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The Growth of Wind Waves Estimated Using a New Irrotational Finite Amplitude Water Wave Model

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ABSTRACT

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A new deep water irrotational water surface wave model based on the stream function expression for a vortex sheet is presented. If it is assumed that there are waves in the air above the sea surface that correspond to the water surface waves then an expression for the growth of the water waves due to the wind is proposed in terms of a Reynolds number $R = U^* \sqrt{LF}/\Gamma$, where U^* is the friction velocity, L is the wavelength, F is the fetch and Γ is the circulation per wave. The proposed model is compared favourably with the *Shore Protection Manual* (1984) methods.

ADDITIONAL INDEX WORDS: *Irrotational water waves, wind, wave growth, vortex sheet.*

INTRODUCTION

A new irrotational finite amplitude water surface gravity wave model is presented. This model is based on the potential flow representation of a vortex sheet containing discrete vortices. Even though it is possible to visualise the boundary between one fluid flowing over another as a vortex sheet (ROBERTSON, 1965, p. 111; PHILLIPS, 1977, p. 121), it does not necessarily follow that an irrotational vortex sheet will be a suitable representation for water surface gravity waves. It will be subsequently shown that it is a useful model. For a related application, LONGUET-HIGGINS (1994) proposed a system of vortices above and below a free surface in his examination of the crest stability of gravity/capillary waves.

In the initial development of the model the air is neglected, and it is assumed that the water continues to flow as if there is an irrotational vortex sheet just above the free surface. The model development follows the usual assumptions for surface waves of a constant form (SCHWARTZ and FENTON, 1982) (DEAN and DALRYMPLE, 1991), *i.e.* the fluid is assumed two dimensional, ideal, with irrotational motion, and the field equation is Laplace's equation $\nabla^2\phi = \nabla^2\psi = 0$, where ϕ is a velocity potential and ψ is a stream function. There is no flow across the bottom boundary and the free surface boundary, there is constant pressure on the free surface, and a periodic solution is specified. These assumptions are found to be an excellent approximation provided that the waves are not too small, or in water that is too shallow so that the thickness of the bottom boundary layer is not too large compared to the wave height (SVENDSEN and JONSSON, 1980, p.62). Only a deep water vortex sheet wave model (VSWM) is presented, however shallow water waves, steep asymmetric waves and standing waves have been modelled in a similar fashion by the writer in CUMMINGS (1996), by using four vortex sheets, two above the free surface, and two below the sea bed as a mirror image.

The VSWM is used to develop a simple model of wave generation due to the wind in deep water. The air is re-introduced into the model at this point, which is assumed to contain the vortex sheet flow pattern assumed in the VSWM. Precedents for this assumption are found in MILES (1957), who envisaged that there would be air waves complementary to the underlying water waves, and BANNER and MELVILLE (1976) and BANNER (1991) who performed flow visualisation and pressure measurements on the air flow above breaking water waves, and found a pattern similar to "Kelvin's cats eyes" in the theory of laminar stability. HARA and MEI's (1994) and GENT and TAYLOR's (1976) numerical models showed similar results.

If it is assumed that the air vortices have rotational cores, then, in the absence of air pressure gradients, it is shown that there is a reduction of the air momentum flux due to the growth and coalescence of the vortices. Analysis by CSANADY (1992) has confirmed that the irrotational water wave flow in the water is due to viscous surface shear forces, these forces not being perfectly uniform over the water surface, hence generating waves, and that the effect of pressure pulses and air flow separation over wave crests ("sheltering") is comparatively minor, unless the waves are steep ($H/L > 0.10$) (GENT and TAYLOR, 1976). The shear at the water surface is estimated by using a sea surface drag coefficient C_D. The KATSAROS and ATAKTURK (1992) expression has been used, which is similar in form to WU (1982), HERBICH (1990, p.219) and LARGE and POND (1981). By equating the reduction in momentum flux with the sea surface drag force, a wave growth model is developed.

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VORTEX SHEET WAVE MODEL DEVELOPMENT

Assumptions

Consider two-dimensional progressive gravity waves propagating without change of form over a layer of ideal fluid. The ideal fluid is assumed to be invicsid, constant density, incompressible, the effects of surface tension are neglected, and the fluid motion is irrotational. The presence of air above the fluid is neglected.

A reference plane is selected such that it moves at the wave's celerity C, thus rendering the wave stationary, with a rectangular coordinate system such that the x-axis is horizontal and the y-axis is vertical, and with corresponding velocities u and v. A similar X,Y coordinate plane, stationary relative to the bottom, with velocities U and V is also present. A horizontal bottom at $y = -\infty$ is assumed.

As the fluid is incompressible, and the motion is irrotational, a stream function ψ, and velocity potential ϕ can be defined such that the horizontal velocity component u, and the vertical velocity component v, are given by:

$$u = \frac{-\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \tag{1}$$

$$v = \frac{-\partial\phi}{\partial y} = \frac{-\partial\psi}{\partial x} \tag{2}$$

ψ and ϕ must satisfy Laplace's equation: $\nabla^2\psi = \nabla^2\phi = 0$.

Using DEAN and DALRYMPLE's (1991) nomenclature, the boundary conditions are:

(1) The Kinematic Free Surface Boundary Condition (KFSBC): there is no flow across the free surface.
(2) The Bottom Boundary Condition (BBC): there is no flow across the bottom.
(3) The Lateral Boundary Condition (LBC): the field extends to $x = \pm\infty$.
(4) The Dynamic Free Surface Boundary Condition (DFSBC): the pressure along the free surface is a constant and equal to atmospheric pressure, *i.e.* Bernoulli's equation:

$$\eta + \frac{(u^2 + v^2)}{2g} = \text{constant} = B = \text{The Bernoulli Sum} \tag{3}$$

where η is the deviation of the free surface from the still water level (SWL), and g is the acceleration due to gravity.

Vortex Sheet Stream Function

From LAMB (1945, p. 224) and MILNE-THOMSON (1968, p. 376) the stream function for an irrotational vortex sheet is:

$$\psi = \frac{-\Gamma}{4\pi}\ln\left[\frac{1}{2}(\cosh ky - \cos kx)\right] \tag{4}$$

where Γ is the vortex circulation, k is the wavenumber $2\pi/L$, and L is the wavelength. A streamline below the x axis is assumed to be the free surface. A deep water wave is assumed with a horizontal bottom located on a streamline at $y = -\infty$. Equation (4) is illustrated in Figure 1, where y_c is the vertical distance from the x axis to the wave crest, and H is the wave height. The vortex sheet is horizontally located on the x axis. The vortices are spaced L apart, and have an anticlockwise rotation. One vortex is located at the origin.

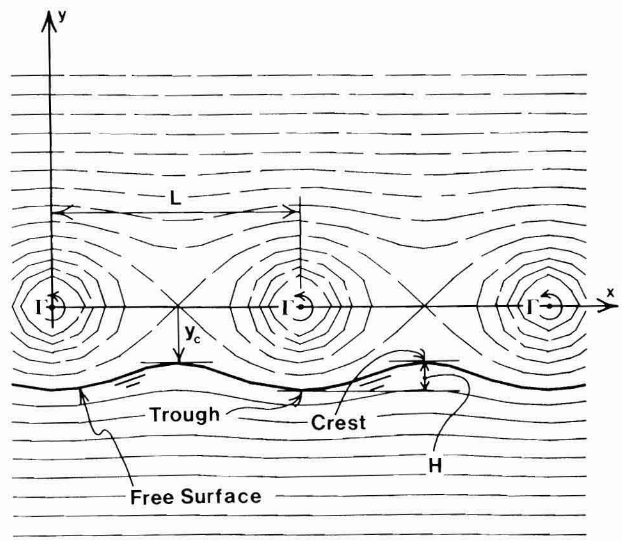

Figure 1. VSWM definition sketch. Moving (x,y) plane stream function.

It is assumed that this stream function is an approximate solution to the surface wave problem. As the vortices are equally spaced L apart, the solution is periodic. From (1, 2 & 4) u and v are:

$$u = \frac{\partial\psi}{\partial y} = \frac{-\Gamma}{2L}\left[\frac{\sinh ky}{\cosh ky - \cos kx}\right] \tag{5}$$

$$v = \frac{-\partial\psi}{\partial x} = \frac{\Gamma}{2L}\left[\frac{\sin kx}{\cosh ky - \cos kx}\right] \tag{6}$$

The KFSBC and BBC are satisfied automatically, as there is no flow across a streamline.

As $y \to \pm\infty$, the apparent flow in the x,y plane is: $u \to \mp \Gamma/2L$, $v \to 0$. In the stationary plane X,Y, the mean horizontal velocity under a real deep water wave is close to zero, therefore the vortex sheet will appear to be translating with the wave form, from right to left at celerity $C = -\Gamma/2L$.

Combining the Field Equation and the Boundary Conditions

As a device to assist the solution it is assumed that the value of the surface streamline ψ_s is proportional to the circulation: $\psi_s = Z\Gamma$, where Z is an arbitrary constant, $Z \leq 0$.

(1) On the free surface, at the wave crest $\psi = \psi_s = Z\Gamma$, $x = L/2$, $y = y_c$, $u = u_c$, $v = 0$. Substitution into (4) and (5) gives:

$$-ky_c = |\cosh^{-1}(2\exp(-4\pi Z) - 1)| \tag{7}$$

$$\frac{u_c}{\Gamma} = \frac{-1}{2L}\left[\frac{\sinh ky_c}{\cosh ky_c + 1}\right] \tag{8}$$

where: "| |" indicates absolute value.

(2) On the free surface, at the wave trough $\psi = \psi_s = Z\Gamma$,

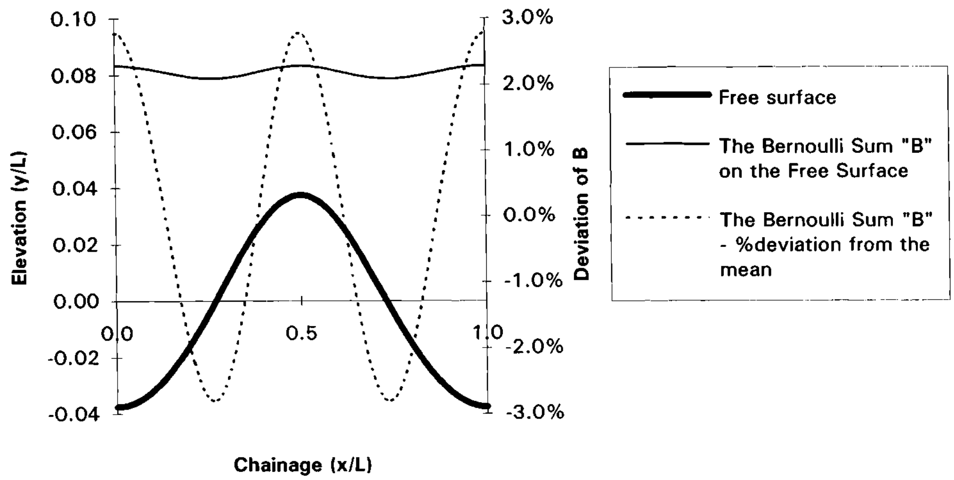

Figure 2. VSWM, Free surface profile, Deep water, $H/L = 0.075$.

$x = 0$, $y = (y_c - H)$, $u = u_t$, $v = 0$. Substitution into (4) and (5) gives:

$$H = y_c + \frac{1}{k}\left|\cosh^{-1}(2\exp(-4\pi Z) + 1)\right| \quad (9)$$

$$\frac{u_t}{\Gamma} = \frac{-1}{2L}\left[\frac{\sinh k(y_c - H)}{\cosh k(y_c - H) - 1}\right] \quad (10)$$

(3) To satisfy the DFSBC, B must be constant on the free surface. Initially B at the wave crest and trough only are equated. Subsequently the value of B along the rest of the free surface is checked.

From equation (3):

$$H = \frac{1}{2g}(u_t^2 - u_c^2) \quad (11)$$

Dividing both sides of (11) by Γ, and re-arranging:

$$\Gamma = \sqrt{\frac{2gH}{(u_t/\Gamma)^2 - (u_c/\Gamma)^2}} \quad (12)$$

Substitution of (7) into (8), and (9) into (10), followed by substitution of both of these combinations into (12), and the utilisation of the identity ($\cosh^{-1}y = \sinh^{-1}\sqrt{y^2 - 1}$) (TUMA, 1987, p. 70) gives:

$$\Gamma^2 = 4gHL^2\exp(-4\pi Z) \quad (13)$$

$$\Gamma = 2gHT\exp(-4\pi Z) \quad (14)$$

$$Hk = \left|\cosh^{-1}(1 + 2\exp(-4\pi Z))\right| - \left|\cosh^{-1}(-1 + 2\exp(-4\pi Z))\right| \quad (15)$$

$$\frac{\omega\sqrt{H/g}}{\exp(-2\pi Z)} = \left|\cosh^{-1}(1 + 2\exp(-4\pi Z))\right| - \left|\cosh^{-1}(-1 + 2\exp(-4\pi Z))\right| \quad (16)$$

where ω = angular frequency = $2\pi/T$, and T = wave period.

If H and L are specified, then Z is obtained iteratively from (15), followed by Γ from (13).

If H and T are specified, then Z is obtained iteratively from (16), followed by Γ from (14).

(4) Once Γ has been obtained, the water particle velocities can be obtained from (5) and (6). The value of the stream function on the free surface ψ_s is obtained from $\psi_s = Z\Gamma$. The wave celerity C, still water level SWL and free surface profile are given by (17), (18) & (19) respectively:

$$C = \frac{-\Gamma}{2L} \quad (17)$$

$$\mathrm{SWL} = y_c + \frac{1}{2g}(u_c^2 - C^2) = y_c + \frac{1}{2g}(C^2 - u_t^2) \quad (18)$$

$$-ky = \left|\cosh^{-1}(2\exp(-4\pi Z) + \cos kx)\right| \quad (19)$$

ACCURACY OF THE VORTEX SHEET WAVE MODEL

DFSBC Compliance

A measure of the theoretical accuracy of the VSWM is the degree to which the DFSBC is satisfied. The free surface profile, the value of B along the free surface, and the deviation of B from the mean $\bar{B}$, for a deep water wave with steepness $H/L = 0.075$, are plotted in Figure 2. The horizontal and vertical dimensions have been non-dimensionalised with respect to L. The value of B is not constant along the free surface, so the DFSBC is not satisfied exactly, and therefore the model is approximate. The deviation of B from $\bar{B}$ has a maximum of about $\pm 3\%$ as shown on Figure 2. A plot of the root mean squared (RMS) deviation of B from $\bar{B}$, vs. H/L is presented in Figure 3.

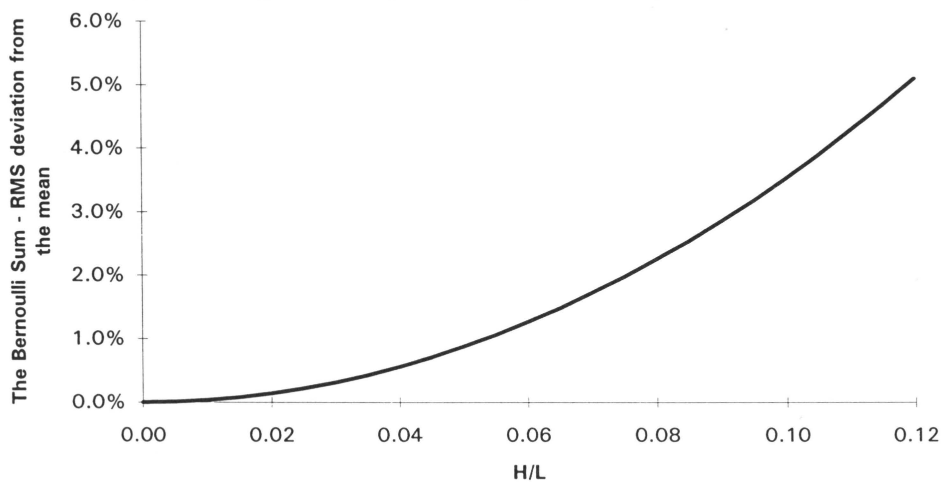

Figure 3. VSWM accuracy in satisfying the DFSBC, deep water.

Comparison with an "Exact" Wave Theory

The wave celerity C has been traditionally used for comparison between wave theories (RIENECKER and FENTON, 1981). A comparison between a high order Stokes solution COKELET (1977) and the VSWM for the deep water case ($\exp(-(kQ/C)) = 0.00$) is presented in Figure 4. Q is the apparent flow rate under the wave form in water of mean depth h. The numerical solution of Cokelet's can be regarded as being very close to an exact solution of the irrotational surface wave problem.

The comparison between the VSWM and Cokelet is close, except near the limiting steepness.

Comparisons with Experiments—Horizontal Velocity Below the Crest

An important test of a wave model is the comparison with measurements of the horizontal velocity under the crest (RIENECKER and FENTON, 1981; HATTORI, 1985; SOULSBY *et al.*, 1993). This is a more sensitive test than the water surface profile (Fenton *priv. comm.* 1994). Most of the water wave horizontal velocity profiles in the literature are for Intermediate ($2 < L/h < 20$) or Long ($L/h > 20$) waves (*e.g.* GRAW, 1994; LE MÉHAUTÉ *et al.*, 1968). Comparisons with the Deep ($L/h < 2$) water experiments of LEE *et al.* (1974) and the Linear (Airy) theory (DEAN and DALRYMPLE, 1991, p. 79) are shown in Figures 5 and 6. In these comparisons T and H were modelled exactly, but the VSWM indicated shorter L values than the experiment or the Linear theory because the later two have a finite depth, whereas the VSWM assumes an infinite depth.

A complication when comparing models and experiments, as pointed out by RIENECKER and FENTON (1981), is that in common with other finite amplitude wave models, the VSWM exhibits a net mass transport in the direction of wave motion ("Stoke's drift"). In a closed system such as used by LEE *et*

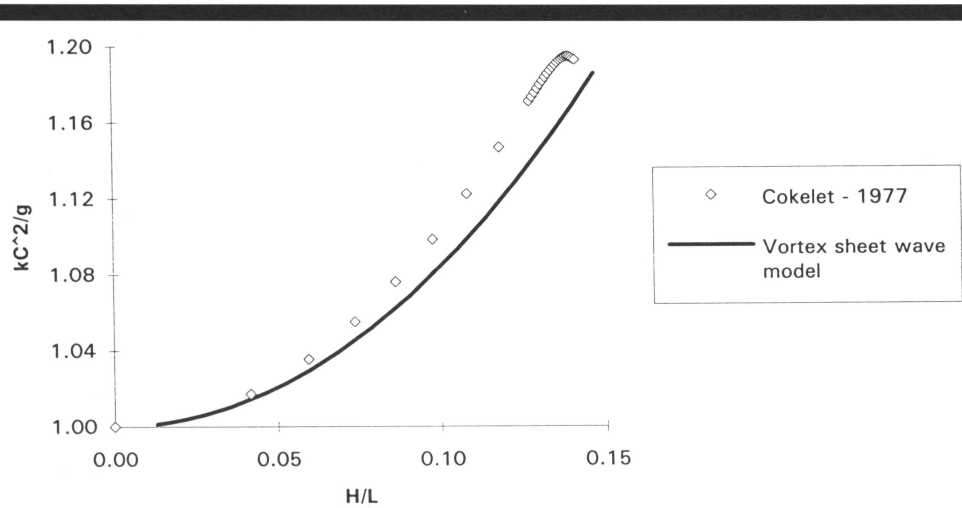

Figure 4. Wave celerity, deep water, VSWM comparison with Cokelet 1977.

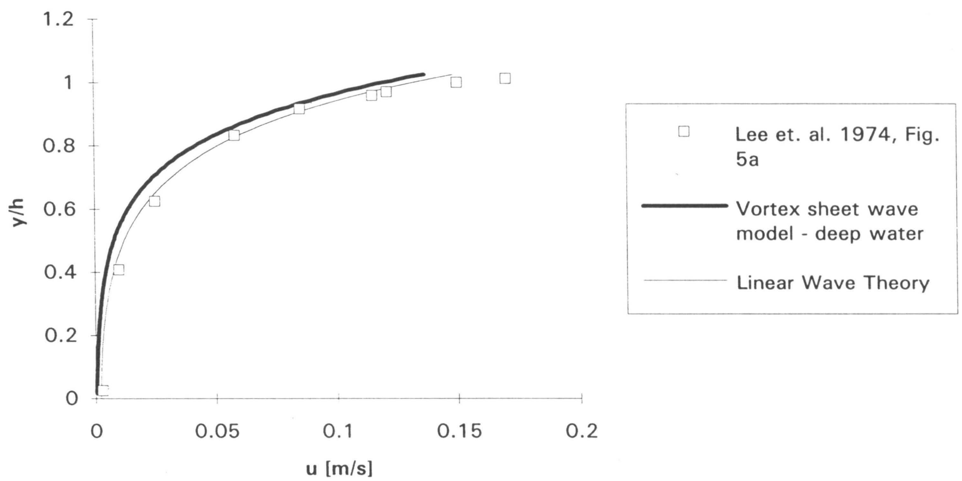

Figure 5. Horizontal velocity under the crest. VSWM comparison with LEE *et al.* (1974), Figure 5a, and the Linear Theory (DEAN and DALRYMPLE, 1991, p. 79). $T = 0.41$ s, $H = 0.0172$ m, $h = 0.24$ m, Approx $L/h = 1.32$.

al., there is no net mass transport. The waves are effectively propagating against a return current induced by their mass transport. As experimental details of this current are not known, it has not been allowed for in the comparison.

WAVE GROWTH MODEL

Introduction and Assumptions

A wave growth model incorporating a sea surface Reynolds number R is obtained from an extension of the VSWM. The wave growth model was suggested by MILNE-THOMSON's, (1968, pp. 380–384) presentation of von Karman's estimate of the drag on a cylinder due to vortex shedding. Refer to Figure 7 for the definition sketch.

The following assumptions are made:

(1) Wave growth and decay are due solely to air drag forces. LAMB (1945, p.624) showed that for waves above the capillary/gravity range, wave decay due to viscosity is very slow.

(2) Apart from the vortex cores and regions very close to the water's free surface it is assumed that the air flowing over the water is a steady ideal fluid, with constant density ρ_a and pressure.

(3) The sea is assumed to have sufficient depth so that the bottom can be neglected.

(4) The vortex system used to model the water waves is present without change in the air above the waves. In other words the water waves induce corresponding air waves and vice versa.

(5) The vortex system changes slowly in the X direction so that locally a uniform vortex sheet expression (LAMB, 1945, p. 224) (similar to (4)) can be used to approximate the flow:

$$\psi = \frac{-\Gamma}{4\pi}\ln\left[\frac{1}{2}(\cosh kY - \cos kX)\right] \qquad (20)$$

(6) In addition to the vortex system a wind is assumed to blow from the land across the sea surface, from right to left. This flow is superimposed onto the air vortex sheet system. The air flow is assumed to have a uniform velocity profile $U_{10} = -\Gamma/L$, where U_{10} is the 10 m height wind velocity, apart from a very thin surface boundary layer.

(7) A large control volume $ABCD$ is placed on the sea surface as shown in Figure 7. At the shore, point B is located the fetch distance F from the origin A. A vortex sheet system is assumed to be generated at point B, as very small vortices, growing and combining as they travel towards A where they exit the control volume.

(8) If the sea is fetch limited then the vortices are assumed to exit at the same rate as they are being generated. Even though the vortices are moving, growing and coalescing, the system is assumed to be quasi-steady.

(9) If the sea is duration limited, it is assumed from dimensional considerations that there is an effective fetch $F = U^*D$, where U^* is the friction velocity, and D is the duration. If F is large then it is assumed that the duration limited system is quasi-steady.

(10) It is assumed that the mean horizontal velocity profile of the vortex system is the horizontal velocity at the quarter wave length ($L/4$) location of the vortex system. This is the same assumption as used in von Karman's analysis. This assumption is used because the vortices are assumed to have rotational cores.

(11) Assuming negligible flow across AB and CD, and a negligible pressure gradient from CD to AB, there is a decrease in momentum from BC to AD due to the vortices, which is balanced by drag forces on the sea surface.

Fetch Limited Wave Growth

The vortex system horizontal velocity U is:

$$U = \frac{\partial\psi}{\partial Y} = \frac{-\Gamma}{2L}\left[\frac{\sinh kY}{\cosh kY - \cos kX}\right] \qquad (21)$$

At $X = (L/4)$ equation (21) becomes:

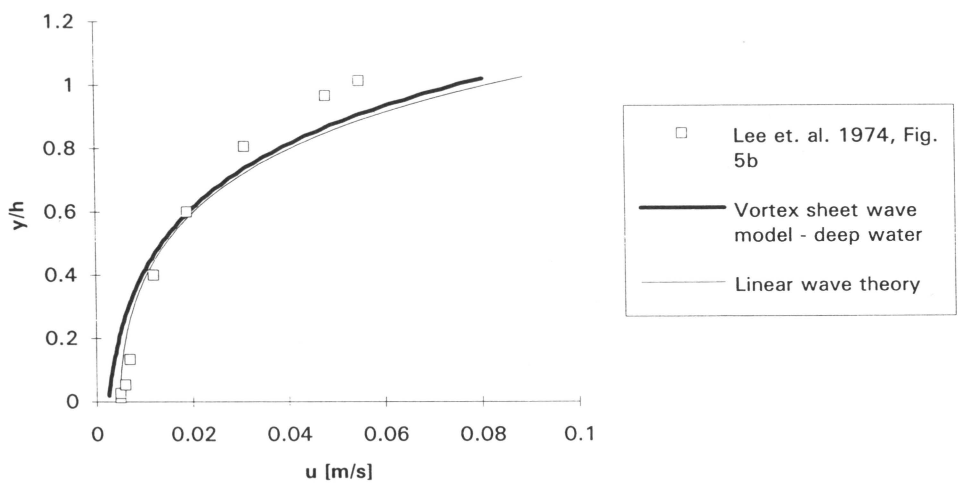

Figure 6. Horizontal velocity under the crest. VSWM comparison with LEE *et al.* (1974) Figure 5b, and the Linear Theory (DEAN and DALRYMPLE (1991, p. 79). $T = 0.41$ s, $H = 0.0106$ m, $h = 0.15$ m, Approx $L/h = 1.80$.

$$U = \frac{\partial\psi}{\partial Y} = \frac{-\Gamma}{2L}[\tanh kY] \tag{22}$$

The change of horizontal momentum flux per unit width ΔM flowing through the control volume from section *CB* to *AD*:

$$\Delta M = \int_A^D \rho_a U^2\, dY - \int_B^C \rho_a U^2\, dY \tag{23}$$

$$\frac{\Delta M}{\rho_a} \cong \int_0^\infty \left(\frac{-\Gamma}{2L} - \frac{\Gamma}{2L}\tanh kY\right)^2 - \left(\frac{-\Gamma}{L}\right)^2 dY \tag{24}$$

$$\frac{\Delta M}{\rho_a} \cong \frac{\Gamma^2}{4L^2}\int_0^\infty (2\tanh kY + \tanh^2 kY - 3)\, dY \tag{25}$$

Integrating (25) (TUMA, 1987, p. 375):

$$\frac{\Delta M}{\rho_a} \cong \frac{\Gamma^2}{4L^2}\left[\frac{2\ln(\cosh kY)}{k} + Y - \frac{\tanh kY}{k} - 3Y\right]_0^\infty \tag{26}$$

$$\Delta M \cong \frac{-\Gamma^2\rho_a}{4L^2 k} = \frac{-\Gamma^2\rho_a}{8\pi L} \tag{27}$$

The wind drag force per unit width on the water surface F_D equals the drag force expression:

$$F_D = \frac{\rho_a F U^{*2}}{2} \tag{28}$$

where:

$$C_D = U^{*2}/U_{10}^{\,2}, \tag{29}$$

The reduction in M is balanced by F_D, therefore:

$$F = \frac{\Gamma^2}{4\pi U^{*2} L} \tag{30}$$

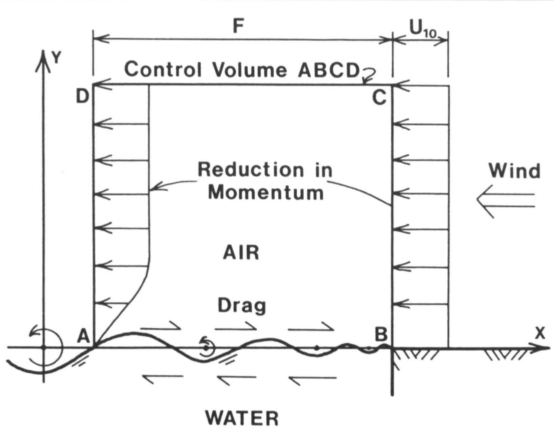

Figure 7. Wave growth due to air drag—definition sketch.

Sea Surface Reynolds Number

Equation (30) is rearranged to give a Reynolds number R in equation (31). The 4π is absorbed into R.

$$R = \frac{U^*\sqrt{LF}}{\Gamma} \tag{31}$$

The usual form of the Reynolds number found in fluid dynamics is the ratio inertia force/viscous force. Similarly the sea surface Reynolds number is the ratio shear force due to the wind/shear force due to wave growth, represented by $R^2 = (\rho_a F U^*2)/(\Gamma^2\rho_a/L)$. The numerator being the "inertia" term, and the denominator being the rotational "viscous" term. In a steady state prototype system this ratio would be expected to be fairly close to unity. If the sea surface shear force due to the wind exceeds the sea surface shear force due to the wave growth, then the growth rate should increase until the forces are similar, and vice versa.

Table 1. *Wave Growth Expressions—SPM (1984) and (VSWM)* H_s = significant wave height, T_m = peak spectral period, $U_A = 0.71U_{10}^{1.23}$

Item	SPM (1984)	VSWM	
H_s fetch limited	$391\cdot10^3 = \frac{U_A^2F}{gH_s^2}$	$Re^2 = \frac{U^{*2}F}{gH_s^2}\cdot\frac{(H_s/L)}{4\exp(-4\pi Z)}$	(32)
T_m fetch limited	$42.9 = \frac{U_AF}{g^2T_m^3}$	$Re^2 = \frac{U^{*2}F}{g^3T_m^4}\cdot\frac{1}{4(H_s/L)^3\exp(-12\pi Z)}$	(33)
H_s duration limited	$49.7\cdot10^{15} = \frac{U_A^5D^3}{gH_s^4}$	$Re^2 = \frac{U^{*3}D}{gH_s^2}\cdot\frac{(H_s/L)}{4\exp(-4\pi Z)}$	(34)
T_m duration limited	$843 = \frac{U_AD}{gT_m^2}$	$Re^2 = \frac{U^{*3}D}{g^3T_m^4}\cdot\frac{1}{4(H_s/L)^3\exp(-12\pi Z)}$	(35)

Comparison with the *Shore Protection Manual* 1984 (SPM, 1984)

Equations (13) & (14), are combined with (31) in Table 1 to obtain four wave growth estimation expressions (32, 33, 34 & 35). These expressions have been re-arranged to present L only in combination with H_s as a wave steepness parameter H_s/L. The SPM (1984) (US Army Corps of Engineers 1984) is a very widely used reference in coastal engineering for the estimation of wave growth due to the wind. Listed in Table 1 for comparison are the corresponding expressions from the SPM (1984), simplified JONSWAP method. The SPM (1984) duration limited expressions have been obtained by combining equations 33-33, 3-34 & 3-35. These expressions have been determined from empirical data using dimensional analysis (BISHOP *et al.*, 1992).

Graphical comparisons of the VSWM and SPM (1984) are shown in Figures 8 and 9. In this comparison the following assumptions have been used:

(1) $C_D = (0.75 + 0.1U_{10})*10^{-3}$ (KATSAROS and ATAKTURK, 1992)

(2) As the wave grows the wave steepness (H_s/L) stays constant due to waves interacting and wave breaking. The assumed value of $H_s/L = 0.025$ is taken from the peak value in Figure 3 in HOLTHUIJSEN and HERBERS (1985). Using this value, from (15), $Z = -0.14771$

(3) $R = 1.0$. This value is based on a visual best fit to the SPM (1984) fetch limited case. Despite the reasoning presented in §4.3, the writer does not imply that R is necessarily exactly unity.

(4) No correction for air-water temperature difference stability effects.

(5) H_m in SPM (1984) = H_s and the significant wave period $T_s = T_m$

Fetch Limited Comparisons

In the H_s case the curves are similar, with the VSWM predicting smaller waves than the SPM (1984). The T_m curves are quite similar, with the SPM (1984) predicting slightly longer periods for the larger fetches.

BISHOP *et al.* (1992) reviewed the wave prediction methods in SPM (1984) for the fetch limited case only, and found that the SPM (1984) over predicts both H_s and T_m. It appears

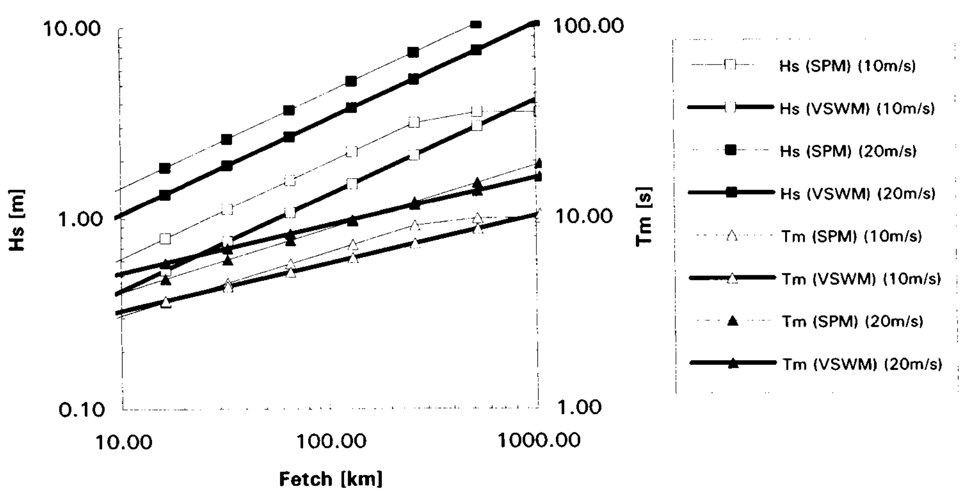

Figure 8. Fetch limited wave generation by the wind. Comparison between SPM (1984) and VSWM.

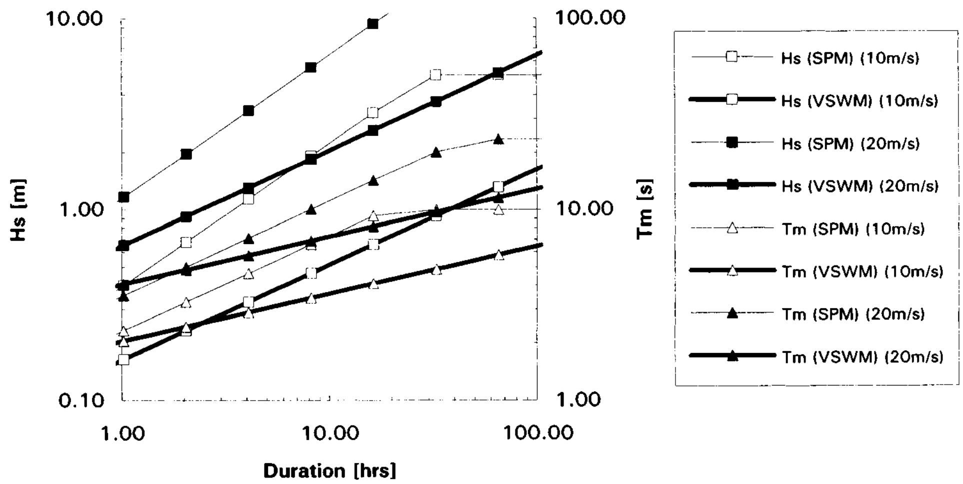

Figure 9. Duration limited wave generation by the wind. Comparison between SPM (1984) and VSWM.

therefore that the VSWM is a reasonable fetch limited wave growth model.

Duration Limited Comparison

Wave growth with wind duration is not as well understood as wave growth with fetch length (SPM, 1984, pp. 3–51), due to the difficulty of obtaining suitable field data. The VSWM under predicts both H_s and T_m compared to the SPM (1984), but in the writer's opinion the VSWM appears to be a reasonable model.

CONCLUSION

Based on the potential flow representation of a vortex sheet, a new finite amplitude irrotational wave model has been developed. The model is shown to be as accurate as the Linear wave model for modelling the horizontal fluid velocity under the wave crest in deep water. The mathematical complexity of the model is comparable to the Linear model and is less than other finite amplitude irrotational models. The model was extended by the introduction of complementary air waves above the water waves in order to model the growth of water waves due to the wind. A sea surface Reynolds number $R = U^* \sqrt{LF/T}$ was obtained. Despite the model's simplifying assumptions it was shown to compare favourably with the SPM 1984 method. The model may offer insights into the mechanisms of wave growth.

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LIST OF SYMBOLS

B = the Bernoulli sum
C = wave celerity
C_D = sea surface drag coefficient
D = wind duration
F = fetch length
F_D = wind drag force per unit width
g = acceleration due to gravity
h = mean water depth
H = wave height
H_s = significant wave height
k = wave number
L = wave length
M = momentum flux per unit width
Q = apparent flow rate under the wave form
R = sea surface Reynolds number
T = wave period
T_m = peak spectral period
u = moving plane horizontal velocity
u_c = moving plane water surface horizontal velocity at the crest
u_t = moving plane water surface horizontal velocity at the trough
U = stationary plane horizontal velocity
U_{10} = 10m height wind velocity
U^* = friction velocity
v = moving plane vertical velocity
V = stationary plane vertical velocity
x = moving plane horizontal axis
X = stationary plane horizontal axis
y = moving plane vertical axis
y_c = vertical distance from the x-axis to the wave crest.
Y = stationary reference plane vertical axis
Z = constant

ϕ = velocity potential
Γ = vortex circulation
η = deviation of the free surface from the still water level
μ = water viscosity
θ = jet angle to the horizontal
ρ_a = air density
σ = surface tension
ω = wave angular frequency
ψ = stream function
ψ_s = surface streamline value
∇^2 = Laplacian operator

LITERATURE CITED

BANNER, M.L., 1991. The importance of wave breaking on the sea surface, *Nonlinear Dynamics of Ocean Waves,* (Proceedings of Symposium, May 30–31, Baltimore), BRANDT, A.; RAMBERG, S.E., and SHLESINGER, M.F., eds., pp. 178–189.

BANNER, M.L. and MELVILLE, W.K., 1976. On the separation of air flow over water waves, *Journal of Fluid Mechanics.* 77(4), 825–842.

BISHOP, C.T.; DONELAN, M.A., and KAHMA, K.K., 1992. Shore protection manual's wave prediction reviewed. *Journal of Coastal Eng.,* 17, 25–48.

COKELET, E.D., 1977. Steep gravity waves in water of arbitrary uniform depth, *Philosophical Transactions Royal Society London.,* Ser. A, 286(1335), 183–230.

CSANADY, G.T., 1992. Wavelets and air-sea transfer, *Air Water Mass Transfer* (2nd Symposium, Minneapolis, U.S.A.), 11–14 Sept, ASCE), WILHELMS, S.C. and GULLIVER, J.S., (eds), pp. 563–581.

CUMMINGS, P.D., 1996. Aeration Due to Breaking Waves. PhD Thesis, University of Queensland, Brisbane, Australia, 9th May, 230p.

DEAN, R.G. and DALRYMPLE, R.A., 1991. *Water Wave Mechanics for Engineers and Scientists.* Advanced Series on Ocean Eng. New York: World Scientific, Vol 2.

GENT, P.R. and TAYLOR, P.A., 1976. A numerical model of the air flow above water waves. *Journal of Fluid Mechanics,* 77(1), 105–128.

GRAW, K.-U., 1994. Comparison of wave theories with velocity measurement. *Intl. Symp.: Waves—Physical and Numerical Modelling* (IAHR, CSCE, UBC, Vancouver, Canada, Aug. 21–24), Vol. 2, pp. 561–569.

HARA, T. and MEI, C.C., 1994. Wind effects on the non-linear evolution of slowly varying gravity-capillary waves, *Journal of Fluid Mechanics,* 267, 221–250.

HATTORI, M. and AONO, T., 1985. Experimental study in turbulence structures under spilling breakers. *In*: TOBA, Y. and MITSUYASU, H., eds., *The Ocean Surface,* Dordrecht: Reidel, pp. 419–424.

HERBICH, J.B., (ed.), 1990. *Handbook of Coastal and Ocean Engineering, Vol. 1, Wave Phenomena and Coastal Structures.* Houston, Texas: Gulf Publishing.

HOLTHUIJSEN, L.H. and HERBERS, T.H.C., 1985. Observed statistics of breaking ocean waves. *In*: TOBA, Y. and MITSUYASU, H., eds., *The Ocean Surface,* Dordrecht: Reidel, pp. 431–436.

KATSAROS, K.B. and ATAKTURK, S.S., 1992. Dependence of wave-breaking statistics on wind stress and wave development. *Breaking waves* (IUTAM Symposium, Sydney, Aus.), BANNER, M.L. and GRIMSHAW, R.H.J., (eds), pp. 119–132.

LAMB, H., 1945. *Hydrodynamics.* New York: Dover.

LARGE, W.G. and POND, S., 1981. Open ocean momentum flux measurements in moderate to strong winds. *Journal of Physical Oceanography,* 11, pp. 324–336.

LEE, A.; GREATED, C.A., and DURRANI, T.S., 1974. Velocities under periodic and random waves, *Proc. 14th Coastal Eng. Conf.* (June 24–28, Copenhagen, Denmark, ASCE), Chapter 31, pp. 558–574.

LE MÉHAUTÉ, B.; DIVOKY, D., and LIN, A., 1968. Shallow water waves: a comparison of theories and experiments, *Proc. 11th Conf. Coastal Eng.,* 1, 86–107.

LONGUET-HIGGINS, M.S., 1994. The initiation of spilling breakers, *Intl. Symp.: Waves—Physical and Numerical Modelling* (IAHR, CSCE, UBC, Vancouver, Canada, Aug. 21–24), Vol. 1, pp. 24–48.

MAAT, N.; KRAAN, C., and OOST, W.A., 1991. The roughness of wind waves. *Boundary-layer Meteorology,* 54, 89–103.

MILES, J.W., 1957. On the generation of surface waves by shear flows, *Journal of Fluid Mechanics,* 3, 185–204.

MILNE-THOMSON, L.M., 1968. *Theoretical Hydrodynamics*, London: Macmillan.

PHILLIPS, O.M., 1977. *The Dynamics of the Upper Ocean*, Cambridge: Cambridge Uni. Press.

RIENECKER, M.M. and FENTON, J.D., 1981. A Fourier approximation method for steady water waves, *Journal Fluid Mechanics,* 104, 119–137.

ROBERTSON, J.M., 1965. *Hydrodynamics in Theory and Application.* Englewood Cliffs, New Jersey: Prentice-Hall.

SCHWARTZ, L.W. and FENTON, J.D., 1982. Strongly non-linear waves. *Annals Reviews Fluid Mechanics,* 14, 39–60.

SOULSBY, R.L.; HAMM, L.; KLOPMAN, G.; MYRHAUG, D.; SIMONS, R.R., and THOMAS, G.P., 1993. Wave-current interaction within and outside the bottom boundary layer. *Coastal Engineering,* 21, 41–69.

SVENDSEN, I.A. and JONSSON, I.G., 1980. *Hydrodynamics of Coastal Regions,* Lyngby, Denmark: Technical University.

TUMA, J.J., 1987. *Engineering Mathematics Handbook.* New York: McGraw Hill, 498p.

U.S. ARMY ENGINEER WATERWAYS EXPERIMENT STATION COASTAL ENGINEERING RESEARCH CENTER, 1984. *Shore Protection Manual.* Washington, D.C.: Dept. of the Army, US Army Corps of Engineers, USA.

WU, J., 1982. Wind-stress coefficients over sea surface from breeze to hurricane. *Journal of Geophysical Research,* 87(C12), 9704–9706.