Journal of Coastal Research	14	1	291–298	Royal Palm Beach, Florida	Winter 1998

Wave Reflection from Beaches: A Predictive Model

A. Baquerizo†, M.A. Losada† and J.M. Smith‡

† Ocean and Coastal Research Group
University of Cantabria
39005 Santánder, Spain

‡ U.S. Army Engineering Waterways Experiment Station and Coastal Engineering Research Center
Vicksburg, MS 39180, U.S.A.

ABSTRACT

BAQUERIZO, A.; LOSADA, M.A., and SMITH, J.M., 1998. Wave reflection from beaches: a predictive model. *Journal of Coastal Research*, 14(1), 291–298. Royal Palm Beach (Florida), ISSN 0749-0208.

A model to predict reflection of random waves on a beach, including dissipation due to breaking, is presented. The evolution of the local reflection coefficient and the incident and total wave height is computed. Two initial conditions are needed; the incident H_{rms} and the reflection coefficient, R, far offshore. The expected value of the wave breaking dissipation is estimated following BATTJES and JANSSEN, (1978). The expected value of the reflected energy flux per unit area of beach profile is obtained hypothesizing that (1) wave reflection may be considered a linear process dependent only on the local profile geometry and wave period and (2) only the non-breaking waves contribute to the reflected flux of energy (BAQUERIZO, 1995). These results are compared with wave parameters measured during the SUPERTANK Project and with the reflection coefficient estimated by three different methods. Measured and computed data agree well for barred and non-barred beach profiles. It is shown that the local reflection coefficient evolves along the beach profile. An overall beach reflection coefficient should be defined only offshore of the depth where waves start to break and where the contribution to the wave reflection process is negligible.

ADDITIONAL INDEX WORDS: *Wave propagation, wave reflection, beach morphodynamics, wave transformation model.*

INTRODUCTION

In the surf zone, wave energy is dissipated and transformed into turbulent energy by wave breaking. BAQUERIZO (1995) and BAQUERIZO *et al.* (1996) showed that under certain conditions wave reflection along the beach profile may not be negligible. They found that the reflection coefficient evolves across the beach profile, implying that reflection takes place along the profile and not entirely at the shoreline. In this paper, a model to predict the evolution of random wave breaking and wave reflection on a beach is proposed. The model is applied to SUPERTANK experiment data and results obtained by BAQUERIZO *et al.* (1996) are predicted.

Quantitative prediction of the random wave train variations in the nearshore region requires specification of the variation of mean wave energy flux. Seeking an approach based on the energy balance equation, in this paper, models for dissipation of wave energy by breaking and for reflection of wave energy by the beach slope are required. Following BATJJES and JANSSEN (1978), hereinafter BJ, THORNTON and GUZA (1983), and DALLY *et al.* (1985), wave decay by breaking in the surf zone can be predicted by solving simultaneously the wave energy and wave momentum balance equations.

All previous energy dissipation models deal only with an incident, regular or irregular, wave train. If the reflected wave train must be included, several approaches may be followed. The most proper way is to solve the elliptic problem of wave propagation, specifying the wave reflection and the wave dissipation terms. This approach is a challenge. A second approach is to apply a splitting method to reduce the elliptic form to two coupled equations describing the forward- and back-scattered wave motion respectively. Again, wave reflection and dissipation terms are needed. This type of approach has been used by several authors: DAVIES and HEATHERSHAW (1984), MEI (1985), and KIRBY (1985). KIRBY and VENGAYIL (1988), for shallow-water motion, derived a set of coupled evolution equations for incident and reflected waves which apply in regions where strong reflection may significantly affect wave evolution. Furthermore, they developed a linear damping model which distributes damping uniformly over all frequencies.

In this paper, a far simplier approach is followed. The incident and reflected wave trains are decoupled by applying the hypothesis, like Miche's wave reflection hypothesis (MICHE, 1951), that wave reflection is a linear process dependent only on local beach geometry and wave period. Under these assumptions, the variation of the flux of reflected wave energy per unit flux of incident wave energy per unit area of beach profile, denoted by $V_R(x)$, can be evaluated. Moreover, based on the definition of V_R, an evolution equation for the reflection coefficient, R, is obtained.

The present model is applied to predict the evolution along the beach profile of the root-mean-square wave height, H_{rms}, and of the local reflection coefficient. The energy equation and the reflection coefficient equation are solved simulta-

96109 received 20 September 1996; accepted in revision 10 January 1997.

neously by an explicit finite difference scheme. The SUPERTANK data are used to validate the model. Two cases are considered: non-barred and barred beach profiles. For both cases the prediction of H_{rms} compares well with measurements. Furthermore, the evolution of the local reflection coefficient along the beach profile is confirmed.

This paper is organized as follows. The Fundamentals section provides the background of the present method. Next, the predictive model for reflection and breaking is developed, with subsections discussing the reflection model, the dissipation model, and the energy balance equation. Model results are then compared with SUPERTANK laboratory data. Finally, conclusions are given.

FUNDAMENTALS OF A PREDICTIVE MODEL FOR WAVE REFLECTION AND WAVE BREAKING

At a distance x measured from the shoreline, the local reflection coefficient, $R(x)$, can be written as the quotient of the local reflected and local incident fluxes of energy:

$$R(x) = \left(\frac{\mathcal{F}_R(x)}{\mathcal{F}_I(x)}\right)^{1/2}. \qquad (1)$$

And for linear theory $\mathcal{F}_R(x)$ and $\mathcal{F}_I(x)$ are given by:

$$\mathcal{F}_R(x) = \frac{1}{8}\rho_w g (H_{rms}{}^2(x))_R C_g(f_p, x) \qquad (2)$$

$$\mathcal{F}_I(x) = \frac{1}{8}\rho_w g (H_{rms}{}^2(x))_I C_g(f_p, x) \qquad (3)$$

where $C_g(f_p, x)$ is the group celerity associated to the peak frequency of the wave spectrum, water density is ρ_w, and g is gravitational acceleration. $(H_{rms})_R$ and $(H_{rms})_I$ are the root-mean-square wave heights of the reflected and the incident wave trains, respectively.

At each location on the beach profile, a function $V_R(x)$ can be defined by,

$$V_R(x) = -\frac{1}{\mathcal{F}_I(x)}\frac{d\mathcal{F}_R(x)}{dx} \qquad (4)$$

which represents the local variation of the flux of reflected energy per unit area of beach profile per unit flux of incident energy.

Substituting (1) into (4), the following differential equation is obtained,

$$2R\frac{dR}{dx}\mathcal{F}_I(x) + R^2\frac{d\mathcal{F}_I}{dx} + V_R(x)\mathcal{F}_I(x) = 0. \qquad (5)$$

Given $V_R(x)$ and $\mathcal{F}_I(x)$, (5) defines the evolution of $R(x)$ along the profile.

Given the incident wave energy flux in deep water and the beach profile, $z = -h(x)$, where $h(x)$ is the local water depth relative to the still water level (SWL), the evolution of the total flux of energy (incident and reflected), $\mathcal{F}_T(x)$, along the profile may be evaluated from the wave energy balance equation,

$$\frac{d\mathcal{F}_T(x)}{dx} + D = 0 \qquad (6)$$

where D is the average value of the dissipated power per unit area. $\mathcal{F}_T(x)$ can be written as

$$\mathcal{F}_T(x) = \mathcal{F}_I(x)(1 - R^2). \qquad (7)$$

Using (4) and (7), (6) can be transformed into the following differential equation,

$$\frac{d\mathcal{F}_I}{dx} + V_R\mathcal{F}_I + D = 0. \qquad (8)$$

Eqs. (5) and (8) form a system of equations for $R(x)$ and $H_{rms}(x)$ which may be solved simultaneously by iteration, if expressions for $D(x)$ and $V_R(x)$ are known. Two boundary conditions for integration of the system are needed: $\mathcal{F}_{I0}$ and R_0 at an offshore location, where the subindex 0 denotes deep-water conditions. The offshore location must be sufficiently deep so that the contribution to the reflection process is negligible, that is the local reflection coefficient is almost constant.

REFLECTION MODEL FOR RANDOM WAVES

As noted above, no model based on the energy balance, for predicting energy variation in random breaking and reflecting waves on a beach is available. Several authors (GORING, 1978; KIRBY and VENGAYIL, 1988) have postulated that reflection may be considered a linear process dependent on the geometry of the slope and the wave length. Then, wave height enters only as a scale parameter. Consequently the following hypothesis is formulated:

The contribution to the flux of reflected wave energy per unit area of beach profile per unit incident wave energy, denoted by $V_R(x)$, depends only on the local geometry, beach slope $\tan\beta$ and depth change Δh, and on the local wave length.

Based on this hypothesis, the function $V_R(x)$ given in (4) may be evaluated along the beach profile without knowing in advance the flux of incident wave energy.

Considering the profile as a series of steps of length Δx (Figure 1a), linear theory (LOSADA, 1991; LAMB, 1932) can be applied to obtain the amplitude of the wave reflected by the change in depth. It is also possible to consider the profile as a series of transitions (Figure 1b) and to apply linear non-dispersive long-wave theory (GORING, 1978), see BAQUERIZO (1995) for details. Appendix A includes the derivation of V_R for a step using linear theory.

Because the approach is linear, these solutions can be used to evaluate V_R for an irregular incident wave train by linearly superimposing a large number of wave components of different frequencies and amplitudes. Thus, the local variation of reflected wave energy flux can be evaluated as the sum of the contributions of each spectral component as follows,

$$V_R\mathcal{F}_I = \sum_i V_{Ri}\frac{1}{2}\rho_w g A_i{}^2 C_{gi}\Delta f \qquad (9)$$

where A_i is the amplitude of the component i with frequency $f_i = (i - 1)\,\Delta f$, C_{gi} is the group celerity for f_i, and Δf is the frequency band discretization. $V_{Ri}\,\Delta f$ denotes the variation of

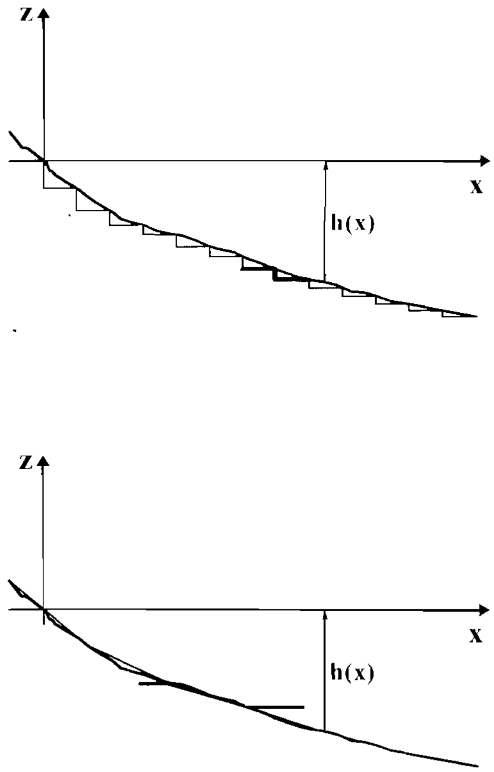

Figure 1 a. Scheme of the beach profile as a series of steps of length Δx. b. Scheme of the beach profile as a series of transitions of length Δx.

the reflected energy flux per unit incident flux of energy for the frequency f_i.

To simplify the calculation, the reflected energy flux is approximated by:

$$V_R\mathcal{F}_I \simeq \beta_R\left[\sum_i V_{Ri}(f_i)\right]\frac{1}{8}\rho_w g H_{rms}{}^2 C_g(f_p) \quad (10)$$

where the coefficient β_R is included to correct errors introduced by the approximation.

Figure 2 shows the evolution of V_R along the beach profile obtained with linear theory (discretization as a series of steps) and eq. (10). Notice that for $(h/L) > 0.15$ the contribution to the reflection process per unit flux of incident wave energy is negligible.

If waves are breaking or broken, it is assumed that they can not contribute to the reflected wave motion. In the application to random waves, we are interested in the expected value of the variation of the reflected flux of energy per unit area. This can be estimated by applying (10) only to the non-breaking waves. Denoting the probability that a wave breaks as Q_b, the probability of non-breaking waves is $1 - Q_b$. Thus, the mean variation of the flux of reflected energy per unit area is given by,

$$\bar{V}_R(x)\mathcal{F}_I = V_R(x)(1 - Q_b)\mathcal{F}_I. \quad (11)$$

Notice that $\bar{V}_R$ is a function of the unknown local H_{rms} and, as will be seen later, Q_b is a function of H_{rmsT}/H_{max} (BJ), where H_{max} is the maximum wave height for a given water depth. When all the waves are breaking or broken, $Q_b = 1$ and $\bar{V}_R\mathcal{F}_I = 0$, and there is no more contribution to the flux of reflected wave energy.

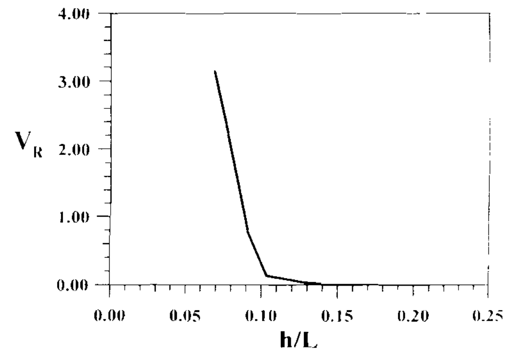

Figure 2. Evolution of $V_R(x)$ along the beach profile for run 1

Dissipation Model For Random Waves

Several analytic and numerical models for the prediction of dissipation of energy in random waves breaking on a beach have been proposed. BJ, THORNTON and GUZA (1983) and DALLY (1990, 1992) are some of the most popular. For the present analysis, the BJ model is adopted. The mean energy dissipation per unit area of beach profile is given by,

$$D = \frac{\alpha_p}{4}Q_b\bar{f}\rho_w g H_{max}{}^2\frac{H_{max}}{h} \quad (12)$$

where $\bar{f}$ is the mean frequency of the irregular wave train and α_p is a constant of order one.

The breaking height may be evaluated following BJ, but working with $H_{rms,T}$ instead of $H_{rms,I}$. The equation for the maximum wave height, including two empirical coefficients, is:

$$H_{max} = \frac{\alpha_p}{k_p}\tanh\frac{\gamma k_p h}{\alpha_p}. \quad (13)$$

BJ suggested $\alpha_p = 0.88$ and $\gamma = 0.8$. Once H_{max} is determined from (13) and $H_{rms,T}$ is known across the profiles, Q_b can be calculated from,

$$\frac{\ln Q_b}{1 - Q_b} = -\left(\frac{H_{max}}{H_{rms,T}}\right)^2. \quad (14)$$

Eq. (14) is similar to BJ's equation for Q_b, developed under the assumption that non-broken waves in the incident wave train are Rayleigh distributed. In (14), this assumption is applied to the total wave train (incident and reflected). BAQUERIZO *et al.* (1996) showed that the wave height distribution of the total wave train may be approximated by a Rayleigh distribution where the parameter of the distribution includes the correlation between both wave trains.

Energy Balance Equation

Having established the dependence of the average dissipation rate, D, and of the average reflection rate, $\bar{V}_R\mathcal{F}_I$, on

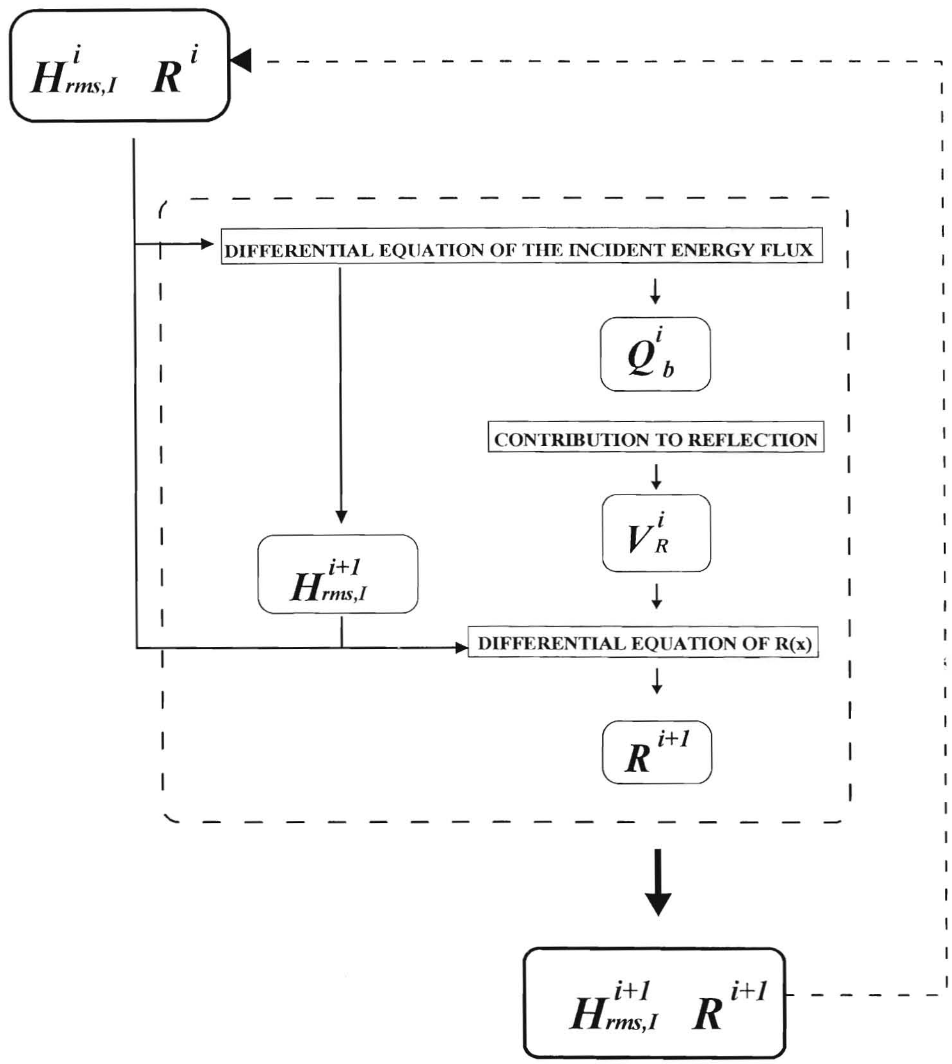

Figure 3. Sketch of the simultaneous solution of the differential equations.

the characteristics of the incident wave train ($H_{rms,\,I}$, f_p, and $\bar{f}$) and on the profile geometry, $h(x)$, the energy balance equation now becomes:

$$\frac{d\mathcal{F}_I(x)}{dx} + \bar{V}_R(x)\mathcal{F}_I(x) + D_*(x) = 0 \qquad (15)$$

where $D_*(x)$ is given by (12) and $\bar{V}_R(x)\mathcal{F}_I$ is given by (11).

Similarly, the differential equation for $R(x)$ now is:

$$2R\frac{dR}{dx}\mathcal{F}_I(x) + R^2\frac{d\mathcal{F}_I}{dx} + \bar{V}_R\mathcal{F}_I = 0. \qquad (16)$$

This equation closes the system of equations for H_{rms} and R. For given depth profile, $h(x)$, and mean and peak period, $V_R(x)$ can be obtained for a frequency band ($f_{\min}$, $f_{\max}$), see (A11); and given incident wave height H_{rms} and a choice of α_p, γ, and β_R, (15) can be solved to find $H_{rms,\,I}(x)$. Finally, given R at a location where the contribution to the reflection is negligible, (16) can be integrated to find $R(x)$. Figure 3 sketches the simultaneous solution of the two differential equations.

THE SUPERTANK EXPERIMENT

Various data sets from the SUPERTANK project conducted in a 104 m long, 3.7 m wide and 4.6 m deep wave tank, have been presented in previous papers (*e.g.*, KRAUS and SMITH, 1994; SMITH, 1994; BAQUERIZO *et al.*, 1996). Only the essential aspects of the present data analysis are described in the following. Table 1 lists the six runs analyzed herein. For simplicity, run number 1–6 is used instead of the lengthy SUPERTANK number. The run duration was 20–70 min. The peak period T_p = 3 or 5 sec, and the peak enhancement factor γ_p = 20 and 100, corresponding to the relatively narrow spectra and narrow spectra, respectively. Sixteen resistances wave gages were

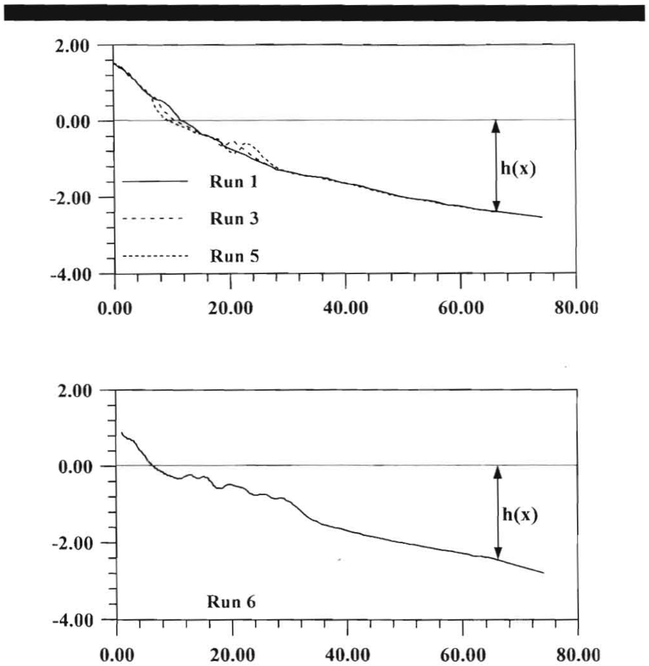

Figure 4. Beach profiles of runs 1 to 5 (a) and of run 6 (b).

used to measure the free surface oscillations. Runs 1–5 were intended to examine the profile evolution of a fine sand beach under the same incident random waves. Figure 4 a shows the beach profiles at the start of runs 1, 3, and 5. The vertical coordinate z in Figure 4 is taken to be positive upward with $z = 0$ at SWL. The beach profiles were surveyed before and after each run. Figure 4 indicates the development of a small bar through the five consecutive runs. On the other hand, Figure 4 b shows the beach profile at the start of run 6 corresponding to the narrower spectrum and longer peak period, as listed in Table 1. This profile exhibits a long flat bar shoreward of gage 5, while the profile seaward of gage 5 is essentially the same as that for runs 1–5.

Baquerizo (1995) and Baquerizo *et al.* (1996) used this data set to calculate the evolution of the reflection coefficient along the beach profile by three different methods. The evolution pattern of R along the beach obtained by the three different methods is the same.

RESULTS

In this section the results obtained with the theoretical model are compared with experimental data from the SUPERTANK Project, measured H_{rms} and computed reflection coefficient, R (Baquerizo *et al.*, 1996). Only results of runs 1, 3, 5 and 6 are shown.

For the non-barred beach profile, run 1, the following parameters are used for comparison: $R(x)$, $H_{rms,I}$, and $H_{rms,T}$ (see Figure 5). All the computations were done with the breaking parameters $\alpha_p = 0.88$ and $\gamma = 0.8$.

For the barred profiles, runs 3 and 5, the evolution of $R(x)$ $H_{rms,I}$, and $H_{rms,T}$ are compared in Figures 6 and 7. For these

Table 1. *Wave characteristics of SUPERTANK runs.*

Run Number (1)	SUPERTANK Run Number (2)	Run Duration (min) (3)	T_p (s) (4)	γ_p (5)
1	A0509A	20	3	20
2	A0510A	40	3	20
3	A0512A	70	3	20
4	A0515A	70	3	20
5	A0517A	70	3	20
6	A2007B	40	3	100

profiles, bed return flow velocities were relatively strong shoreward of the bar crest and weak seaward of the bar where the depth increases (Smith, 1994). The strong current appears to occur offshore of the inception of wave breaking. Parameter values $\alpha_p = 0.5$ and $\gamma = 0.88$ over the bar, were found to represent the breaking process. This result is supported by the experimental work of Sakai *et al.* (1988), who showed that the presence of a current flowing against wave propagation on a sloping bottom affects the depth of breaking. All other parameters used in the calculations are the same as for the non-barred beach profile.

The evolution of $H_{rms,T}$ is well predicted, at least until the region where most of the waves break. The agreement between $R(x)$ computed by the three methods and predicted $R(x)$ is fairly good for all the cases. $R(x)$ evolves monotonically along the beach profile, from a constant value offshore of the breaking zone.

In Figure 8 the evolution of predicted and measured $H_{rms,I}$ and $H_{rms,T}$ for run 6 are given. Moreover, the evolution of $R(x)$ obtained by the prediction and the separation methods in Baquerizo *et al.* (1996) are shown. It is clearly seen that the

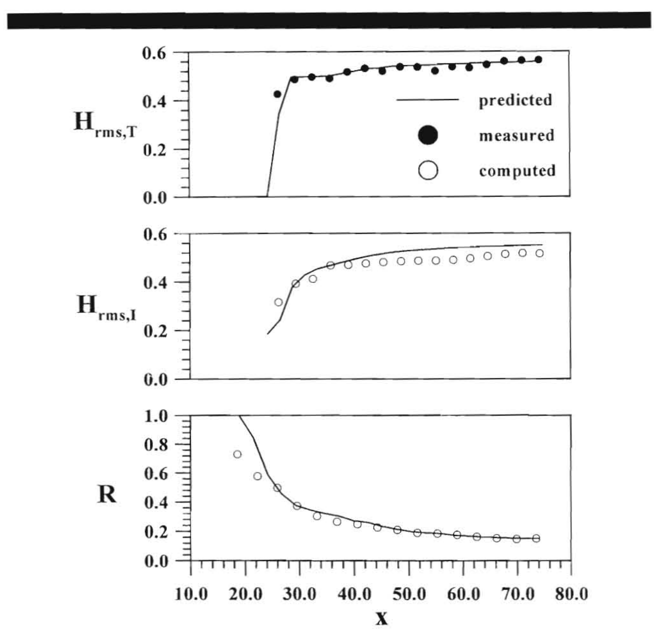

Figure 5. Evolution of R, $H_{rms,I}$, and $H_{rms,T}$ along the beach profile (run 1).

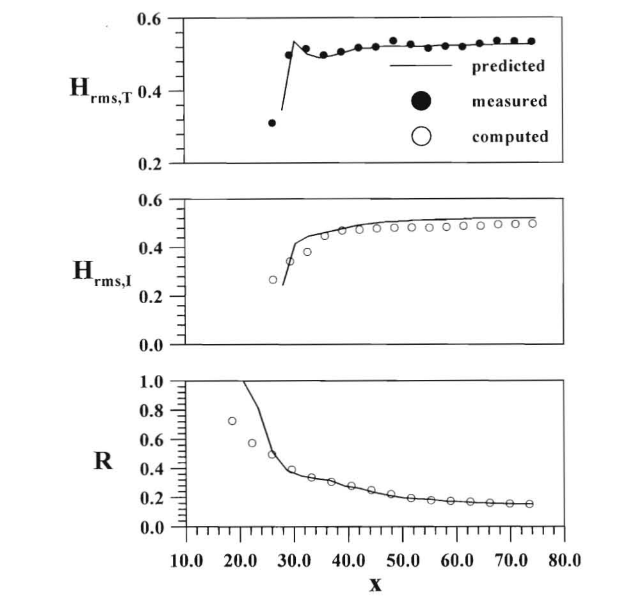

Figure 6. Evolution of R, $H_{rms,I}$, and $H_{rms,T}$ along the beach profile (run 3).

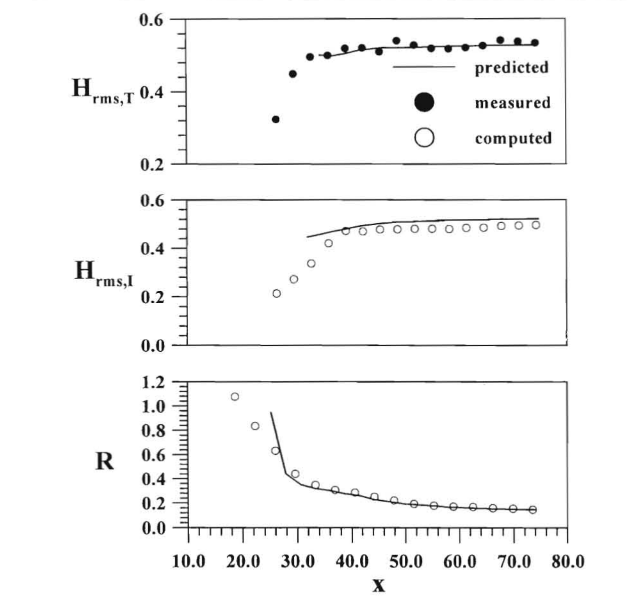

Figure 7. Evolution of R, $H_{rms,I}$, and $H_{rms,T}$ along the beach profile (run 5).

predictive model reproduces the overall behavior of $H_{rms,T}$ fairly well, but does not match the oscillating pattern. Again the evolution of $R(x)$ is adequately predicted by the present method.

CONCLUSIONS

A model which predicts dissipation due to breaking and reflection of random waves on a beach is presented. The evolution of the incident and total H_{rms} and the local reflection coefficient, R, is computed. Two initial conditions are needed: the incident H_{rms} and the overall beach reflection coefficient. The expected value of the wave breaking dissipation is estimated following BJ. For the non-barred beach profiles, the value of the breaking parameter α_p proposed by BJ is used. For barred beach profiles, α_p has to be decreased about 40 percent over the bar. The bed return flow seems to induce wave breaking in deeper depths.

The hypothesis that reflection may be considered to be a linear process, dependent only on the local profile geometry (slope and water depth variation) and wave period, provides a good estimation of the local rate of reflected energy flux. The local contribution to the reflected flux of wave energy may be obtained by modeling the beach profile with a series of steps or transitions and applying linear theory to each of them. For random waves, the expected value of the reflected flux of wave energy per unit area may be obtained by applying the above hypothesis to the non-breaking waves only. Model comparison to data measured in the SUPERTANK project show good prediction results.

The predictive model confirms the results obtained by BAQUERIZO *et al.* (1996) that the local reflection coefficient, R, grows toward the shoreline. An overall beach reflection coefficient may be defined offshore of the depth where waves start to break (h_b) and where the contribution to the process of wave reflection is negligible (h_R). These two conditions may be matched by depths $h_b > 1.5\ H_{max}$ and $h_R > 1/4\ L_p$.

ACKNOWLEDGEMENTS

Funding support for the first and second authors was provided by the Commission of the European Communities in

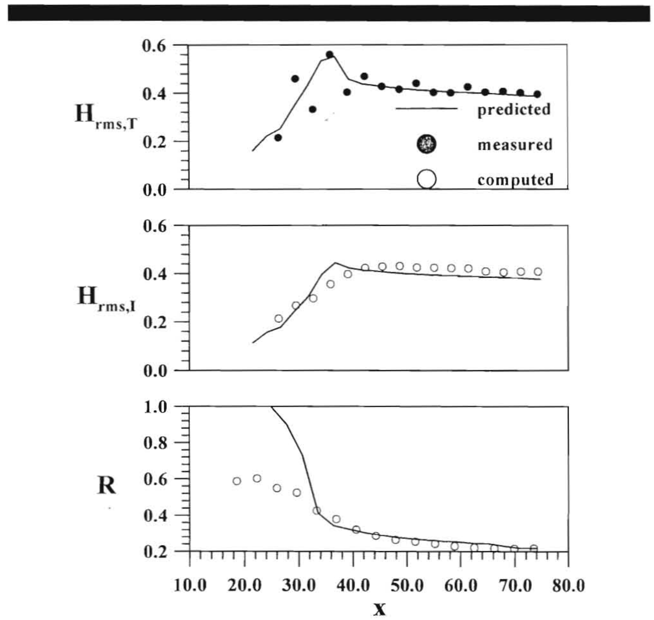

Figure 8. Evolution of R, $H_{rms,I}$, and $H_{rms,T}$ along the beach profile (run 6).

the framework of the Marine Science and Technology Programme (MAST) under contract no. MAS2-CT92-0030. The paper was developed under a joint research effort between the Ocean and Coastal Research Group from the University of Cantabria, Spain, and the USAE Waterways Experiment Station, Coastal Engineering Research Center, Vicksburg, Miss. Permission was granted by the Chief of Engineers, U.S. Army Corps of Engineers, to publish this information. We acknowledge D. Huntley for helpful discussions.

LITERATURE CITED

BAQUERIZO, A., 1995. Reflexión del Oleaje en Playas. Métodos de Evaluación y de Predicción. Ph.D. Dissertation, Universidad de Cantabria, Santander, Spain, 180 p.

BAQUERIZO, A.; LOSADA, M.A.; SMITH, J.M., and KOBAYASHI, N., 1996. Cross-shore variations of wave reflection from beaches. *Accepted for publication* in ASCE Journal Waterway Port Coastal Ocean Engineering.

BATTJES, J.A. and JANSSEN, J.P.F.M., 1978. Energy loss and set-up due to breaking of random waves. *Proceedings 16th Coastal Engineering Conference* (ASCE) 569–587.

DALLY, W.R.; DEAN, R.G., and DALRYMPLE, R.A., 1985. Wave height variation across beaches of arbitrary profile. *Journal Geophysical Research*, 90(C6), 11,917–11,927.

DALLY, W.R., 1990. Random breaking waves: a closed-form solution for planar beaches. *Coastal Engineering*, 14, 233–263.

DALLY, W.R., 1992. Random breaking waves: field verification of a wave-by-wave algorithm for engineering application. *Coastal Engineering*, 16(4), 369–397.

DAVIES, A.G. and HEATHERSHAW, A.D., 1984. Surface propagation over sinusoidally varying topography. *Journal of Fluid Mechanics*, 144, 419–443.

GORING, D.G., 1978. *Tsunamis, the propagation of long waves onto a shelf.* Rep. No. K.H.-R.38 W.M. Keck Laboratory of Hydraulics and Water Resources, California Inst. Tech., 337p.

GONZÁLEZ, E.M., 1995. Morfología de Playas en Equilibrio: Planta y Perfil. Ph.D. Dissertation, Universidad de Cantabria, Santander, Spain, 270p.

KIRBY,J.T., 1985. A general wave equation for waves over rippled beds. *Journal of Fluid Mechanics*, 162, 171–186.

KIRBY, J.T. and VENGAYIL, P., 1988. Non resonant and resonant of long waves in varying channels. *Journal of Geophysical Research*, 93(C9), 10,782–10,796.

KRAUS, N.C. and SMITH, J.M., 1994. *SUPERTANK Laboratory Data Collection Project, Volume I: Main Text*, Technical Report CERC-94-3, US Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, 274p.

LAMB, H., 1932. *Hydrodynamics*. Cambridge Univ. Press.

LOSADA, I.J., 1991. Estudio de la Propagación de un Tren Lineal de Ondas por un Medio Discontinuo. Ph.D. Dissertation, Universidad de Cantabria, Santander, Spain, 183p.

MEI, C.C., 1985. Resonant reflection of surface waves by periodic sand-bars. *Journal of Fluid Mechanics*, 152, 315–335.

MICHE, R., 1951. Le pouvoir réfleéchissant des ouvrages maritimes exposés à l'action de la houle. *Ann. Ponts Chaussées*, 121, 285–319.

SAKAI, S.K.; HYTAYAM, K., and SAEKI, M., 1988. A new parameter for wave breaking with opposing current on a sloping sea bed. *Proceedings 21st Coastal Engineering Conference* (ASCE), pp. 1035–1042.

SMITH, J.M., 1994. Undertow at SUPERTANK. *Coastal Dynamics '94*, pp. 220–231.

THORNTON, E.B. and GUZA, R.T., 1983. Transformation of wave height distribution. *Journal of Geophysical Research*, 88(C10), 5925–5938.

□ RESUMEN □

Se presenta un modelo para predecir la reflexión del oleaje en playas, incluyendo la disipación producida por la rotura. El modelo calcula el coeficiente de reflexión local y la altura de ola de los trenes incidente y total. Se necesitan dos condiciones iniciales: la H_{rms} del tren incidente y el coeficiente de reflexión en un punto alejado de la costa. La disipación media producida por la rotura se estima siguiendo a BATTJES y JANSSEN, 1978. El flujo medio de energía reflejada por unidad de área de perfil de playa se obtiene bajo las hipótesis: (1) la reflexión puede considerarse un proceso lineal que depende únicamente de la geometría local del perfil y del período del oleaje y (2) sólo las olas que no han roto contribuyen al flujo de energía reflejada. El modelo se aplica con los datos medidos durante el Proyecto SUPERTANK. Los resultados se comparan con los parámetros del oleaje medidos y con el coeficiente de reflexión estimado por tres métodos diferentes. El ajuste es bueno tanto para perfiles con barra como sin ella. Se demuestra que el coeficiente de reflexión local evoluciona a lo largo del perfil de playa. Puede definirse un coeficiente de reflexión total de la playa en un punto alejado de la posición en la cual las olas empiezan a romper y donde la contribución al proceso de la reflexión es despreciable.

APPENDIX I. LINEAR WAVE PROPAGATION ON A STEP

Predictions of the function V_R for a single step is based on wave propagation using the velocity potential function as follows (see Figure A1),

$$\frac{\partial^2\Phi_i}{\partial x^2} + \frac{\partial\Phi_i}{\partial z^2} = 0 \qquad 0 < z < h \qquad i = 1, 2 \tag{A1}$$

$$\frac{\partial\Phi_i}{\partial z} + \frac{\sigma^2}{g}\Phi_i = 0 \qquad z = 0 \qquad i = 1, 2 \tag{A2}$$

$$\frac{\partial\Phi_i}{\partial z} = 0 \qquad z = -h_i \qquad i = 1 \tag{A3}$$

where $\sigma = (2\pi/T)$, T = wave period, and the coordinate system is fixed on the SWL, with z pointing positive upward. The system of equations can be applied to the upstream ($i = 1$) $x < 0$, and downstream region ($i = 2$) $x > 0$.

At the step, $x = 0$, matching conditions must be applied, specifying the continuity of velocity and pressure at the step interface, $x = 0$:

$$\frac{\partial\Phi_1}{\partial x} = 0 \qquad -h_1 < z < -h_1 + \Delta h; \qquad x = 0 \tag{A4}$$

where Δh is the height of the step.

$$\frac{\partial\Phi_1}{\partial x} = \frac{\partial\Phi_2}{\partial x} \qquad -h_1 + \Delta h < z < 0; \qquad x = 0 \tag{A5}$$

$$\Phi_1 = \Phi_2 \qquad -h_1 + \Delta h < z < 0. \tag{A6}$$

The solution to this problem is given elsewhere (LOSADA 1991, GONZÁLEZ 1995),

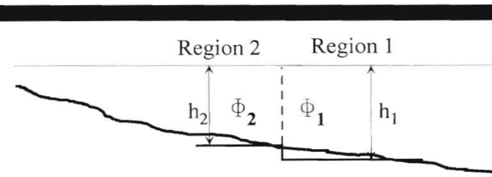

Figure A1. Configuration of the problem for each step.

$$\Phi_1 = I_{11}(z)e^{-ik_{11}x} + \sum_{n=1}^{\infty} R_n I_{1n} e^{ik_{1n}x} \qquad \text{(A7)}$$

$$\Phi_2 = \sum_{n=1}^{\infty} I_{2n}(z) T_n e^{-ik_{2n}x} \qquad \text{(A8)}$$

where R_1 and T_1 are the reflected and transmitted coefficients and,

$$I_{in} = \frac{-ig}{2\pi f_n} \frac{\cosh k_{in}(h + z)}{\cosh k_{in} h} \qquad i = 1, 2. \qquad \text{(A9)}$$

The indices $n = 1$ and $n > 1$ correspond to the real, k_{i1}, and imaginary roots, k_{in}, of the dispersion equation,

$$(2\pi f_n)^2 = g k_{in} \tanh k_{in} h_i \qquad \text{(A10)}$$

respectively. Notice that because of the linear character of the problem, the incident wave train is assumed to have unit amplitude, without loss of generality.

The system of equations established using the matching conditions may be solved numerically by truncating each of the infinite sums (A7) and (A9) to order N. A scattering matrix of (2N + 2) × (2N + 2) is obtained which can be solved by standard subroutines.

Next, the function V_R can be evaluated by:

$$V_R = \frac{R_1^2}{\Delta x} \qquad \text{(A11)}$$