Friction Factors for Wave Uprush

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ABSTRACT ▉

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The non-linear shallow water theory contains a set of solutions for the problem of swash following bore collapse on a hydraulically smooth and impermeable beach. These have recently been compared with field data from a number of natural sandy beaches (HUGHES, 1992). The comparison between the inviscid equations and the data was generally favourable; however, the parameters measured were consistently over-predicted by the theory. It is assumed here that this discrepancy is due to energy dissipation effects not originally represented in the theory. The inviscid equation of motion for the shoreline following bore collapse is expanded to include a shear stress term to account for bed friction. This equation is then solved for the time-history of the shoreline position and the maximum swash height. Field measurements of the maximum swash height, initial shoreline velocity, swash depth, beach slope and grain size are used with these equations to determine the inferred friction factor for the uprush. The magnitude of the friction factor is found to be of the order of 0.1 for the sandy beaches considered here. A recent model for the bottom boundary layer in the presence of sheet flow is capable of predicting the magnitude of the observed friction factor.

ADDITIONAL INDEX WORDS: *Non-linear shallow water theory, swash height, shoreline displacement, friction, infiltration, sheet flow, sandy beaches.*

INTRODUCTION

Wave motion in the nearshore zone shapes beach morphology via the transfer of fluid momentum to the bed sediments. The rate and intensity of this momentum transfer and concomitant sediment transport is dependent on both the velocity of the fluid and the flow resistance offered by the bed configuration. A complete understanding of the morphological behaviour of beach systems requires detailed investigation of these complex fluid and sediment interactions in a variety of shallow water environments. The study reported here contributes to this field of investigation by addressing the problem of energy dissipation in the swash zone.

The investigation reported here is restricted to swash motion produced by the collapse of fully developed surf zone bores at the shoreline (see HUGHES, 1992). The approach adopted to address this problem is based on the non-linear shallow water theory. Traditionally this approach treats the surf zone bore as a propagating discontinuity; the shallow water equations are applied on either side of the discontinuity and the bore equations are used to describe the propagation of the discontinuity. The bore can be modelled this way until it reaches the undisturbed shoreline. At this point a singularity exists in the equations. The singularity is interpreted, in a physical sense, as bore collapse and the point of transformation from the bore to the swash phase of the wave runup process (MEYER and TAYLOR, 1972). Following bore collapse, the shallow water equations can again be used to describe the fluid motion during the swash phase (*e.g.*, SHEN and MEYER, 1963; MEYER and TAYLOR, 1972; HIBBERD and PEREGRINE, 1979). The swash phase is very distinct from the surf zone bore phase in both appearance and fluid behaviour (FREEMAN and LEMEHAUTE, 1964; HUGHES, 1992) and can therefore be studied separately, with some information from the bore phase providing the seaward boundary conditions for the swash. Internal turbulent energy dissipation within the bore is important during its propagation across the surf zone and its collapse at the undisturbed shoreline. Once the initial shoreline is crossed and the swash phase begins, energy losses due to local bed friction and infiltration become paramount since bore related dissipation has ceased (FREEMAN and LEMEHAUTE, 1964).

The effects of friction and infiltration on the swash process have been modelled numerically (*e.g.*, FREEMAN and LEMEHAUTE, 1964; KIRKGOZ, 1981; MATSUTOMI, 1983; PACKWOOD, 1983; KOBAYASHI *et al.*, 1988) and measured in a number of laboratory experiments (*e.g.*, KISHI and SAEKI, 1966; MILLER, 1968; KIRKGOZ, 1981), but to date no studies have attempted to specifically address these processes in detail using field data.

Field measurements of swash made on sandy beaches where fully developed bores provided the initial impetus for the swash process have recently been reported in HUGHES (1992). It was found that the non-linear shallow water theory correctly predicted many features of the observed swash behaviour, an observation also supported by the laboratory data reported in YEH *et al.* (1989). The most accurate field measurements obtained by HUGHES were of the time-history of shoreline position during the uprush and the maximum swash height. Although the inviscid theory described the behaviour of these parameters well, their magnitudes were over-predicted. It was found that the maximum swash height reached only 65% of that expected from theory.

Several arguments were presented to suggest that the flow resistance imparted by the bed was responsible for the discrepancy between the inviscid swash equations and the field data. Observations of intense sediment transport, attesting to significant bed shear, and the presence of a blunt leading edge profile for the swash lens were considered to be important in this regard (see also FREEMAN and LEMEHAUTE, 1964; MATSUTOMI, 1983).

The inviscid non-linear shallow water theory predicts that the shoreline motion following bore collapse on a beach can be derived by considering the balance of forces on a small fluid element representing the front of the swash lens (SHEN and MEYER, 1963; HO *et al.*, 1963). The forces previously considered for the inviscid case were the initial acceleration of the moving shoreline, induced by bore collapse, and the gravitational acceleration. Given that the available field data seem to indicate that the inviscid theory can predict the gross physical behaviour of the uprush, it now seems reasonable to expand the original concepts to investigate friction effects. Following KIRKGOZ (1981), a shear stress term will be incorporated into the inviscid equation of motion for the shoreline to derive analytical equations for the time-history of shoreline position and the maximum swash height. The available data set will then be used in conjunction with these equations to provide an estimate of the friction factor for swash on sandy beaches.

It should be stated at the outset that the discrepancy between the measured value of the swash parameters reported in HUGHES (1992) and the predictions of the inviscid theory must be attributable to both friction and infiltration. While the former can be accounted for by introducing a shear stress term into the equation of motion for the shoreline, the latter is less easily dealt with. The loss of fluid into the permeable beach face is expected to contribute to the total flow resistance in a different manner to the energy dissipation effects represented in a shear stress term. The loss of fluid from the swash lens alters the dimensions of the flow and must therefore be functionally related to the swash volume. Swash volumes were not measured, thus the individual contributions of friction and infiltration to the total flow resistance are intractable from the data set.

Despite the poor parameterisation of infiltration in the data set, an analysis of the friction effects is still worth pursuing for two reasons. Firstly, the results from a numerical model developed to study swash following bore collapse on a permeable beach composed of fine-medium sand suggests that the effects of infiltration on the maximum swash height are minimal, although the effect on the backwash is found to be substantial (PACKWOOD, 1983). Secondly, a 'slick' zone produced by the intersection of the water table with the beach face was typically observed during the field experiments. It usually extended across a substantial portion of the active swash zone and must have in effect reduced infiltration to zero.

THEORETICAL BACKGROUND

Equations for Swash on a Natural Beach

The effect of the bed roughness on a natural beach is to produce a shear stress that dissipates energy contained in the flow. This stress acts parallel to the beach and in the opposite direction to the uprush (Figure 1). For hydraulically rough, fully turbulent flows it is often written as

$$\tau = \frac{1}{8}\rho f u |u| \qquad (1)$$

where ρ is the fluid density, f is the friction factor and u is the horizontal water velocity.

If this shear stress is incorporated into the equation of motion for the moving shoreline or leading edge of the swash lens climbing a beach, we have

$$m\frac{d^2X_s}{dt^2} + mg\sin\beta + \tau\delta = 0 \qquad (2)$$

where m is the mass per unit width of the fluid element representing the moving shoreline, X_s is the shoreline position relative to the initial shoreline position at time $t = 0$, g is the gravitational acceleration, β is the beach slope and δ is a nominal length for the fluid element (Figure 1). The beach slope in this analysis is a measure for the active swash zone; typically between the berm crest and beach step. It should be noted that (2) is intended to describe the flow landward of the point of bore collapse and does not, therefore, require a term to describe bore related energy dissipation.

If the horizontal water velocity in the fluid element is assumed to be closely approximated by the shoreline velocity, then (1) can be re-written for the problem at hand;

$$\tau = \frac{1}{8}\rho f\left(\frac{dX_s}{dt}\right)^2. \qquad (3)$$

The modulus has been removed since only the uprush is being considered here. Substituting (3) into (2) and dividing throughout by $m = \rho\delta h_\delta$, where h_δ is the water depth within the fluid element, we have

$$\frac{d^2X_s}{dt^2} + g\sin\beta + \frac{f}{8h_\delta}\left(\frac{dX_s}{dt}\right)^2 = 0. \qquad (4)$$

If g, β, f and h_δ are assumed to be constants, then (4) can be integrated using the separation of variables technique to yield the shoreline velocity U_s;

$$\frac{dX_s}{dt} = U_s(t) = \sqrt{\frac{8gh_\delta\sin\beta}{f}}\tan(F + G) \qquad (5)$$

where F and G are respectively

$$F = -t\sqrt{\frac{gf\sin\beta}{8h_\delta}} \qquad \text{and}$$

$$G = \tan^{-1}\left(\frac{u_o\sqrt{f}}{\sqrt{8gh_\delta\sin\beta}}\right) \qquad (6)$$

and u_o is the initial shoreline velocity at the point of bore collapse ($x = 0$, $t = 0$). Since $X_s = 0$ when $t = 0$, integration of (5) yields
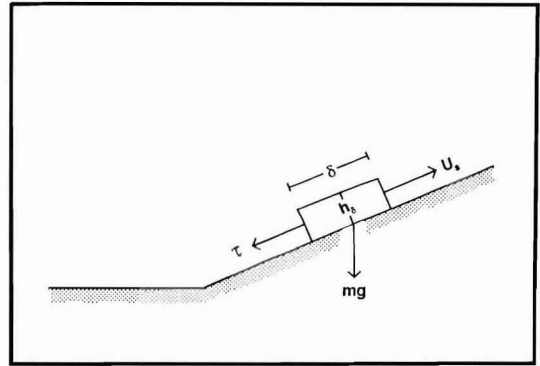

Figure 1. Balance of forces acting on the small fluid element representing the moving shoreline or leading edge of the swash lens.

$$X_s(t) = \frac{8h_\delta}{f}\ln\left(\frac{\cos(F + G)}{\cos G}\right). \qquad (7)$$

The qualitative behaviour of $U_s(t)$ and $X_s(t)$ is the same as that predicted by the inviscid version of the theory (see Ho et al., 1963; HUGHES, 1992), the only effect that the inclusion of the shear stress produces is a reduction in the magnitude of U_s and X_s. When $U_s = 0$ the shoreline is at its maximum landward displacement, and from (5) this occurs when

$$t = \frac{G}{\sqrt{\frac{gf\sin\beta}{8h_\delta}}}. \qquad (8)$$

Substituting (8) into (7) and employing some trigonometry yields the maximum swash height Z_s;

$$Z_s = \frac{-8h_\delta\sin\beta}{f}\ln\cos\left(\tan^{-1}\frac{u_o\sqrt{f}}{\sqrt{8gh_\delta\sin\beta}}\right). \qquad (9)$$

These equations describing the shoreline behaviour on a hydraulically rough beach were first presented by KIRKGOZ (1981), but he used a Chezy Coefficient to formulate the friction effects. The equations assume that the presence of a shear stress at the bed does not alter the gross behaviour of the flow from that predicted by the inviscid non-linear shallow water theory. This assumption now seems justified in the light of recent laboratory and field data available for beaches where fully developed bores provided the initial impetus for the swash cycle (YEH et al., 1989; HUGHES, 1992).

Estimating the Friction Factor

The approach used when calculating the friction factor for most practical applications begins by assuming that it is a function of the relative roughness of the bed. Following the work of Nikuradse the ratio of an equivalent roughness length to the flow depth is generally used. Several types of bed roughness contribute to the friction factor. The most widely considered include: 1) the roughness of individual sediment grains (skin friction), 2) the roughness created by sediment being transported by the flow, and 3) the roughness created by perturbations in the bed surface, such as ripples and dunes (YALIN, 1977). The equivalent roughness length is usually considered to represent a simple addition of these individual contributions, and then some empirical relationship is often used to relate the relative roughness length to the friction factor (e.g., GRANT and MADSEN, 1982; NIELSEN, 1985).

For the data considered here, the third contribution to bed roughness can be conveniently ignored, since no bedforms were observed during the experiments. Although the critical Froude Number for ripple development is typically exceeded during a single uprush, the duration of exceedence is apparently insufficient for the bed to respond completely (NELSON and MILLER, 1974). BROOME and KOMAR (1979) have reported the formation of ripples beneath hydraulic jumps in the backwash, that may produce some roughness effects on particular beaches. However, these backwash ripples are restricted to the wide Dissipative Beach Type described by WRIGHT and SHORT (1984), which is beyond the scope of the data set reported here. Some measurements of small amplitude bedforms have been reported for similar beach conditions to those considered here, but they have wave lengths in excess of the swash length (e.g., SALLENGER and RICHMOND, 1984; HOWD and HOLMAN, 1987). The roughness contribution of this type of bedform can therefore be considered negligible, since the active beach face for any one swash cycle remains planar.

For a clear fluid flowing over a fixed bed, the only source of flow resistance is considered to be skin friction due to the roughness of individual grains. If the flow is hydraulically rough and turbulent, which the swash is expected to be for most of its advance up the beach, then the velocity distribution in the boundary layer of the flow may be described by

$$\frac{u}{u_*} = 2.5 \ln\left(\frac{z}{z_o}\right) \qquad (10)$$

where u_* is the horizontal shear velocity ($u_* = \sqrt{(\tau/\rho)}$), z is the elevation above the bed and z_o is the hydraulic roughness length of the bed. If it is assumed that the boundary layer occupies the entire swash depth, then (10) can be re-written to yield the water velocity at the surface of the fluid element and hence an approximation to the shoreline velocity:

$$\frac{U_s}{U_{s*}} = 2.5 \ln\left(30\frac{h_b}{k_s}\right). \qquad (11)$$

The parameter k_s is the equivalent bed roughness length, and relates z_o to the grain diameter. Several laboratory studies indicate that for a fixed bed the value of k_s is constant for a given grain size; typically

$$k_s = 3D_{90} \qquad (12)$$

(e.g., VAN RIJN, 1982) where D_{90} is the grain diameter for which 90% of the bed material is finer. The friction factor for the uprush over a fixed bed, using (3) to formulate U_{s*} in (11), is given by

$$f = \frac{8}{\left(2.5 \ln\left(30\frac{h_b}{k_s}\right)\right)^2}. \qquad (13)$$

If the bed is not fixed, as in the case of a natural beach, then a two phase flow exists in the presence of sediment transport. Typically a relatively clear fluid phase interacts with an underlying phase of mixed fluid and granular material. Even if the granular-fluid phase becomes dominated by intergranular contact, it continues to display fluid-like behaviour and is therefore still considered to be part of the flow (HANES and INMAN, 1985; HANES and BOWEN, 1985; WILSON, 1988). The physics of this type of flow is more complex than that of a clear fluid over a fixed bed, hence there is less consensus in the literature regarding its effect on the friction factor (cf. HANES, 1984; WILSON, 1988). The energy dissipation caused by the granular-fluid phase is believed to result from the turbulent wakes behind saltating grains and the transfer of momentum from the flow to the stationary bed due to the transported grains impacting with the bed (OWEN, 1964; GRANT and MADSEN, 1982).

Cursory observations of natural sandy beaches show that the bed shear stress is sufficient for the leading edge of the swash lens to transport sedi-

ment during most of the uprush. Moreover, the preservation of the swash mark on beaches testifies to the fact that transport continues almost to the point of maximum uprush, otherwise the mark would be destroyed by the subsequent backwash. The transport of sediment in the swash as bedload typically appears like sheet flow, with the whole bed mobilised to a depth of several grain diameters. Laboratory experiments reported by NELSON and MILLER (1974) substantiate these field observations.

WILSON (1988) argues that (11) is not appropriate during conditions of such intense sediment transport. Instead, the flow has a characteristic friction length that is proportional to the shear layer thickness. Other than that, the velocity gradient is still proportional to the shear velocity and inversely dependent on a mixing length. Thus the form of the vertical velocity distribution in the presence of sheet flow is similar to (11), but appears as

$$\frac{U_s}{U_{s*}} = 2.5 \ln\left(53.2\frac{h_s}{\gamma}\right) \qquad (14)$$

(after WILSON, 1988), where the water velocity and water depth notation have been presented for the problem considered here and γ is the thickness of the sheet flow layer.

WILSON's laboratory experiments show that

$$\gamma = 10\theta D \qquad (15)$$

where D is the mean grain diameter, θ is the Shield's Parameter;

$$\theta = \frac{\tau}{\rho g D(s - 1)} \qquad (16)$$

and s is the ratio of sediment to fluid density. The friction factor for the uprush, in the presence of sheet flow, is therefore

$$f = \frac{8}{\left(2.5 \ln\left(5.32\frac{h_s}{\theta D}\right)\right)^2}. \qquad (17)$$

FIELD DATA AND METHODS

The data used in the analysis here is a sub-set of that presented in HUGHES (1992). Full details of the field sites and methods are reported in the original paper. The morphodynamic conditions that existed during the experiments were similar to WRIGHT and SHORT's (1984) low to moderate energy Intermediate Beach Types, where fully de-

veloped bores provide the initial impetus for the swash. Only swash cycles relating to individual bores are considered, interacting swash from two or more bores are not included here. The range of experimental bore heights, beach slopes and grain sizes measured were 0.1–0.8 m, 0.093–0.15 and 0.31–2.00 mm respectively.

The experimental setup consisted of a shore-normal line of range poles placed in the inner surf zone and six swash probes (capacitance-type water level probes) placed in the swash zone. Sediment samples were collected for grain size analysis and the positions of the range poles and swash probes were surveyed together with the beach profile prior to each experiment. As a bore propagated towards the beach face, its progress past the range poles was filmed to provide a measurement of its height and velocity immediately seaward of the shoreline. Filming continued during bore collapse and the early stages of the uprush to provide a visual record of the bore collapse process and an estimate of the initial swash velocity. The swash probes were activated at the time of bore collapse and recorded the movement of the leading edge of the swash lens and the time-history of the local swash depth. When the point of maximum uprush was reached, the shoreline position was marked on the beach by an observer. After the backwash receded, the maximum swash length was measured with a tape.

Due to the variable location of the point of bore collapse for different waves the swash probe furthest seaward, located near the still water shoreline, was used as the reference point for measuring swash parameters. That is to say the initial swash velocity is taken as the shoreline velocity recorded by the camera at the most seaward swash probe, and both the maximum swash length and height are measured relative to the position of the most seaward swash probe. The nature of the theoretical equations suggests that this approach causes no limitations, since the equations describe the behaviour of the swash relative to any choice of reference point on the beach face provided that the shoreline velocity is known at the chosen point (HUGHES, 1992).

The initial swash velocity was calculated from the number of frames in the film record required for the leading edge of the swash lens to travel the last 0.5 m before reaching the reference swash probe. The maximum swash height was calculated trigonometrically using the maximum swash length and the measured beach slope. The swash
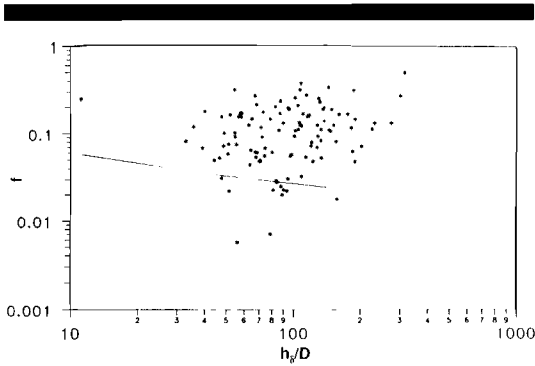
Figure 2. The inferred friction factor as a function of the relative roughness length of the bed. The theoretical estimate for the friction factor assuming a fixed bed, (13), is shown as a solid line.
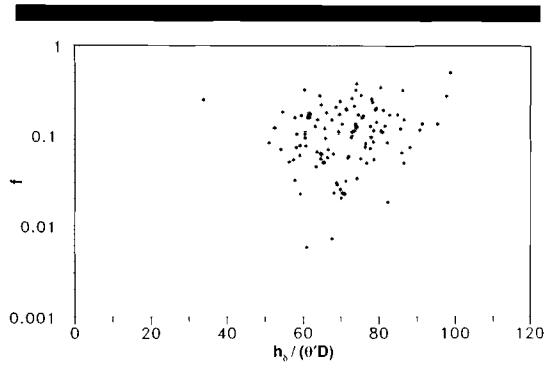

Figure 3. The inferred friction factor as a function of the relative roughness length of the bed. The theoretical estimate for the friction factor in the presence of sheet flow, (21), is shown as a solid line.

depth was determined from the difference between the varying capacitance measured by the swash probes when they were immersed by the swash lens and the constant, ambient capacitance of the wetted beach face measured between swash cycles. The average shoreline velocity during the uprush was calculated by dividing the swash length by the time taken for the shoreline to travel from the reference swash probe to the point of maximum uprush.

RESULTS

Before (9) can be used with the available field data to determine the inferred friction factor, an estimate for h_δ is required. Since h_δ is taken to be constant in the derivation, the value of the maximum swash depth at the mid swash position will be used here. For the existing data set the maximum swash depth, $h_{s(max)}$, can be estimated using the empirical equation

$$h_{s(max)} = \frac{u_o^2}{2g}(0.21 - 0.48x_* + 0.32x_*^2) \quad (18)$$

(after HUGHES, 1992) where x_* is the non-dimensional distance from the initial shoreline position. To determine the maximum swash depth at the mid swash position and thus h_δ, $x_* = 0.5$ is substituted into (18) together with the initial swash velocity. Clearly this approach will under-estimate the thickness of the fluid element representing the leading edge of the swash lens in the early stages of the uprush and over-estimate it in the later stages. The sensitivity of the theoreti-

cally predicted swash height to h_δ is discussed later.

Measured values of the swash height, beach slope, initial swash velocity and maximum swash depth at the mid swash position were substituted into (9), which was then solved for the friction factor. These inferred values for the friction factor are shown in Figure 2 as a function of the ratio of mean grain size to swash depth. The relationship expected for flow over a fixed bed, using (12) and (13), is also shown. The inferred friction factor for the sandy beaches represented in the data set is of the order of 0.1 and generally an order of magnitude larger than that expected for flow over a fixed bed. Interestingly, this result is consistent with the difference in friction factors measured over fixed and movable beds under oscillatory flow (see GRANT and MADSEN, 1982; NIELSEN, 1983).

Direct measurements of the shear stress are unavailable from the data set, thus in order to test the ability of WILSON's sheet flow model for predicting the friction factor a skin friction formulation for the Shield's Parameter was employed;

$$\theta' = \frac{0.125\rho f U_s^2}{\rho g D(s - 1)}. \quad (19)$$

The value of f in the numerator of (19) is calculated using (12) and (13). The use of θ' requires some estimate of the shoreline velocity. Since the level of analysis to this point assumes that the friction factor is constant for the entire uprush, a constant value for the shoreline velocity is required here. The average shoreline velocity, de-

fined as the maximum swash length divided by the time taken to reach the point of maximum uprush, has been adopted here and can be calculated using the empirical equation

$$\bar{U}_s = 0.39u_o \qquad (20)$$

(HUGHES, 1989).

Clearly the skin friction formulation for the Shield's Parameter will underestimate the parameter's true value. Consequently, the constant in the logarithmic term of (17) is expected to be too large in this analysis, which was indeed found to be the case. The best fit to the data was found when a value of 0.5 was used (Figure 3). Hence, the following formula for the friction factor provides a practical alternative to (17) in the absence of direct measurements of the shear stress:

$$f = \frac{8}{\left(2.5 \ln\left(0.5 \dfrac{h_\delta}{\theta' D}\right)\right)^2}. \qquad (21)$$

DISCUSSION

In order to derive the equations for shoreline displacement (7) and swash height (9), it was necessary to assume that the swash depth is constant throughout the uprush. Since it has been shown that the swash depth decreases as the shoreline advances up the beach (HUGHES, 1992), this assumption requires further discussion. Figure 4 shows that the predicted swash height is quite sensitive to the choice of swash depth. Choosing a representative swash depth that is too small would lead to an under-estimation of the inferred friction factor, whereas a swash depth that is too large would lead to an over-estimation. In the absence of any physical reasoning to direct the choice of a representative swash depth, (18) was used here largely for convenience. Although this choice led to inferred friction factors that were physically explainable, it is worth keeping in mind that another choice for the representative swash depth would have produced significantly different results.

WILSON's (1988) model for bed friction in the presence of sheet flow was used here to predict the value of the inferred friction factor for two reasons. The first was that visual observations indicated that sediment transport occurred as sheet flow most of the time for most of the uprush, and the second was that it was capable of predicting the correct order of magnitude for the
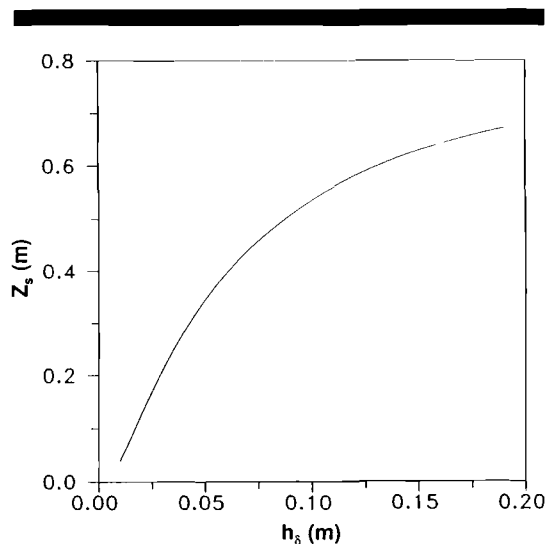

Figure 4. Theoretical relationship between swash height and swash depth from (9), demonstrating the sensitivity of the predicted swash height to the choice of representative swash depth. Initial conditions for the example shown are: $u_o = 4$ m s^{-1}, $D = 0.0005$ m and $\beta = 0.1$.

inferred friction factor. It should be noted that WILSON's model is only valid during conditions when the Shield's Parameter is greater than about 0.8. For the range of grain sizes and swash velocities represented in the data set reported here, HUGHES (1989) showed that the Shield's Parameter is likely to have remained above this value for most of the uprush. When conditions are such that the Shield's Parameter is less than 0.8 for most of the uprush, due to small fluid velocities or large grain sizes, then sheet flow will not be of overriding importance. In these cases, bed friction effects may be incorporated more appropriately by using a more conventional model for the bottom boundary layer (i.e., the Law of the Wall) with weak sediment transport (e.g., GRANT and MADSEN, 1982).

The large inferred friction factors reported here are physically reasonable in the context of WILSON's (1988) model for bed friction in the presence of sheet flow. It should be pointed out, however, that there are at least three factors unrelated to bed shear that may also have contributed to the value of the inferred friction factor. The first is the presence of turbulence in the swash lens that is antecedent from the bore stage. The bore and the swash are different hydrodynamic stages in the wave runup process and are sepa-
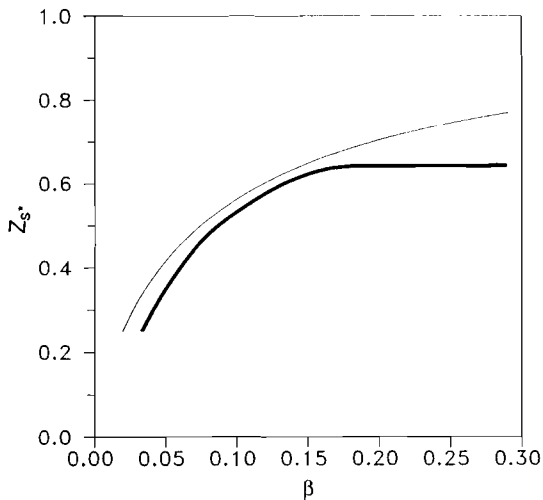
Figure 5. Theoretical relationship between the non-dimensional swash height and beach slope from (9); initial conditions for the example shown are: u_o = 4 m s $^{-1}$, D = 0.0005 m and h_s = 0.075 m (thin line). Also shown is MILLER's (1968) laboratory data (thick line).

rated at the point of bore collapse (FREEMAN and LEMEHAUTE, 1964; YEH and GHAZALI, 1988; HUGHES, 1992); theoretically bore related energy dissipation should not exist in the swash since the bore no longer exists. However from visual observations, it seems probable that turbulence present in the bore prior to its collapse is advected through the bore collapse region and into the swash. Thus in addition to the turbulent energy dissipation related to local bed shear in the swash zone, there will also be energy dissipation which is related to this antecedent turbulence. The second factor unrelated to local bed shear that may have contributed to the inferred friction factor is infiltration, which might in fact be significant during the uprush. Apart from the numerical modelling of infiltration effects reported by PACKWOOD (1983), there is little other work in the field or the laboratory that can shed further light on the issue. The third factor is that the flow is unsteady, which will generally tend to produce larger friction factors than those expected for steady flows (F. GERRITSEN, *personal communication*, 1993).

Since the advection of turbulence from the bore to the swash, infiltration and unsteady flow effects all potentially influence the swash height; they are all represented in the magnitude of the inferred friction factor. If these individual effects

can be isolated in the future and are found to be significant, then the empirical constant in (21), which was used to provide a match between WILSON's model and the data presented here, will probably need to be increased towards WILSON's original value of 5.32 to more accurately represent the frictional energy dissipation produced solely by local bed shear.

Since MILLER (1968) used one of the few data sets on swash dimensions available at the time to dismiss the inviscid non-linear shallow water theory as a satisfactory model for describing the swash process, it is instructive to compare his data with the theoretical approach that now includes friction. MILLER dismissed the theory in its inviscid form because it did not reproduce the relationship between maximum swash height and beach slope that he observed in his laboratory data. The theoretical relationship between the non-dimensional swash height, Z_s*, and beach slope, β, predicted by (9) is shown in Figure 5 together with the laboratory data of MILLER (1968). Both the theoretical relationship and the laboratory data have been non-dimensionalised by dividing by $u_o^2/2g$. Apparently, when friction effects are included in the non-linear shallow water theory (*e.g.,* Equation 9) it is capable of at least qualitatively modelling the dependence of swash height on beach slope that was observed in MILLER's laboratory data. It is not evident why MILLER's fixed bed data lies below the prediction for a set of typical field conditions in which a movable bed is accounted for.

WILSON's (1988) laboratory experiments and his formulation for the equivalent roughness length suggest that for sand size material undergoing sheet flow the friction factor is independent of grain size. This result is consistent with the apparent lack of grain size effect between field sites reported in HUGHES (1992), but it also poses an intriguing question. If the uprush process is mostly independent of grain size, then what is the mechanism responsible for the frequently observed relationship between grain size and equilibrium beach slope (*e.g.,* BASCOM, 1951; SUNAMURA, 1984)? One possible answer is that the backwash process must be sensitive to the grain size. If we assume that the most important effect of grain size is to increase the infiltration capacity, then our answer implies that the effects of infiltration are negligible during the uprush and significant during the backwash. PACKWOOD's (1983) numerical model for swash on a permeable beach produced just such a result. It was found that the

thickness of the swash lens during the uprush was sufficient to ensure that fluid loss into the beach had a negligible effect on the flow, whereas the relatively thin backwash responded noticeably to infiltration. Although the variation in grain size between sandy beaches apparently has little effect on the uprush, the ratio of uprush to backwash volume is still likely to increase with grain size due to the enhanced infiltration effects during the backwash. Intuitively, this process will produce the velocity magnitude asymmetry in favour of the uprush that is required to balance the enhanced capacity of backwash flows to transport sediment via the effects of gravity acting down slope. Since the velocity asymmetry in favour of the uprush increases with grain size due to infiltration in the backwash, the equilibrium beach slope that is possible also increases with grain size.

CONCLUSION

A method has been presented here to model wave uprush on a sandy beach using a non-linear shallow water theory that includes the effects of bed friction. The approach is restricted to the situation where uprush follows bore collapse at the still water shoreline and where significant transport of bed sediment as sheet flow occurs during the uprush. Field data and the theoretical equations were used to infer the magnitude of the friction factor for the uprush. The inferred values of the friction factor obtained for the sandy beaches studied were of the order of 0.1. This magnitude is larger than that expected for flow over a fixed bed but is consistent with WILSON's (1988) model for bed shear in the presence of sheet flow.

There is a great deal of scatter apparent in the inferred friction factors and although measurement error may provide some explanation, the advection of turbulence from the bore phase into the swash phase, swash infiltration, and unsteady flow effects may also have made a contribution that was not accounted for. Despite the uncertainty surrounding these contributions to energy dissipation during wave uprush, WILSON's model does seem appropriate to describe the contribution from bed friction. It is clear that a great deal of further work is required before a complete understanding of energy dissipation in the swash zone is achieved. Nevertheless, it is encouraging that the non-linear shallow water theory, with the inclusion of bed friction, is now capable of quantitatively predicting many features of swash be-

haviour (HUGHES, 1992; YEH et al., 1989) and is at least qualitatively beginning to model morphodynamic relationships such as the positive relationship between swash height and beach slope.

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