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Wave Directions in a Groin Field

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ABSTRACTI

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The time series of the onshore-offshore and the alongshore velocities obtained at different locations within a groin field and at a deeper location have been analysed for wave directionality. For this purpose, seven different available methods along with a methodology proposed in this paper have been adopted to draw a comparative study, the results of which are presented and discussed in this paper. The variation of wave direction inside the groin field at a particular time and its variation along the depth are studied. The seneral wave climate and the flow characteristics in the groin field are also reported.

ADDITIONAL INDEX WORDS: Particle kinematics, flow field, groin field, linear wave theory.

INTRODUCTION

In recent years, wave directional information has become important and sometimes essential in coastal and ocean engineering design and practice. Waves normally approach the coast at an angle; once they break, the sediments in the nearshore zone are driven by the longshore currents. The magnitude and direction of these currents and the resulting sediment transport rate are dependant on the wave direction near the breaker zone. Estimation of wave direction is critical for the calculation of the alongshore component of radiation stresses (LONGUET-HIGGINS, 1970a,b), particularly for small values of wave approach angles. GALVIN (1986) has reported that when the angle is 5 degrees, a 1 degree error in the estimates of the angle results in a twenty percent error in the radiation stress computation. This error leads to under- or overestimation of the littoral drift.

The importance of wave directionality in studies related to wave diffraction and refraction has been illustrated by GODA *et al.* (1967). The need for the wave directional information in the design of offshore structures and motions of floating structures has been discussed in detail by SAND *et al.* (1981). Though the importance of good estimates of the wave directions are well known, there are still difficulties in their predictions.

FIELD DATA

The time histories of the wave elevation and the two horizontal velocity components measured at one elevation above the sea bed at eight different locations (M1, M2, M3, O1, O2, O3, W1 and W3) inside a groin field off the East Frisian Island of Norderney in the North Sea measured from 7 November 1988 to 21 November 1988 have been analysed for the prediction of the wave directions. For each location, about twenty five records were analysed. The length of the record considered for analysis was 20 minutes with a sampling interval of 0.17 sec and 0.085 sec for the wave elevation and velocity records respectively. These data referred hereafter to as (OLD data) are used to study the variability in wave directions within the groin field.

The wave elevations were measured with pressure gauges and the two velocity components with electromagnetic flowmeters. The location map of the study area and the measurement locations are given in Figure 1 and Figure 2, respectively. The average water depths and the distance of the flowmeters from the sea bed for the different measurement locations are given in Table 1.

In addition to the above data, the wave elevation and the two horizontal velocity components at locations M1 and M2 along with the data from a deeper location (VS obtained during 05 April 1991 to 19 April 1991) have been analysed for the

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				Loca	ation			
	M1	M 2	M 3	01	02	03	W1	W 3
Distance between seabed and flowmeters	0.38	0.24	0.15	0.18	0.21	0.23	0.04	0.12

Table 1. Average distance between sea bed and flowmeters (m).

wave directions. The velocities were measured at three elevations at these three locations. The positions of the flowmeters at these locations are shown in Table 2. These data hereafter will be referred to as (NEW data). About 112 records for each of the locations were analysed. However, only typical results are herein reported.

METHODOLOGIES

General

A search of the literature revealed a number of methods for the prediction of wave directions in both time and frequency domains. The information on the wave directional spread in the frequency domain is more useful for the behaviour of fixed and floating structures in the offshore. The time domain analysis in the form of probability density or mean directions would be normally needed for sediment transport rate assessment, weather window calculations, determination of the orientation of structures in coastal waters, *etcetera*. The different methods considered in the present study are discussed below.

Methods 1 to 4

These methods proposed (MIZUGUCHI *et al.*, 1980) employed the wave elevation and the two horizontal component velocities time histories to determine the wave direction. The first two methods are given by

$$\theta_{i_{i}} = \tan^{-1} \left(\frac{\mathbf{v}_{\max_{i}} - \mathbf{v}_{\min_{i}}}{\mathbf{u}_{\max_{i}} - \mathbf{u}_{\min_{i}}} \right)$$
(1)

Table 2. Position of the pressure gauges and flowmeters (m).

Loca tion	Averag Flow	e Distance B meter and Se	Average Distance of Pressure Gaug from the	
	S1	S 2	S3	Seabed
vs	1.00	2.85	4.65	1.32
M 1	0.20	0.55	1.96	0.77
M 2	0.20	0.50	1.26	0.77

$$\theta_{2_{i}} = \tan^{-1}(v|_{u=u_{max}}/u_{max_{i}}),$$

i = 1, 2, ..., NW (2)

Herein, the angle, θ_1 is for a wave defined by the zero down crossing method and is assumed to be determined by the wave front characteristics. The angle, θ_2 is assumed to give the propagating direction of the wave crest. The subscript, max and min, indicates the maximum and minimum of the corresponding velocity component. The onshore-offshore velocity is denoted by u and the along-shore velocity by v and NW is the number of waves in the record. The numerator of Equation 2 is the alongshore velocity at the time corresponding to the maximum u.

The expression given by Equation 2 was slightly modified by the investigators and is given by Equation 3

$$\theta_{a_i} = \tan^{-1} \left(\frac{\mathbf{v} |_{\mathbf{u} = \mathbf{u}_{\text{max}_i}} - \overline{\mathbf{v}}}{\mathbf{u}_{\text{max}_i} - \overline{\mathbf{u}}} \right) \mathbf{i} = 1, 2, \dots, \text{NW}$$
(3)

where '-' represents time mean values.

An alternate expression given by MIZUGUCHI et al. (1980) is

$$\theta_{4_i} = 0.5 \tan \left(\frac{2\overline{u_i v_i}}{\overline{u_i^2 - v_i^2}}\right), \quad i = 1, 2, \dots, N$$

in which N is the number of data points in the corresponding velocity records. Angle θ_{4} , is akin to a principal direction of the wave and considered to be an average direction of the fluctuating motion.

The directions obtained by the first three equations are averaged over the number of waves to give us $\overline{\theta_1}$, $\overline{\theta_2}$ and $\overline{\theta_3}$, whereas, the direction obtained by the fourth equation is averaged over the number of data values to yield the average value, $\overline{\theta_4}$.

The methods to determine the angles θ_1 , θ_2 and $\overline{\theta_4}$ were compared by MIZUGUCHI *et al.* (1980) with that derived from wave crest lines recorded on 16 mm films, and it was reported that the agreement with $\overline{\theta_2}$ was the best.



Figure 1. Location map of the island of Norderney.

Methods 5 and 6

These two methods for the mean direction proposed by BUCHAN *et al.* (1984) and based on the assumption that linear theory describes the particle velocity satisfactorily are represented by Equations 5 and 6.

$$\overline{\theta}_{5} = \tan^{-1} \left(\frac{c_{\eta v}}{c_{\eta u}} \right)$$
(5)

$$\overline{\theta_6} = 0.5 \tan^{-1} \left(\frac{2c_{uv}}{c_{uu} - c_{vv}} \right)$$
(6)

in which

$$\begin{split} \mathbf{c}_{\eta \mathbf{v}} &= \frac{1}{N} \sum_{i=1}^{N} (\eta_i - \overline{\eta}) (\mathbf{v}_i - \overline{\mathbf{v}}), \\ \mathbf{c}_{uu} &= \frac{1}{N} \sum_{i=1}^{N} (\mathbf{u}_i - \overline{\mathbf{u}}) (\mathbf{u}_i - \overline{\mathbf{u}}) \\ \mathbf{c}_{\eta u} &= \frac{1}{N} \sum_{i=1}^{N} (\eta_i - \overline{\eta}) (\mathbf{u}_i - \overline{\mathbf{u}}), \\ \mathbf{c}_{vv} &= \frac{1}{N} \sum_{i=1}^{N} (\mathbf{v}_i - \overline{\mathbf{v}}) (\mathbf{v}_i - \overline{\mathbf{v}}) \\ \mathbf{c}_{uv} &= \frac{1}{N} \sum_{i=1}^{N} (\mathbf{u}_i - \overline{\mathbf{u}}) (\mathbf{v}_i - \overline{\mathbf{v}}) \end{split}$$

The above estimates have been derived by BUCHAN et al. (1984) from measurements of wave kinematics which have suffered frequency dependant depth attenuation; that is, as the depth of measurement increases, these estimates become more biased towards longer period waves. The deviation of the mean values of η , u and v are directly taken into account while deriving the mean direction.

Method 7

Outside the surf zone where the longshore currents and turbulence are not predominant, KIM $et \ al.$ (1986) have suggested two methods to determine the wave direction. However, only the expression as given below is considered, since it has been mentioned by them that both their methods yielded identical values.

$$\theta_{\gamma_i} = \tan^{-1}(v_i/u_i), \quad i = 1, 2, ..., N$$
 (7)

The average value for directions by methods 1, 4 and 7 are obtained and considered for comparison. Further, KIM *et al.* (1986) have pointed out that inside the surf zone where the turbulence due to breaking and longshore current is predominant, the direction cannot be predicted by the above expression.



Figure 2a. Locations of measurements in the groin field.

Present Method

The present method for the average wave direction is based on the linear regression analysis to obtain the correlation between the two measured velocity components u and v.

Table 3a. Average wave angle in degree calculated by different methods. Date of measurements: 15 November 88. Time of measurement: 23:55.

Loca- tion	$ar{ heta}_1$	$ar{m{ heta}}_2$	$ar{m heta}_{\gamma}$	$ar{ heta}_{_4}$	$ar{m{ heta}}_{5}$	$\bar{ heta_6}$	$ar{m{ heta}}_{7}$	(Pres- ent) $\bar{\theta}_{8}$
M1	341	314	331	339	337	338	325	333
M2	344	287	320	344	309	312	321	313
M 3	340	304	317	326	293	313	312	314
01	334	285	298	311	294	296	304	306
O2	346	290	311	324	292	298	319	308
O3	351	286	311	342	307	309	321	313
W1	348	307	324	328	315	316	319	315
W3	353	327	316	326	318	316	316	316



Figure 2b. Sea bed profile of locations of measurement.

Two linear regression equations for the two variables were determined. First, the inline velocity u is considered as the fixed independent variable and the corresponding alongshore velocity v as the dependent variable. This would yield an equation of the form

$$v_i - \bar{v} = \beta(u_i - \bar{u}), \quad i = 1, 2, ..., N$$
 (8)

From the above expression the average angle of incident waves is calculated as

$$\bar{\theta} = \tan^{-1}\beta = S_{uv}/S_{1}^{2}$$

Considering v as fixed independent variable and u as dependent variable an equation of the form below can be obtained

$$u_i - \bar{u} = \beta(v_i - \bar{v}), \quad i = 1, 2, ..., N$$
 (9)

The average angle of θ is than calculated as

$$\bar{\theta} = \tan \beta = S_{uv}/S_2^2$$

Table 3b. Average wave angle in degree calculated by different methods. Date of measurement: 16 November 88. Time of measurement: 02:58.

Loca- tion	$ar{ heta}_1$	$ ilde{oldsymbol{ heta}}_{2}$	$ar{ heta}_{.s}$	$\bar{\theta}_{4}$	<i>Ē</i> .,	$\bar{ heta_6}$	$ar{ heta}_7$	(Present) $ar{ heta_s}$
M1	344	322	332	344	343	344	327	334
M2	335	313	319	321	317	318	316	317
M 3	336	327	312	313	314	315	309	315
01	334	290	306	322	307	306	309	310
O2	344	302	308	318	304	306	311	310
O 3	342	307	315	324	308	317	318	316
W1	343	316	321	326	325	322	322	320
W3	346	323	321	323	326	318	320	317



Figure 3. Wave direction obtained by the present method (OLD data).

in which

$$\begin{split} \mathbf{S}_{uv} &= \frac{1}{N-1} \Bigg[\sum_{i=1}^{N} \, \mathbf{u}_i \mathbf{v}_i \, - \, \frac{1}{N} \Biggl(\sum_{i=1}^{N} \, \mathbf{u}_i \Biggr) \Biggl(\sum_{i=1}^{N} \, \mathbf{v}_i \Biggr) \Bigg] \\ \mathbf{S}_{1}^{-2} &= \frac{1}{N-1} \Bigg[\sum_{i=1}^{N} \, \mathbf{u}_i^{-2} \, - \, \frac{1}{N} \Biggl(\sum_{i=1}^{N} \, \mathbf{u}_i \Biggr)^2 \Bigg] \\ \mathbf{S}_{2}^{-2} &= \frac{1}{N-1} \Bigg[\sum_{i=1}^{N} \, \mathbf{v}_i^{-2} \, - \, \frac{1}{N} \Biggl(\sum_{i=1}^{N} \, \mathbf{v}_i \Biggr)^2 \Bigg] \end{split}$$

The lines obtained from Equations 8 and 9 are the two regression lines. The average of these two lines is defined as the angle of wave propagation, $\overline{\theta_8}$.

RESULTS AND DISCUSSION

The values of the average directions computed are all with respect to Geographic North. The groin field is inclined at an angle of about 325 degrees with respect to North.



Figure 4. Wave direction obtained by the present method (NEW data).

The wave directions by different methods were computed for the entire number of records for both OLD and NEW data sets. In order to make the presentation simple, only typical results are given in this paper. The average directions based on the eight methods discussed earlier for the OLD data for two instances of measurements for the different locations inside the groin field are given in Table 3a and 3b. In this paper, the average wave direction computed by the method proposed by the investigators is kept as the base for comparison. The directions computed based on methods 3, 5, 6 and 7 yield almost identical values with that obtained with the present method. However, method 7 yields slightly higher values for locations O2 and O3 as can be seen in Table 3a. $\overline{\theta_1}$ and $\overline{\theta_4}$ are seen to be consistently higher than the present method



for all locations, in which $\overline{\theta_1}$ is higher than $\overline{\theta_4}$. In general, method 2 yielded lower values except for location W3.

The slight difference in the methods with respect to the different locations may also be due to several other effects; *i.e.*, contamination of the wave and velocity records with surf beat, reflections from groins, cross waves, *etcetera*, in addition to the basic assumptions in each of the meth-

 Table 4a.
 Average wave angle in degree calculated by different methods. Date of measurement: 06 April 91. Time of mea

Lo- ca- tion	Level	$ar{ heta}_1$	$ar{m{ heta}}_2$	$ar{m{ heta}}_3$	$ar{ heta}_4$	$ar{ heta}_5$	$ar{ heta}_6$	$ar{ heta}_7$	(Pres- ent) $\bar{\theta}_8$
VS	s1	364	332	325	325	316	317	325	310
	s2	365	314	320	322	319	317	323	310
	s3	364	336	322	324	322	319	324	306
M 1	s1	360	320	325	325	321	321	324	316
	s2	360	320	317	325	318	317	321	316
	s 3	362	314	323	329	322	318	323	318
M2	s 1	412	388	352	420	307	481	399	320
	s2	406	375	354	397	310	486	392	317
	s3	412	366	345	405	303	478	382	316

surement: 10:05.

ods. It is further seen from Tables 3a and 3b that location M1 yields a higher value of $\bar{\theta}$ compared to all other locations.

The results on the average wave directions for the NEW data with the velocity components measured at three different elevations from sea bed at each of the three locations, VS, M1 and M2, are depicted in Tables 4a and 4b.

These tables reveal that methods 1 and 4 show

Table 4b. Average wave angle in degree calculated by different methods. Date of measurement: 06 April 91. Time of measurement: 13:06.

Level	$ar{ heta}_1$	$ar{ heta}_2$	$ar{ heta}_3$	$\bar{ heta}_{_4}$	$ar{ heta}_5$	$\bar{\theta}_6$	$\bar{\theta}_7$	(Pres- ent) $\bar{\theta}_8$	-
s1	365	328	332	324	319	319	326	311	
s2	364	311	328	332	319	318	329	313	
s3	363	332	327	329	322	323	326	306	
s1	356	323	326	326	321	323	326	318	
s2	360	324	317	318	314	317	322	318	
s3	361	321	327	326	319	320	325	321	
s1	412	306	359	428	299	480	382	323	
s2	414	306	343	461	302	482	339	320	
s3	417	317	383	382	375	376	381	317	
	Level \$1 \$2 \$3 \$1 \$2 \$3 \$1 \$2 \$3 \$1 \$2 \$3 \$1 \$2 \$3 \$3 \$1 \$2 \$3 \$3 \$1 \$2 \$3 \$3 \$1 \$2 \$3 \$3 \$1 \$2 \$3 \$3 \$3 \$1 \$2 \$3 \$3 \$3 \$1 \$2 \$3 \$3 \$3 \$1 \$2 \$3 \$3 \$3 \$3 \$3 \$3 \$3 \$3 \$3 \$3	Level $\bar{\theta}_1$ s1 365 s2 364 s3 363 s1 356 s2 360 s3 361 s1 412 s2 414 s3 417	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$ \begin{array}{c c c c c c c c c c c c c c c c c c c $



Figure 6. Variations of wave direction and resultant velocity.

a similar trend of higher values for $\bar{\theta}$ as that obtained with the OLD data. So is the case with methods 6 and 7 yielding higher values and greater for location M2. In general, all the methods except method 5 vield a higher value compared to that obtained by the present method. The results have thus shown that method 5 in general is consistent in its agreement with the present method especially at s1. The vertical variation of the values of $\bar{\theta}$ is small but quite significant for values of $\tilde{\theta}$ obtained based on method 2. Typical results of the directions based on the present method for all locations for the OLD data showing the respective average directions (only for the results reported in Table 3a) are depicted in Figure 3. The angle between the two regression lines, α , at the different locations is also indicated in the plot. The probable reason for this trend could be that the locations M3, O3 and W3 are in shallower



Figure 7. Variation of wave direction and resultant velocity.

waters and thus leads to nonlinearities in the velocity profiles. The reason for a larger angle between the regression lines for locations O1 and O2 could be that the waves attack the eastern groin resulting in reflections from the groin.

A typical plot showing the above features at the three different levels for locations VS, M1 and M2 for the NEW data are presented in Figure 4. The scatter in the results does not vary along the depth of measurements; however, the deviation is seen to be more for location VS. The angle between the regression lines, α , is seen to be more for location VS and found to slightly decrease nearer to the free surface. However, a reverse trend is seen in its variation along the depth for the other two locations. It is further observed that α is less for location M2 and found to increase towards offshore. It is to be recalled that a reasonable value for α for the OLD data for locations M1 and M2

Table 5. Magnitude of the resultant velocity (cm/sec).

	Location							
Date and Time	M 1	M2	M3	01	02	O3	W1	W 3
15 November 88 (23:55)	14.2	37.0	6.3	32.0	53.9	28.3	15.2	30.4
16 November 88 (02:58)	2.7	10.3	5.7	13.1	19.4	27.3	3.2	21.6





Figure 8b. Variation of H_{1/3}.



(effect of groins assumed to be less significant) was obtained, and here we observe a similar trend for locations M1 and M2. It is felt that the present method could be adopted for the determination of the average shallow water wave direction.

The daily variation of the mean wave directions estimated with the NEW data by the present method is given in Figure 5. The results obtained are for the measurements taken once in three hours from 05 April 1991 at 03 hr 57 min. The mean directions have been evaluated with the velocity components measured at level s3; *i.e.*, nearer to the free surface. The mean direction ranges from 300 to 330 degrees with respect to North. The angle at VS is generally found to be less compared to the other two locations indicating that the waves approach this location obliquely with respect to the shore normal, and approach the other two locations M1 and M2 almost normal to the shore.

The magnitudes of the resultant velocity for the OLD data given as $\sqrt{u^2 + v^2}$ within the groin field depicted in Table 5 show that the resultant velocity is generally small at locations M1 and M3. A higher value for the locations O1, O2, O3 should be due to the direct wave attack on the eastern

groin leading to reflections. Though one might expect a low value for directions at location W3, this is surprisingly not the case.

Furthermore, it is seen in this table that the resultant velocities change considerably within a few hours. These findings explain the complexities in understanding the flow field within a groin field.

Typical plots of the resultant velocity and its directions calculated for two days (6 April 1991 and 16 April 1991) are illustrated in Figures 6 and 7, respectively. The above two parameters have been calculated for all the three elevations from the sea bed. In the first plot we see that the waves approach the locations from different directions and that the resultant velocity near the sea bed can be greater than that observed near the free surface. It is further seen that the velocity reduces towards the shore. Similar results are revealed in Figure 7, the only difference being that the waves have a well defined predominant direction. It is to be mentioned here that the measured velocities are not strictly induced by the waves alone.

The variation of mean wave height, \hat{H} , and the significant wave height, $H_{1/3}$, for the three loca-

tions plotted in Figure 8a and b reveal that the \hat{H} vary from 0.1 to 1.0 m and that the $H_{1/3}$ vary from 0.2 to about 1.5 m. The variation in the wave characteristics with the locations is found to be less than the velocity variations.

The variation of the mean wave period illustrated in Figure 9 shows that it ranges from about 3.5 to about 10 sec, and that its variation with respect to measurement locations is not found to be significant. The individual wave heights and periods have been obtained by zero upcross analysis.

SUMMARY AND CONCLUSION

The methods available to determine wave direction in shallow waters with measured wave elevation and the two horizontal velocity components have been reviewed. A new technique is introduced to determine the average directions only. A comparative study is drawn between all the methods by determining the average directions at different locations in a groin field. The wave directions change drastically from the deeper location M1 whereas its change at other locations is not significant. This conclusion is based on the analysis of OLD data.

The analysis of the NEW data has shown that the waves approach at an angle with shore normal at deeper location VS which reduces with shore normal. The analysis for $\bar{\theta}$ with the velocity profiles measured at three elevations above the sea bed has shown that the variation in $\bar{\theta}$ along the depth is insignificant. The trend is similar for all three locations in the groin field considered in this study. The resultant velocity at the sea bed is found to be almost equal or even higher compared to that obtained nearer to the free surface. The resultant velocity within the groin field is found to vary drastically. There is no appreciable variation in average wave characteristics (\bar{H} , $H_{1/3}$ and \bar{T}) within the groin field.

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