 106–112.

CARTWRIGHT, D.E. and LONGUET-HIGGINS, M.S., 1956. The statistical distribution of the maxima of a random function. *Proceedings Royal Society of London,* A 237, 212–232.

CAVANIE, A.; ARHAN, M., and EZRATY, R., 1976. A statistical relationship between individual heights and periods of storm waves. *In: 1st Int. Conf. on Behaviour of offshore structures,* BOSS'76, Trondheim, vol. 2, pp. 354–360.

CHANDRAMOHAN, P.; SANIL KUMAR, V., and NAYAK, B.U., 1991. Wave statistics around the Indian coast based on ship observed data. *Indian Journal of Marine Sciences,* Vol. 20(2), 87–92.

DATTATRI, J.; RAMAN, H., and JOTHI SHANKAR, N., 1979. Height and Period Distribution for Waves off Mangalore Harbour—West coast of India. *Journal of Geophysical Research,* 84 (C7), 3767–3772.

DATTATRI, J.; JOTHI SHANKAR, N., and RAMAN, H., 1977. Comparison of Scott spectra with ocean wave spectra. *Journal of Waterways Port Coastal and Ocean Engineering* (ASCE), 103, 375–378.

DOERING, J.C. and DONELAN, M.A., 1993. The joint distribution of heights and periods of shoaling waves. *Journal of Geophysical Research,* 98, 12543–12555.

EZRATY, R.; LAURENT, M., and ARHAN, M., 1977. Comparison with observation at sea of period or height dependent sea state parameters from a theoretical model, OTC 2744. *Nineth Offshore Technical Conference* (Houston) pp. 149–154.

GODA, Y., 1970. Numerical experiments on waves statistics with spectral simulation, Report, *Port and Harbour Research Institute* (Japan), 9, 3–57.

GODA, Y., 1978. The observed joint distribution of periods and heights of sea waves. *Proceedings of 16th International Conference on Coastal Engineering* (Sydney) pp. 227–246.

HARISH, C.M. and BABA, M., 1986. On spectral and statistical characteristics of shallow water waves. *Ocean Engineering,* 13(3), 239–248.

KUIK, A.J.; VLEDDER, G., and HOLTHUIJSEN, L.H., 1988. A Method For The Routine Analysis of Pitch and Roll Buoy Wave Data. *Journal Of Physical Oceanography,* 18, 1020–1034.

LINDGREN, G. and RYCHLIK, I., 1982. Wave characteristic distribution for Gaussian waves—wave-length, amplitude and steepnes. *Ocean Engineering,* 9(5), 411–432.

LONGUET-HIGGINS, M.S., 1975. On the joint distribution of the periods and amplitudes of sea waves. *Journal of Geophysical Research,* 80(18), 2688–2694.

LONGUET-HIGGINS, M.S., 1983. On the joint distribution of wave periods and amplitudes in a random wave field. *Proceedings Royal Society* London, A, 389, 241–258.

MATHEW, J.; BABA, M., and KURIAN, N.P., 1995. Mudbanks of the southwest coast of India. I: Wave characteristics. *Journal of Coastal Research,* 11(1), 168–178.

MEMOS, C.D., 1994. On the theory of the joint probability of height and periods of sea waves. *Coastal engineering,* 22, 201–215.

MYRHANG, D. and KVAALSVOLD, J., 1995. Comparative study of joint distributions of primary wave characteristics. *Journal of Offshore Mech. Arct. Engng.,* 117(2), 91–98.

PRASADA RAO, C.V.K. and BABA, M. 1996. Observed wave characteristics during growth and decay: a case study. *Continental Shelf Research,* 16(12), 1509–1520.

SCOTT, J.R., 1965. A sea spectrum for model test and long-term ship prediction. *Journal of Ship Research,* 9, 145–152.

SHAHUL HAMEED, T.S. and BABA, M., 1995. Joint distribution of shallow water wave heights and periods. *Int. Conf. on Coastal and Port Engineering in Developing Countries (COPEDEC-IV),* Brazil, Vol. III, pp. 1785–1795.

SHUM, K.T. and MELVILLE, W.K., 1984. Estimate of the joint statistics of amplitude and periods of ocean waves using an integral transform technique. *Journal of Geophysical Research,* 89(C4), 6467–6476.

SROKOSZ, M.A. and CHALLENOR, P.G., 1987. Joint distributions of wave height and period: A critical comparison. *Ocean Engineering,* Vol. 14(4), pp. 295–311.

STEPHEN, F.B. and TOR KOLLSTAD., 1991. Field Trials Of The Directional Waverider. *Proceedings of The First International Offshore And Polar Engineering Conference,* III, 55–63.

SWAIN, J.; ANANTH, P.N., and RAO, C.V.K.P., 1993. Shallow water wave characteristics off Cochin during monsoon 1986. *Indian J. of Marine Sciences,* 22(4), 256–262.

WIEGEL, R.L., 1980. Design wave, *Lecture notes of short term course on small harbour engineering.* Bombay: Indian Institute of Technology, Vol. I, 4.1–4.55.