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A Fractal Approach to Sea-Surge Occurrences in the Northern Adriatic Sea

A. Mazzarella

Department of Geophysics and Volcanology
University of Naples, Federico II
Largo S. Marcellino, 10
Naples 80138, Italy

ABSTRACT

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The underlying complex physical processes associated with the generation of sea-surges in Venice and in Trieste, located in the Northern Adriatic Sea, are found to be sufficiently scale invariant in respect of their magnitude-time patterns. This provides a rational basis (i) for the computation of the relative sea-surge risk normally underestimated when it is obtained through the classical exponential distributions and (ii) for the identification of fractal time clustering patterns not obtainable from a simple linear approach.

ADDITIONAL INDEX WORDS: *Sea-surge, fractals, Cantor dust.*

INTRODUCTION

The genesis of the sea-surges in the Northern Adriatic Sea is very complex depending on different forcing factors that, interconnected among themselves, are responsible for large increase in sea level with dramatic consequences for the sea cities, vegetation, acquifers, estuaries and sedimentation. The difficulty in finding a simple linear relationship which links all these different factors is the main reason to follow a non-linear approach (MANDELBROT, 1983). The surges in periods of maximum sea level have a greater probability of turning into flooding. An application to the sea-surge occurrences in the cities of Venice and Trieste, located in the Northern Adriatic Sea and subjected to frequent sea-floodings, is provided. Extreme annual sea-surges and the relative times of occurrence recorded in Venice from 1872 to 1995 (PIRAZZOLI, 1982, and updates) and in Trieste from 1875 to 1995 (FERRARO, 1972, and updates) are analysed. The values in Venice are referred to the mean of low and high tides occurred during the 1884–1909 interval, while those of Trieste are referred to 399.3 cm under the benchmark of the National Geographic and Geodetic Service. The distribution of sea surges in Venice and in Trieste is found to be sufficiently scale-invariant in respect to magnitude and times of occurrences. This provides a rational basis for the computation of the sea-surge risk normally underestimated when it is obtained through the classical exponential distribution.

CAUSES OF SEA-SURGES IN THE NORTHERN ADRIATIC SEA

Relatively small increases in sea level could greatly expand the land area subjected to flooding, and thus increase the coastal hazard. The factors responsible for the Northern Adriatic sea-surges are here summarized.

(1) When low pressure passes over the western and central Mediterranean (and particularly over the Ligurian or Tyrrhenian Sea) and then moves toward the north–east generates the Sirocco wind which blows from SE along the main axis of the Adriatic Sea and this is the main source of transport of sea water. This adverse situation may last for several days when a cyclonic depressure is generated in the lee of the Alps over the Ligurian Sea, or in the cold season, when the sea waters are warmer than the air masses and thus generate cyclogenesis over the Ligurian Sea by supplying the air masses with a large amount of heat and moisture. The aerodynamic interaction between the Sirocco and the Alpine chain generates a "Dark Bora" (*i.e.* a fresh wind from NE associated with heavy rainfall which blows below the Sirocco flowing through a gate in the Dinaric Alps, and this causes the additional transport of sea water across the minor axis of the basin. When the Sirocco blows, the rise Dh in sea level ranges from Dh = 20 cm with a wind-speed of 10Km/h to Dh = 100 cm with a wind-speed of 60Km/h (CALOI, 1973); the Bora is responsible for approximately half of this rise.

(2) The Adriatic is a semi-closed sea, and a non-homogeneous distribution of the atmospheric pressure over its surface causes a rise in sea level when the pressure is lower; *i.e.*, as 10.33 m of water corresponds to 1013hPa, in a space gradient of pressure, 1hPa depression causes, locally, the sea level to rise by almost 1 cm. The usual range of Dh induced by this forcing is $10 < Dh < 20$cm, although an upper limit Dh = 45cm may be possible. The pressure pattern over the Mediterranean varies daily, but some weather types are more recurrent in certain seasons.

(3) Rapid changes in the atmospheric pressure pattern cause

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free oscillations in the Adriatic Sea (named "seiches") where the first two resonant periods are of 23 hours and 11.7 hours, respectively both close to the dominant tidal diurnal and semidiurnal periods, and the seiches may last for several days always keeping the same phase (CALOI, 1973). The average rise in the level of the sea caused by the seiche is Dh = 45 cm, although the maximum expected value is double. It was very lucky that, during the extreme surge occurred in Venice in 1966, the seiche was in opposition of phase with the tide, otherwise the situation would have been extremely critical for the survival of the city and its inhabitants.

(4) The tide in the Adriatic is a mixed tide, the semidiurnal and diurnal components being the most important followed by the fortnightly (14.28 days) and nodal (18.6 years) modes (MAZZARELLA and PALUMBO, 1994). In this century, the astronomic lunar-solar factor may be responsible for Dh = 25–80cm.

(5) The mass balance among rainfall, evaporation and the local effect of Atlantic inflow through the Gibraltar strait is responsible for a seasonal variation of about 15 cm in sea-surges with a maximum in the period from October to December, and a mimimum from January to March (MAZZARELLA and PALUMBO, 1991). Again, a secular increase in sea level, associated to sea-water expansion, melting of continental ice and subsidence of the soil, is responsible for an increase of sea level equal to about 4mm/year (MAZZARELLA and PALUMBO, 1991).

OUTLINE OF FRACTAL APPROACH

Just as Poisson distributions model purely random processes, fractal distributions model processes that exhibit scale invariance (self-similarity). If the number of objects N with a characteristic linear dimension greater than r satisfies the relation:

$$N = Cr^{-D} \qquad (1)$$

a fractal distribution is defined, where C is a constant of proportionality and D is the fractal dimension that provides a measure of the clustering of the objects versus r. The more isolated the clusters the smaller the value of D. The basic concept of a fractal distribution is that a phenomenon will be repeated on different scales in the same manner, the major variable being the fractal dimension which is used as a measure of the nature of the phenomenon. D is always higher (provided it is a fractal set) or equal (if it is not a fractal, but a simple Euclidean subset) with the topological dimension D_T of the set where $D_T = 0$ (point), $D_T = 1$ (line), $D_T = 2$ (surface), $D_T = 3$ (space) *etc*, for higher dimensions. Also, D is lower or equal to topological dimension of metric space in which the set belongs (MANDELBROT, 1983). As mathematical representation, the relation (1) could be valid over an infinite range; however, for physical applications, there will be upper and lower limits on the applicability of the fractal distribution. Appreciating the nature of the scaling region of a fractal process could be the key to understanding why the system exists. It is here investigated the applicability of the power law (1) relating the number of the sea-surges to their linear dimension r separately chosen as being the magnitude and the time of occurrence.

Sea-Surges and Magnitude

An important question for estimating the sea surge risk concerns the assumption of the relative frequency distribution to be followed. In the widely used traditional exponential models (GUMBEL, 1958), the magnitude m of the sea-surges is hypothesized to be a random variable distributed with a cumulative distribution function:

$$N = C \exp(-bm)$$

where C is a constant of proportionality. The specific limits, inside which N is related to m according to an exponential law, are obtained verifying the confidence level of the relationship:

$$\ln(N) = \ln(C) - bm \qquad (2)$$

In a fractal approach, the magnitude of the investigated sea-surges is assumed to be distributed with a cumulative distribution function:

$$N = Cm^{-b}$$

where C is a constant of proportionality. The specific limits, inside which N is related to m according to a power law, are obtained verifying the confidence level of the relationship:

$$\log(N) = \log(C) - b\log(m) \qquad (3)$$

When the sea-surge magnitudes are treated as continuous functions, the fractal dimension D can be obtained through the exponent b (TURCOTTE and GREEN, 1993):

$$D = 2 - 1/b$$

A scale-invariant distribution of sea-surges can be also expressed in terms of the index of severity for future sea-surges:

$$F = 10^{(2-D)} \qquad (4)$$

(TURCOTTE and GREEN, 1993): the higher the value of F the more likely that severe sea surges will occur.

Sea-Surges and Time of Occurrence

The fractal relationship of time distribution of sea-surge magnitudes verified to be scale invariant is investigated on the basis of the Cantor dust model (MANDELBROT, 1983; LUONGO *et al.*, 1986a; 1986b). The interval t_o, over which the series of N events occurs, is divided into a series of n smaller intervals $t = t_o/n$ with n = 2, 3, 4 .. and the fraction Pr = N/n of intervals of length t occupied by events is computed. If the distribution of events has a power-law structure then one obtains:

$$Pr = N/n = Ct^{(1-D)}$$

or, equivalently, on a log-log scaled plane:

$$\log(Pr) = \log(C) + (1-D)\log(t) \qquad (5)$$

where C and D are constants. D is estimated from the regression coefficient of linear relationship (5) when it is found to be confident at not less than 99% and to deviate signifi-

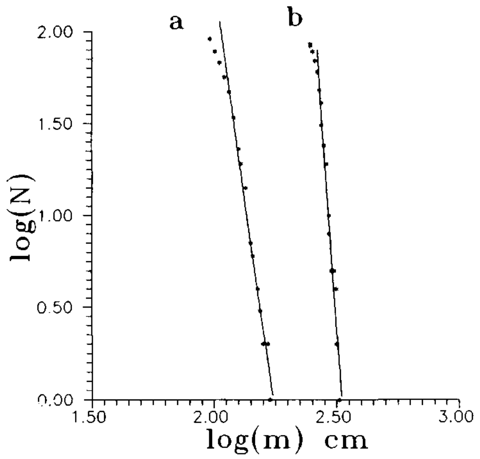

Figure 1. Observed cumulative frequencies (star points) (with an increment of 5cm) of yearly extreme sea level data in Venice (interval 1872–1995) and in Trieste (b) (interval: 1875–1995). The best fitting regression line of log(N) on log(m) is also reported.

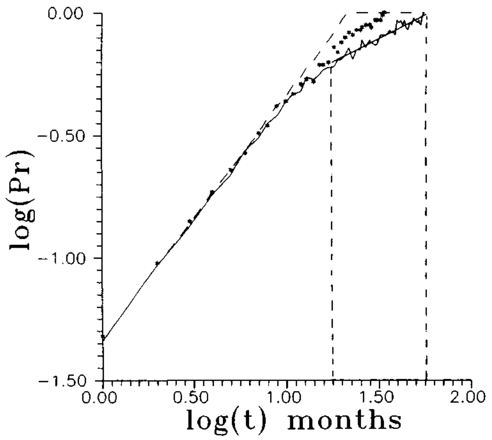

Figure 2. The logarithm of the fraction Pr of time intervals of length t (in months) including the sea-surge events in Venice as a function of log(t). Continuous line links the observed data while the star and dashed lines represent the relative random and uniform simulations, respectively. Vertical dashed lines represent the lower and upper limits of log(Pr) inside which the linear slope provides the best fitting to the data.

cantly, in a specific time range, from the correspondent simulations obtained when the same number N of events is equally or randomly spaced in time (FELLER, 1968). Such a time range is the scale region over which the time structure of N events is scale-invariant. It is easy to show that D is a fractal dimension (MANDELBROT, 1983) for which a regular time distribution of events causes an increase of D from zero that is the dimension of a point up to the limiting value D = 1 that is the dimension of a line.

RESULTS

Sea-Surges and Magnitude

In calculating the cumulative frequency distribution in the log-log plane, the values of the yearly extreme values of sea level are grouped into intervals which are sufficiently large to account for the error due to the method of estimating the sea level height. After preliminary tests, an interval of 5 cm yields the best constrained shapes both for Venice and for Trieste data sets (Figure 1) and the best linear fitting occurs for 67 events of magnitude greater than 105 cm in Venice and for 60 events greater than 260 cm in Trieste. The parameters of the exponential (2) and of the fractal relationship (3), both computed at a confidence level greater than 99% level, are reported on Table 1 together with the values of the index F computed according to (4). The values of the mean periods of return computed according to power-law relationship for some damaging sea-surges in Venice and in Trieste are found systematically shorter than those computed according to exponential one.

Sea-Surges and Time of Occurrence

The Cantor dust model is applied to the 67 sea-surges in Venice and to the 60 sea-surges in Trieste whose magnitude structure is verified to be reasonably scale invariant. The fraction Pr of time intervals including a sea-surge is plotted as a function of the interval size t on a log-log scaled plane (Figures 2, 3). The smallest time interval is chosen to be 1 month. For values of t ranging from 1 to 10 months, Pr in Venice and in Trieste is found to be not significantly different from the correspondent random and uniform simulations; single sea-surges therefore occur in each time interval and no clustering is observed (D = 0); for t > 56 months in Venice and t > 126 months in Trieste each interval contains an event (D = 1); for 18 < t < 56 months a fractal relationship is observed with a fractal dimension D = 0.62 ± 0.01 in Venice and 0.63 ± 0.01 in Trieste, confident at a level greater than 99% and significantly different from the correspondent uniform and random simulations.

DISCUSSION

The complexity of a natural phenomenon does not depend on the number of causes that govern it but essentially on the number of their interconnections, on the magnitude of such linkages and on the feed-back processes. In this case, the whole system is more than the sum of its parts and its re-

Table 1. *Values of parameters of power-law and exponential models for sea-surge events greater than 105 cm and 260 cm occurred in Venice and in Trieste, respectively. Here N is the total number of events with magnitude greater than m; b and C are constant; D is the value of fractal dimension and F is the severity index of sea-surge computed according to (4).*

Fractal Model: log(N) = log(C) − blog(m)	
Venice: log(C) = 19.95 ± 0.70	Trieste: log(C) = 47.52 ± 2.43
b = 8.89 ± 0.33	b = 18.85 ± 0.98
D = 1.89	D = 1.95
F = 1.30	F = 1.13
Exponential Model: ln(N) = ln(C) − bm	
Venice: ln(C) = 11.21 ± 0.24	Trieste: ln(C) = 21.59 ± 0.78
b = 0.065 ± 0.005	b = 0.066 ± 0.005

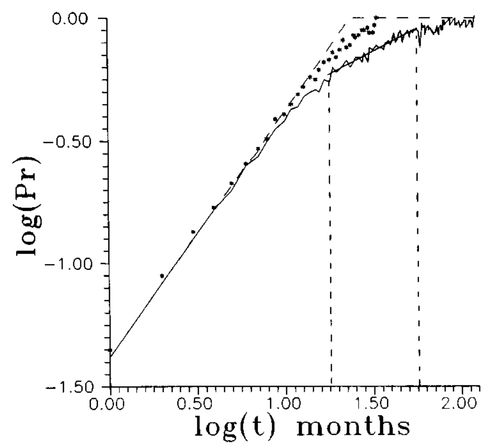

Figure 3. The logarithm of the fraction Pr of time intervals of length t (in months) including the sea-surge events in Trieste as a function of log(t). Continuous line links the observed data while the star and dashed represent the relative random and uniform simulations, respectively. Vertical dashed lines represent the lower and upper limits of log(Pr) inside which the linear slope provides the best fitting to the data.

Table 2. *Values of mean return periods provided by power-law and exponential models for different damaging sea-surge intensities recorded in Venice and in Trieste.*

	Venice	
	Power-law Model	Exponential Model
Intensity	Return period	Return period
190 cm	243 years	375 years
200 cm	385 years	719 years
	Trieste	
	Power-law Model	Exponential Model
Intensity	Return period	Return period
350 cm	232 years	385 years
360 cm	477 years	744 years

sponse to an external forcing does not follow a linear but a power-law relationship. The sea surges occurring in the Northern Adriatic Sea are due to several forcing factors: a low pressure passing over the Mediterranean and generating a Sirocco wind; the barometric effect associated with a gradient of atmospheric pressure over the sea waters; free oscillations in the Adriatic sea; solar and lunar influences; mass balance among evaporation, rainfall and Atlantic inflow; sea water expansion and melting of continental ice; subsidence of the soil. So, the identification of fractal behaviours in the magnitude-time pattern, at least over a limited range of scales, can be taken as evidence of the complexity of the sea-surge phenomenon in the Northern Adriatic Sea. An important question for estimating the sea-surge risk concerns the assumption of the relative frequency distribution to be followed. Traditional exponential sea-surge models assume extreme annual sea level data to be independent, so neglecting the large non stationarity related to the systematic presence of biennal, 11-yr, 18-yr and secular signals (MAZZARELLA and PALUMBO, 1991). On the contrary, the power-law sea-surge models allow to reduce such a non-stationarity for which the sea-surge risk estimated through an exponential model is largely underestimated in respect to that computed according to the fractal model (Table 2). The index of severity F found to be higher in Venice (1.30) than in Trieste (1.13) indicates that more severe future sea-surges will occur in Venice than in Trieste. The application of the Cantor dust model to the occurrence-times of sea-surges allows to identify different patterns of clustering: a region of large clustering with a dimension equal to zero (that is an Euclidean dimension of a point) for $1 < t < 10$ months; a region of weak clustering with a fractal dimension equal to 0.6 (*i.e.* with an Euclidean dimension intermediate between that of a point and that of a line) for $18 < t < 56$ months; a region of no clustering with a dimension equal to 1 (that is an Euclidean dimension of a line) for $t > 56$ months in Venice and for $t > 100$ months in Trieste.

CONCLUSIONS

The results obtained in this paper should be considered as being preliminary because relatively few stations have been considered. Many geophysical phenomena governed by different factors interconnected among themselves obey power-law statistics over relatively wide range of scales (MANDELBROT, 1983). It should not be surprising that the variety of factors that determine sea-surges in the Northern Adriatic Sea also produces fractal behaviours with no particular scale connected with their magnitude-time patterns, at least over limited range of scales.

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