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Ice Ride-Up and Pile-Up on Shores and Coastal Structures

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ABSTRACT

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Simple techniques for selecting design ice ride-up and ice pile-up phenomena on beaches and coastal structures, such as breakwaters and bridge abutments, are outlined. The size of the ice pieces breaking off when the floe interacts with the slope is determined in a static analysis. Ride-up and instabilities leading to pile-up are examined. Two conceptually different relations between driving force and pile-up height arise from different mechanical models of the piling-up process. Design values are estimated by limiting the pressure within the advancing ice sheet to the horizontal failure pressure, which is generated as a floe when significant kinetic energy impacts a shore or a sloping coastal structure. Subsequent use of ALLEN'S (1970) formula gives a good estimate of maximum pile-up heights for given ice conditions. This implies that the entire pile-up must be pushed upward. Limiting heights based on the forces required to overcome gravity of only the advancing ice and friction, as proposed by KOVACS and SODHI (1980), give unrealistic pile-up heights. This presumes that the horizontal ice failure pressure is substituted for the unit driving force, *and* no other limitations or instabilities in the piling-up process are considered. Their model is suited for plane situations, where the ice velocity is nearly zero and the pressure is limited by driving forces rather than by the failure pressure. A limit must be added to their pile-up process model corresponding to buckling failure of the train of prebroken ice blocks riding up the seaward face of the ice pile-up.

ADDITIONAL INDEX WORDS: *Ice pressure, ice ride-up, ice pile-up, shores, coastal structures, design value.*

INTRODUCTION

When a drifting ice sheet or ice floe comes in contact with a continuous slope, such as a beach or a mound breakwater, it begins to bend where it is pushed up on the slope. If the driving forces are sufficiently large, the ice will break, and the ice sheet will continue to push broken-off pieces up the slope. These may either form an ice pile-up or continue to ride up and eventually overtop the mound. Figure 1 shows an ice pile-up completely covering a breakwater. SPEERSCHNEIDER (1927, p. 61) refers to a description from 1901 of ice piling-up at the harbour of Dragør immediately south of Copenhagen, Denmark. BRUUN (1989) offers an English translation of this description:

> "... slowly, silently, the floes pressed up over the stonework of the breakwater. Ice soon reached the timberwork, stopped for a while as if to gather strength, and then with two-three cracks and crashes, the planking, framing and bolts shattered like glass and broke like matchsticks, packice pressing in everywhere, piled 20 feet high. Then the packice went over the breakwater, and, with a hollow, snarling sound, fell upon the ships and boats moored within. It was all over within a quarter of an hour. The packice came to a stop as suddenly as it had started, yet in that short while had wreaked a havoc of destruction. Mounds of ice still cover the outer part of the breakwater, and the total extent of the damage cannot yet be assessed."

This description captures well the brief and violent ice pile-up formation process, which is typical of most piling-up events.

Ice pile-ups are formed when an ice cover is forced against a coast line. The forcing towards the coast line can be generated by stresses due to wind and/or current, by thermal expansion of a relatively large coherent cover or by a drifting ice floe with substantial kinetic energy. Ice pile-ups are often formed in a few minutes and can cause considerable loading on the underlying base. When the base is a natural or similar beach, the loading is usually irrelevant, while issues such as sediment transport can be of interest (BARNES *et al.*, 1993; DIONNE, 1993).

Based on observations in the Beaufort Sea, KOVACS and SODHI (1980) stated that, "Pile-up seldom occurs more than 10 m inland from the sea, but ride-up frequently extends 50 m or more

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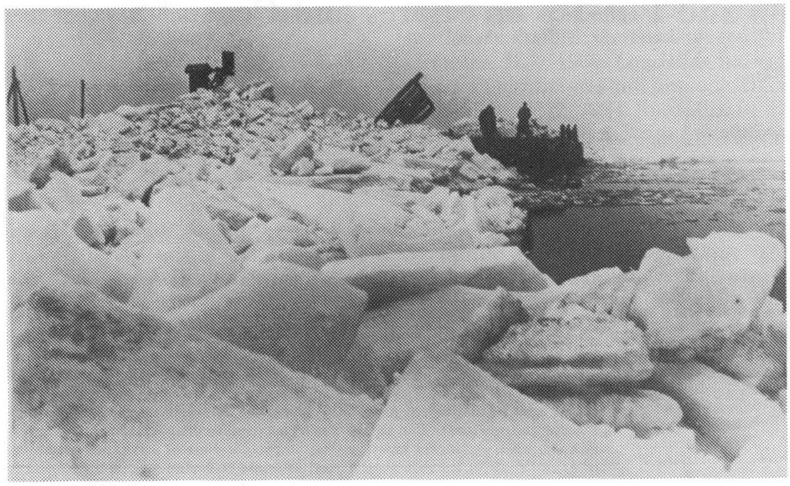

Figure 1. Ice pile-up covering the old breakwater at Knudshoved on the island of Fyn, Denmark, on 22 March 1940. This breakwater was located a few hundred meters south of the present day western landfall of the Great Belt Link. Photo courtesy of Danish Hydraulic Institute.

inland, regardless of ice thickness". The 50 metres should be seen in conjunction with the very flat shores of Alaska's north coast. KOVACS (1983, 1984) even mentions inland ice thrusts extending 100 m behind the shoreline. KOVACS and SODHI (1988) show a range of interesting pictures of ice ride-up and pile-up.

The primary point of interest in design is the magnitude of the protrusion of sea ice in any form behind the shoreline, and/or the elevation reached by the sea ice behind the shoreline. Also of interest is the necessary clearance of low bridges, typically near bridge abutments, to avoid ice pile-up impact on the superstructures.

In the following, the size of the broken-off pieces is first determined. Ride-up is then analyzed with particular emphasis on instabilities leading to ice piling. Finally, the sizes and associated forces of ice pile-ups are outlined. Two conceptually different approaches are outlined, and a simple method for deterministic design is recommended.

More complex approaches are needed in research, *e.g.*, by YEAN *et al.* (1981). The objective of the following is to outline a simple method. Probabilistic methods are not included in this paper, although they may be of interest (HOMMEL and BERCHA, 1983). New numerical methods, *e.g.*, by HOPKINS (1992, 1993), might also be used as a basis for rational design approaches, although this would require some adaptation efforts.

Ride-up and pile-up phenomena are of importance to structures placed in ice-infested waters such as the Beaufort Sea (CROASDALE and MARCELLUS, 1978; CROASDALE, 1980; CROASDALE *et al.*, 1988) and the Bering Sea (ETTEMA *et al.*, 1983; SACKINGER *et al.*, 1983). Ride-up can take place also in the more pronouncedly seasonal ice regions, such as Scandinavia, and design measures against overriding may be necessary. This is true for *e.g.*, many breakwaters in subarctic waters, and the writer has proposed design ride-up and pile-up sizes for fill islands incorporated in two Scandinavian strait crossings, the Great Belt Link in Denmark and the planned Sound Link between Denmark and Sweden.

GENERAL EQUATIONS

Consider an ice floe of relatively large extent in contact with a slope as shown in Figure 2. The situation is considered to be plane, and the length of the breakwater is infinite. If the coefficient of friction between the ice and the slope is zero, the horizontal and vertical reactions, H_0 and V_0, are related by:

$$H_0/V_0 = \tan \alpha \tag{1}$$

where α is the slope angle with horizontal. A triangular volume of height z_v and length z_h is crushed during the initial contact. The reaction forces are assumed to be evenly distributed over the result-

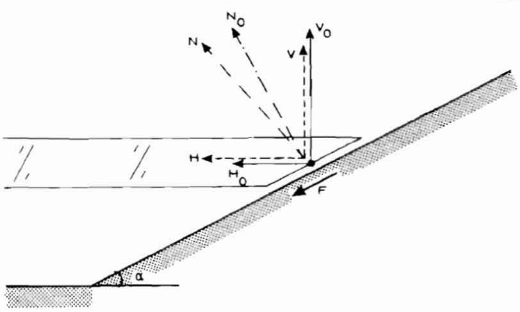

Figure 2. Ice sheet in contact with slope.

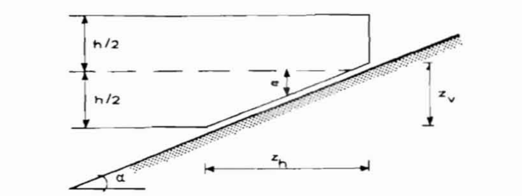

Figure 3. Localized crushing causing eccentricity of the horizontal reaction component.

ing inclined sea surface. The friction force acting on the ice, parallel to the slope in the downward direction, equals the product of the normal force and the friction factor, μ:

$$F_{\mu} = \mu(H_0^2 + V_0^2)^{0.5} \tag{2}$$

When this friction force is included in the force balance, the reactions become:

$$H = H_0(1 + \mu \cot \alpha) \tag{3}$$

$$V = V_0(1 - \mu \tan \alpha). \tag{4}$$

By use of equation (1) the relation between H and V becomes:

$$H/V = \tan \alpha(1 + \mu \cot \alpha)/(1 - \mu \tan \alpha)$$

$$= (\mu + \tan \alpha)/(1 - \mu \tan \alpha). \tag{5}$$

The horizontal driving force on the floe can be calculated as:

$$\bar{F} = \bar{F}_{wind} + \bar{F}_{water} \tag{6}$$

$$F_{wind} = 0.003\rho_a V_{10}^2 A \tag{7}$$

$$F_{water} = 0.003\rho_w u^2 A \tag{8}$$

where A is the top surface area of the ice floe, V_{10} is the wind speed 10 metres above the floe, u is the current speed 1 metre below the floe, and ρ_a and ρ_w are the densities of air and water, respectively. Note that these two driving forces should be added vectorially. For small driving forces, F and H might be in equilibrium, and no further development takes place. For large driving forces, the ice will ride up the slope and potentially form an ice pile-up as explained below.

The following analyses are best understood if one considers a vast ice floe pushed against a slope with a constant velocity sufficiently small that dynamic effects on ice deformations can be disregarded. Upstream of the slope and outside the interaction zone, an imaginary pressure sensor records the horizontal compressive interaction force in the ice. When this force reaches the magnitude of the external driving force, *F*, the interation stops. In reality, it will continue until the kinetic energy of the floe has dissipated. For the purpose of making the analysis clearer, this is disregarded at first.

DETERMINATION OF MAXIMUM ICE PIECE LENGTH

The size of the ice pieces breaking off as the ice sheet pushes onto the slope is of interest for two reasons. The piece size affects the stability of the ride-up process, particularly at points where the slope angle changes. This will be described later. The piece size may affect the piling-up process, although this is currently not well understood.

By using the reactions H and V determined above, the stress in the critical cross-section *i.e.*, the cross-section with maximum combined stresses, can now be determined as:

$$\sigma_{cr} = (H/Bh) + 6(V\ell_p - Hfh)/(Bh^2) \tag{9}$$

where B is the ice floe width, σ_{cr} is the stress in the critical cross-section from combined compression and bending, ℓ_p is the distance from the floe edge to the critical cross-section, and f is a dimensionless eccentricity factor defined as the ratio of vertical eccentricity to thickness, $f = e/h$, where e is the eccentricity. After determining the eccentricity coefficient, f, the piece length, ℓ_p, can be found from the above equation. When the ice floe first meets the slope some initial localized crushing will take place. This is shown in Figure 3. If, for simplicity, it is assumed that the height of the crushed zone equals:

$$z_v = H/(\sigma_u B) \tag{10}$$

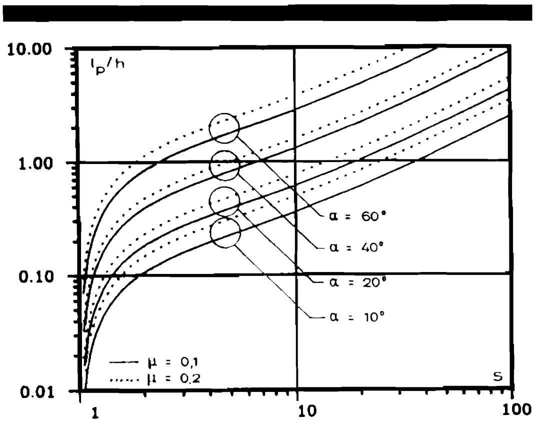

Figure 4. Dimensionless piece length as a function of the dimensionless parameter S, the shore slope angle α, and the friction coefficient μ, based on a static analysis.

where σ_u is the uniaxial compression strength, the eccentricity can be found as:

$$e = \tfrac{1}{2}(h - z_v) \tag{11}$$

$$f = e/h = \tfrac{1}{2}(1 - z_v/h) = \tfrac{1}{2}(1 - S^{-1}) \tag{12}$$

$$S = \sigma_u hB/H \tag{13}$$

where S is a dimensionless parameter expressing the ratio of the compressive ice failure load and the external forcing. The last parameter needed in order to determine ℓ_p is the ice strength under combined bending and compression, σ_{bc}. It may be calculated from:

$$\sigma_{bc} = \sigma_u(1 - \tfrac{1}{2}(1 - S^{-1})^2) \tag{14}$$

Failure takes place when the stress given by equation (9) reaches the magnitude of the combined bending-compression strength given in equation (14). This leads to a formula for the length of the broken-off ice pieces, *cf.* TRYDE (1972, 1973), who also referenced KORZHAVIN (1962) for inspiration:

$$\frac{\ell_p}{h} = \frac{\tan\alpha}{12} \cdot \frac{(1 + \mu\cot\alpha)}{(1 - \mu\tan\alpha)}\left(S + 6 - \left(\frac{7}{S}\right)\right). \tag{15}$$

Note that for S = 1 the piece length is zero. The relationship (15) is shown in Figure 4. It is valid only when an ice sheet is in contact with the slope *while at rest* with external driving forces acting upon it. As a static analysis, it fails to predict the correct piece size when the velocity of the ice floe is significant. A dynamic analysis can predict the piece size correctly (REEH, 1972), regardless of the velocity of the incoming ice floe. This quite complicated analysis has been conducted by SØRENSEN (1977, 1978).

As an example, consider a nearly circular ice floe of 1,000 m diameter forced against a slope by a 16 m/sec wind, and in contact with that slope over a width of 200 m. The driving force on the floe is 3.11 MN according to equation (7). The horizontal ice pressure per unit width is then 15.6 kN/m. With an ice thickness of 0.5 m and a uniaxial compressive ice strength of 2 MPa, the S parameter in equation (13) becomes 64.3. On a 35° slope with a 0.1 friction coefficient this leads to an ice piece aspect ratio, ℓ_p/h, of 5, corresponding to a piece length of 2.5 m.

KOVACS and SODHI (1988) offered an approximation which is quicker to calculate:

$$\ell_p/h = (\sigma_f/(3\gamma_i h))^{0.5} \tag{16}$$

where σ_f is the flexural ice strength and γ_i the specific weight of the ice. The above 0.5 m thick ice floe could reasonably have a flexural strength of say 500 kPa, corresponding to 25% of the compressive strength. With a specific weight of 8.9 kN/m^3, the aspect ratio of the ice pieces becomes 6 according to equation (16). This is a little higher than the value of 5 estimated by equation (15) but within reasonable uncertainty.

The technique of equations (9) through (15) appears to give reasonable results, but some reservations are necessary. The unit force of 15.6 kN/m is far from the limiting horizontal failure pressure within the ice sheet. The larger pile-ups are most likely formed through focusing of force or energy so that a large floe causes a highly variable pressure along the shore. When a point experiences a large pressure, the pile-up may increase at this point in time and space, while other points along the shore experience only modest pressure. After a failure of the sheet ice, different points may experience high pressure and pile-up formation. In this way, a modest average unit force can actually build a relatively high ice pile-up through a non-simultaneous process along a shore. This explanation would indicate that the average unit force is not representative of actual contact stresses. The use of average unit force to calculate piece sizes is only suitable for situations of perfect contact. This may be the case where an intact ice sheet is frozen onto *e.g.*, a breakwater, and the wind subsequently increases. This may lead to a nearly plane two-dimensional stress state. This distinction will be applied again for determination of ice pile-up heights.

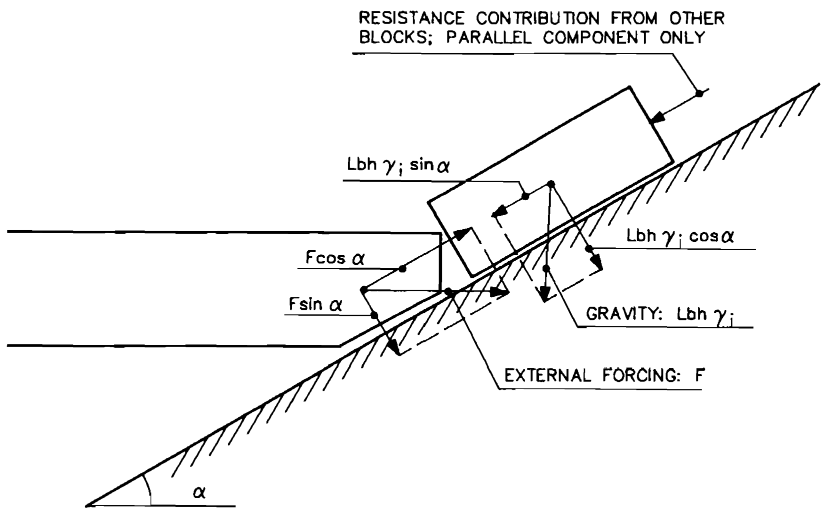

Figure 5. Schematic ice ride-up.

ICE RIDE-UP

The broken-off ice pieces are pushed against the slope by the advancing ice sheet behind them. They continue up the slope and may even reach levels as high as 10 to 12 metres above water level, although levels about 3 metres are more common.

Cox *et al.* (1983) added an upper limit to the aspect ratio of the broken-off pieces by considering the bending of a piece resting with one end on the slope and the other end on the edge of the advancing ice sheet. By demanding that the maximum bending moment in the piece, $\gamma_i bh\ell_p^2/6$, be less than or equal to the breaking moment, $\sigma_f bh^2/6$, the limiting aspect ratio becomes:

$$\ell_p/h < (\sigma_f/(\gamma_i h))^{0.5} \tag{17}$$

If the piece length found from equation (15) is larger than the maximum permissible length according to equation (17), the piece will break in half during the initial part of the ride-up.

If the driving force on the floe is larger than the resistance offered by the pieces on the slope, ride-up may occur. The resistance may be found from:

$$R = L\gamma_i bh(\sin\alpha + \mu\cos\alpha) \tag{18}$$

where R is the resistance parallel to the slope, L is the length of the slope covered by ice pieces, γ_i is the specific weight of the ice, b is the width of ice on the slope, h is the ice sheet thickness, α is the slope angle (with horizontal), and μ is the friction coefficient. This formula implies a continuous cover of ice pieces over the length L. If the available horizontal driving force, F, has a component parallel to the slope, F cos α, larger than the parallel resistance, R, ride-up may occur. This happens when:

$$F\cos\alpha > R$$

$$F > L\gamma_i bh(\mu + \tan\alpha) \tag{19}$$

where L is the slope length above water. (Note that some authors have wrongly projected onto a horizontal line and arrived at $F > R\cos\alpha$ instead). The criterion (19) is in principle conservative as the normal reaction at the contact point at the water line, *cf.* Figure 5, has been neglected. It would cause an additional resistance of $\mu(R/\cos\alpha)\tan\alpha$. The parentheses in equation (19) would then contain the terms $(\mu + \tan\alpha + \mu\tan^2\alpha + \mu^2\tan\alpha)$, where the two latter terms are of a smaller order of magnitude. If condition (19) is not satisfied, a partial ride-up occur instead. The length of the slope that might become overridden by ice in a partial ride-up can be found by isolating the slope length in condition (19). This yields:

$$L_p = F/(\gamma_i bh(\mu + \tan\alpha)) \tag{20}$$

in which L_p is the length of the slope that is covered with ice. If an ice floe has a relatively limited

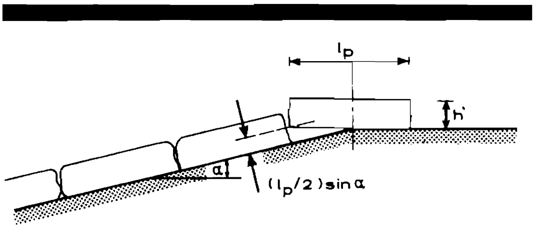

Figure 6. Ice pieces passing a decrease in slope steepness.

extent but a considerable kinetic energy, it may still ride up the slope turning some of its kinetic energy into potential energy and dissipating some by overcoming frictional resistance. The work done during the ride-up may be expressed as:

$$W = \tfrac{1}{2} L^2 \gamma_i bh(\sin\alpha + \mu\cos\alpha) \qquad (21)$$

in which the first term expresses the work to overcome gravity and the second term the work to overcome friction. The kinetic energy of the ice floe immediately prior to impact is:

$$E_k = (1 + C_m)\pi D^2 h \rho_i v^2/8 \qquad (22)$$

where E_k is the kinetic energy of the ice floe, C_m is the added mass coefficient, D is the diameter of the ice floe, ρ_i is the density of the ice, and v is the ice floe velocity prior to impact. The floe may cause a ride-up to the top of the slope if the E_k/W ratio exceeds one:

$$E_k/W = \frac{(1 + C_m)\pi D^2 V^2}{4bL^2 g(\sin\alpha + \mu\cos\alpha)} > 1 \qquad (23)$$

where g is the acceleration of gravity and b the width of the ride-up zone. The value of the added mass coefficient depends on water depth, floe, velocity, *etc.* With lack of accurate expressions, a central value of 0.2 may be used. The criterion (23) does not consider the work done in breaking off pieces of the ice floe. This amount of work may be significant on a relatively steep mound breakwater, while it may be of no importance on a long shallow beach. Ignoring it makes the criterion conservative.

In order to determine whether a given ice floe can cause a ride-up to the top of a given slope, it is necessary to check both criteria (19) and (23), *i.e.*, whether the environmental driving forces can push the ice up the slope, *and* if not, whether the floe possesses enough kinetic energy to ride-up while dissipating this kinetic energy.

Complex profiles with one or more slope changes between the water line and the crest may warrant detailed investigations in case ride-up and pile-up are critical. One such example is the breakwater in Nome, Alaska, which was studied in a physical model, *cf.* ETTEMA *et al.* (1983) and SACKINGER *et al.* (1983). Complex profiles have also been used in the Beaufort Sea, *cf. e.g.*, CROASDALE and MARCELLUS (1978). Physical model studies were used here as well, *cf. e.g.*, ABDELNOUR *et al.* (1982).

ICE PILING-UP

Ice Pile-Up Initiation

On shores and breakwaters an ice pile-up may form as a result of an instability in the ride-up process. Designs favouring instabilities may therefore be desirable. They can limit the maximum ride-up by ensuring that a pile-up is initiated close to the water line. Instabilities usually occur where there is a change of slope angle and where there is sufficient compression between the individual ice pieces.

Forces and deformations are governed by the ice strength rather than the available driving force for ice that is on the slope. The following equations consequently focus on reactions, H and V, for given deformations, rather than on the external driving force, F.

The in-plane forces in the ice on the slope are largest at the ice edge at the water line. The horizontal and vertical force components here, *cf.* Figure 5, are:

$$H = R\cos\alpha \qquad (24)$$

$$V = R\sin\alpha \qquad (25)$$

in a simple static analysis. Note that $H = F\cos^2\alpha$, where F is the horizontal driving force, and H is the horizontal projection of the slope parallel resistance. If the height from the water line to the point that the ice has reached, is denoted Z, with $Z = L\sin\alpha$, and the expression (18) for R is used, the force components become:

$$H = Z\gamma_i bh\cot\alpha(\sin\alpha + \mu\cos\alpha) \qquad (26)$$

$$V = Z\gamma_i bh(\sin\alpha + \mu\cos\alpha). \qquad (27)$$

If the in-plane (horizontal) force is ignored in the calculation of bending and the theory for a simple beam on an elastic foundation (HETENYI, 1946) is used, the critical vertical load at the edge of the ice becomes:

$$V_{cr} = 0.68\sigma_f b(\gamma_i h^5/E)^{0.25} \qquad (28)$$

If the vertical edge load expressed by equation (27) exceeds the critical load expressed by equation (28), the floating ice floe will break, and ride-up becomes impossible. For a ride-up to be possible, the vertical edge load must be less than the critical load *i.e.*, the V_{cr}/V ratio must exceed one:

$$V_{cr}/V = 0.68(\sigma_f/Z)(h/E)^{0.25}\gamma_i^{\ 0.75} \cdot(\sin\alpha + \mu\cos\alpha)^{-1} > 1 \qquad (29)$$

If this criterion is not fulfilled, a pile-up will form at the water line provided adequate driving forces are present. The ignoring of in-plane forces is not necessarily a good assumption, and caution is recommended in applying this criterion. A "conservative" criterion may require either higher or lower values depending on the problem in question.

On a very shallow beach the edge load on the floating floe may not be able to cause breaking, because the floe will touch the bottom. On mound breakwaters, the above criterion (29) is applicable. In its simplicity the criterion is a handy rule-of-thumb, but more detailed analyses are warranted if pile-ups constitute a governing criterion.

As the ice rides up a slope, it may eventually reach a change of slope angle, typically at the "top" where the surface slope changes to zero, *i.e.*, horizontal, as shown in Figure 6. If the change in slope angle is too large, an ice piece which has tilted to horizontal but still overhangs the slope with half the piece length will be tilted over by the following piece.

This may take place if $h > \ell_p \sin\alpha/2$ assuming no friction between the individual pieces on the slope. In reality, a piece will ride slightly more than $\ell_p/2$ over the top before tilting to horizontal. A local failure near the corner of the piece may occur if the contact point is too close to the corner. It may be more realistic to use a criterion like:

$$h > 0.6\ell_p \sin\alpha \rightarrow \text{ride-up continues} \qquad (30)$$

$$h < 0.6\ell_p \sin\alpha \rightarrow \text{pile-up forms.} \qquad (31)$$

The magnitude of this correction was suggested by CROASDALE *et al.* (1978).

Another form of instability may occur if there is a bump in the slope as shown on Figure 7. If the compressive force in the ice is sufficiently large, the bump will initiate an ice piling when ice pieces ride over it. With $\alpha = 0$ for simplicity a balance of moments yields:

$$P = \frac{\ell_p b h \gamma_i}{2\tan\phi} \qquad (32)$$

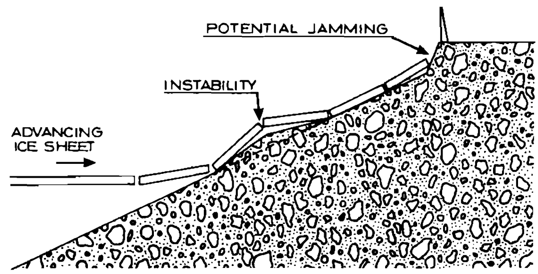

Figure 7. Instability at a bump in the slope.

$$P \approx \frac{\ell_p^{\,2} b h \gamma_i}{2e} \qquad (33)$$

where in the second expression the approximation $\tan\phi \approx \sin\phi$ has been applied. The parameters ϕ and e are the inclination angle of a floe on the bump and the maximum height of the bump, respectively. If the compressive force is known, for example from equation (18), the necessary height of the bump to cause an ice piling may be found by inverting equation (33) to read:

$$e \approx \frac{\ell_p^{\,2}}{2L(\sin\alpha + \mu\cos\alpha)} \qquad (34)$$

with the expression (18) inserted. This instability mechanism may be utilized by the designer to reduce the risk of ice riding over the top of the slope. It is important that the bump has a horizontal extent along the breakwater larger than the width of the ice pieces. This means that the bump must be part of the cross-section at any point of the breakwater. Local bumps with limited extent will not necessarily be effective, *cf.* ABDELNOUR *et al.* (1982).

Ice Pile-Up Height

Once the pile-up is initiated, a complex variety of processes takes place as the pile-up grows. If the ice sheet rides up over the seaward face of the pile-up, it will fragment at the top. Most fragments will fall on the landward face of the pile-up, thereby adding to the height and the landward horizontal extent of the ice pile-up. As a limiting height is reached, fragments will fall on the seaward face eventually leading to a disruption of the sheet ride-up by breaking apart the pieces. The advancing floating ice sheet may push itself either on top of or into this new rubble, adding to the seaward horizontal extent of the ice pile-up.

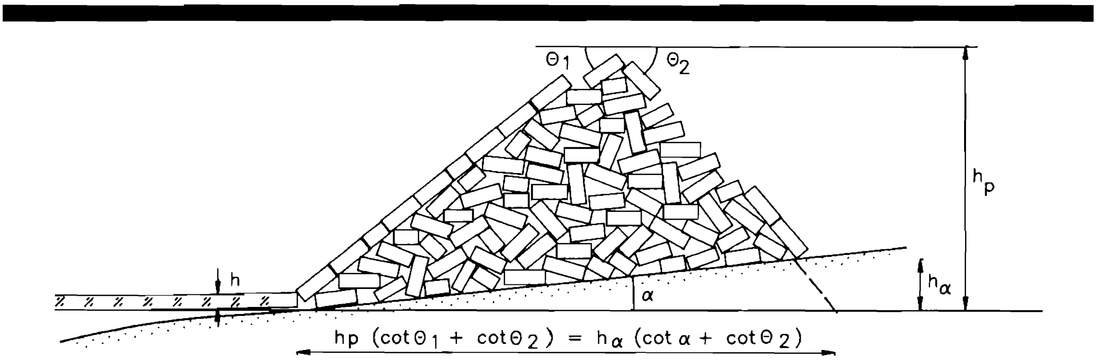

Figure 8. Geometry of the ice piling process in KOVACS and SODHI's (1980) model.

A calculation of the limiting maximum ice pile-up height requires knowledge of the forces involved. Two different approaches can be taken, based on either ALLEN (1970) or KOVACS and SODHI (1980). KOVACS and SODHI gave equations for the forces required to overcome gravity of the advancing ice sheet itself and friction. Adding these terms to obtain the total piling force implies that the ice sheet is riding up over the front (seaward) face of the pile-up in what may be characterized as a very steep ride-up over ice rubble. Allen assumed a somewhat more complex rubble building process in which the advancing ice sheet must essentially lift the existing pile-up in a pushing-up process. The former approach leads to limiting heights proportional to the ice thickness, whereas the latter approach leads to limiting heights proportional to the square root of the ice thickness. Both pile-up process models appear to describe actually observed mechanisms.

It is of interest to compare these conceptually different approaches to see which one is best suited for determination of pile-up heights to be used in design of coastal structures. Each approach is therefore outlined in more detail below.

Ice Pile-Up Heights Based on Kovacs and Sodhi's Model

KOVACS and SODHI (1980) described the forces required to overcome gravity of the advancing ice itself and ice-to-ice friction in a ride-up over the seaward face of an ice pile-up. The force required to overcome gravity, F_{gr}, is found from a balance of the work done by the force and the increase in potential energy of the ice piling. The geometry of the ice pile-up is shown in Figure 8. The volume per unit width of the ice pile-up is:

$$V/b = \tfrac{1}{2}(h_p^2(\cot\theta_1 + \cot\theta_2) - h_\alpha^2(\cot\alpha + \cot\theta_2)) \quad (35)$$

where V is the volume of the ice piling, b is the width of the ice piling (plane situation), h_p is the height of the piling above water level, h_α is the beach elevation increase under the ice piling, θ_1 is the slope of the seaward face of the ice piling, θ_2 is the slope of the landward face of the ice piling and α is the slope of the beach. The beach elevation increase, h_α, can be expressed in terms of h_p, α, θ_1 and θ_2. Consequently, expression (35) may be reformulated to avoid using h_α and only use h_p. It becomes:

$$V/b = (h_p^2/2)G(\alpha, \theta_1, \theta_2) \quad (36)$$

$$G(\alpha, \theta_1, \theta_2) = \frac{(\tan\theta_1 - \tan\alpha)(\tan\theta_1 + \tan\theta_2)}{(\tan\alpha + \tan\theta_2)\tan^2\theta_1}. \quad (37)$$

By relating the mass transport in an incremental advance of the ice sheet, $\rho_i hdx$, to the associated incremental increase in mass in the ice piling, $\rho_p dV$, the expression for an increment in height, dh_p, becomes:

$$dh_p = (\rho_i/\rho_p)(h/h_p)(1/G(\alpha, \theta_1, \theta_2))\,dx \quad (38)$$

The potential energy per unit width in the ice piling is established from simple geometrical considerations:

$$E_p/b = (1/6)\gamma_p(h_p^3(\cot\theta_1 + \cot\theta_2) - h_\alpha^3(\cot\alpha + \cot\theta_2)) \quad (39)$$

where γ_p is the specific weight of the pile-up. It is again possible to eliminate h_α by expressing it in terms of h_p, α, θ_1 and θ_2. The expression then reads:

$$E_p/b = (1/6)\gamma_p h_p^3 Q(\alpha, \theta_1, \theta_2) \quad (40)$$

$$Q(\alpha, \theta_1, \theta_2) = \frac{(\tan\theta_1 + \tan\theta_2)}{(\tan\theta_1 \tan\theta_2)} - \frac{(\tan\theta_1 + \tan\theta_2)^3 \tan^2\alpha}{(\tan\alpha + \tan\theta_2)^2 \tan^3\theta_1 \tan\theta_2}. \quad (41)$$

Finally, by equating the increment in potential energy, dE_p, to the work done by the driving forces to overcome gravity, $F_{gr}dx$, these may be expressed:

$$F_{gr} = \frac{1}{2}\gamma_i bhh_p \frac{Q(\alpha, \theta_1, \theta_2)}{G(\alpha, \theta_1, \theta_2)}. \quad (42)$$

In the area of interest ($o < \alpha < \theta_1$) the ratio Q/G generally varies between 1 and 2. For $\alpha = 0$ it is equal to one. It is shown in Figure 9.

Friction must also be overcome in order to add to the pile-up. The necessary force was expressed by KOVACS and SODHI as:

$$F_{fr} = \mu\gamma_i bhL \cos\theta_1 \quad (43)$$

where μ is the ice-to-ice friction coefficient, γ_i is the specific weight of ice, b is the width of the ride-up zone, h is the ice sheet thickness, L is the length of ice on the slope, and θ_1 is the slope angle of the seaward face.

By using the relationship $L = h_p/\sin\theta_1$ assuming that the ride-up continues to the top of the ice pile-up, equation (43) can be rewritten:

$$F_{fr} = \mu\gamma_i bhh_p \cot\theta_1 \quad (44)$$

and eventually the total force to add to the ice piling becomes:

$$F_p = F_{fr} + F_{gr}$$

$$F_p = \gamma_i bhh_p\left(\mu\cot\theta_1 + \frac{1}{2}\frac{Q(\alpha, \theta_1, \theta_2)}{G(\alpha, \theta_1, \theta_2)}\right). \quad (45)$$

If the height of the pile-up is only limited by the magnitude of the driving forces, it may be calculated from:

$$h_p = \frac{F_p}{\gamma_i bh[\mu\cot\theta_1 + \tfrac{1}{2}\{Q(\alpha,\theta_1,\theta_2)/G(\alpha,\theta_1,\theta_2)\}]} \quad (46)$$

which is a simple inversion of equation (45) in order to isolate h_p. The driving forces, F_p, normally consist of wind shear forces on the ice sheet and current drag on the underside of the ice cover. The current drag is normally small for directions perpendicular to a coast, but may have significant along shore components, that are relevant for *e.g.*, breakwaters. The height of the pile-up predicted by equation (46) is directly proportional to the magnitude of the driving forces, F_p. This reflects the fact that the equation limits the height by equating the driving forces and the force required to push the "last" piece to the top of the pile-up.

Ice Pile-Up Heights Based on Allyn and Charpentier's Formula

ALLYN and CHARPENTIER (1982) further developed a somewhat similar model originally created by ALLYN. They developed it to take temperature and salinity effects into consideration, but the original model is of primary interest here. ALLYN and CHARPENTIER's formula for pile-up height is:

$$h_p = \frac{(\cos\theta_1 - \mu\sin\theta_1)F - (\sin\theta_1 + \mu\cos\theta_1)V}{\gamma_i h(1 + \mu\cot\theta_1)(1 + (s'/s))} \quad (47)$$

where F is the external force and V the vertical edge load on the incoming ice sheet. The assumed pile-up formation process is quite similar to that described by KOVACS and SODHI (1980), but a correction factor is added to account for ice blocks riding on top of the primary train of ice blocks. The length of the primary train of ice blocks in the ride-up over the front face is denoted s, and s′ is the cumulative length of ice blocks riding on top of the primary train. Allyn and Charpentier quote $s'/s = 0.5$ as a typical value. While this correction factor accounts for ice blocks on top of the primary train, it does not apply to situations where the ice sheet pushes deep into the pile-up. Because of the similarities with Kovacs and Sodhi's expressions, notably the linear relationship between external force and pile-up height, equation (47) is not analyzed any further here.

Ice Pile-Up Heights Based on Allen's Formula

A formula was derived by ALLEN (1970) under the assumption that the driving forces had to push the entire front part of the pile-up upwards, rather than just the outermost layer. Figure 10 shows an impression of how the piling-up process takes place in Allen's model. More precisely, Allen assumes that the ice sheet pushes horizontally into the pile-up. He views the volume of ice above

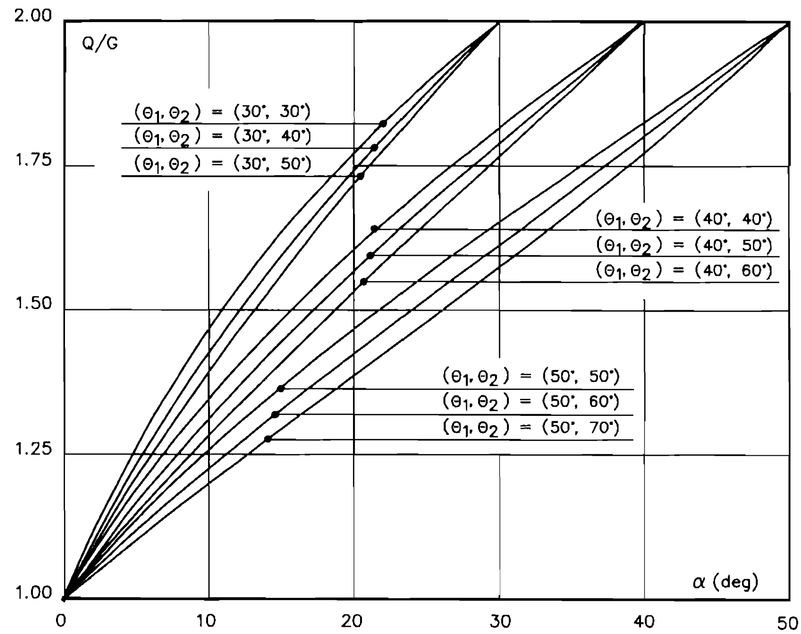

Figure 9. Plot of Q/G as a function of α for different values of pile-up slopes (θ_1, θ_2).

water level and in front of the top as a number of slices, each of which has to be pushed upward by the incoming ice sheet. Figure 10 actually shows the ice sheet pushing somewhat upward after having entered the pile-up and as such the Figure depicts a process between the two extremes represented by ALLEN (1970) and by KOVACS and SODHI (1980). ALLEN's (1970) formula is derived from fairly straightforward balances of driving forces, gravity, and friction. The formula reads:

$$h_p = \left(\frac{2F_p}{\gamma_p b(1 + (f/\tan\theta_1))}\right)^{0.5} \quad (48)$$

where γ_p is the specific weight of the pile-up *i.e.*, ice and air, and f is a coefficient of internal friction in the pile-up. This formula is conceptually different from equation (46) in that the predicted height is now proportional to the square root of the driving force. Like equation (46), however, it also assumes that the available driving force is the limiting factor. For design calculations, ice strength should be the limiting factor. This will be explained in the next section. Allen's formula, equation (48), is based on a balance between the force, F_p, which in the horizontal plane acts against the base of the ice pile-up, and the force required to further pile up the ice. Other parameters are the specific gravity of the pile-up (ice and air-filled voids), γ_p, the coefficient of internal friction, *f*, and the slope of the pile-up with horizontal θ_1. The reader should confer with the original reference for a detailed account of how the formula is derived. ALLEN (1970) stated that his formula is conservative because most observed heights are smaller than predicted by his formula. One very plausible explanation is that the entire ice volume behind the pile-up top *i.e.*, the landward side, is disregarded in his analysis. Some force will be required for redistribution of ice blocks there. Another important point is that γ_p, f, and α are not necessarily completely independent (BRUNN and JOHANNESSON, 1971). Weaknesses normally occurring in ice sheets may furthermore be responsible for the observations being generally smaller than his predictions. A wider base of observations is necessary.

LIMITING ICE PILE-UP HEIGHTS FOR DESIGN

The formulas discussed above relate the height of an ice pile-up to the available driving force. However, owing to typical ice floe shapes, large ice floes can interact with shores or coastal structures over widths many times smaller than the

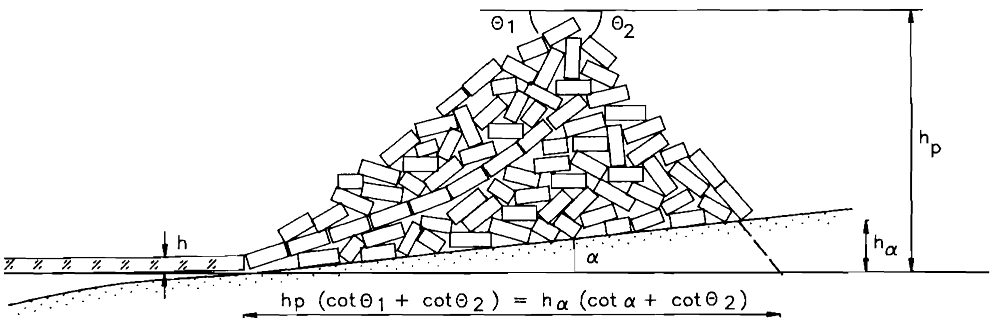

Figure 10. Geometry of the ice piling process in ALLEN's (1970) model, slightly modified (see text).

floe diameter. The interaction *i.e.*, the piling-up process, may thus be sustained past the limiting height as the floe energy is dissipated in a relatively narrow interaction zone. The floe or structure shape causes a focusing of the energy.

In general, there are three different limitations applicable in most ice engineering analyses, *cf.* CROASDALE (1984), *viz.*:

limited driving force,
limited ice strength, and
limited kinetic energy.

The pile-up heights, or *e.g.*, an ice load, may naturally be limited by the available driving forces. If the driving forces are unlimited, the ice itself will limit the load through its strength, whether compression strength associated with crushing or flexural strength associated with bending, *etc.* Thirdly, with the driving forces limited *e.g.*, because of a small surface area of a single floe, kinetic energy might still be sufficient to generate a ride-up or pile-up event of a certain magnitude.

The theoretical upper limit for horizontal ice pressure corresponds to the critical failure pressure in the ice in front of the pile-up. The ice cannot impose more pressure on the pile-up than that which corresponds to this critical failure pressure. More pressure would lead to failure of the ice in front of the pile-up, extending the pile-up seawards instead of increasing the height. In cases of a moderate ice thickness, the pile-up height is often limited by the ice buckling pressure. For large ice thicknesses, the limitation may correspond to ice crushing.

For the case of a *beam* on an elastic foundation (HETENYI, 1946; ASHTON, 1986), the critical buckling pressure may be estimated as $ck\ell^2$ where k is the density of the sea water and ℓ is the characteristic length of the ice defined as the fourth root of the ratio between the plate stiffness and the foundation stiffness. The reaction of the water to a given downwards deflection of the ice is equal to the buoyancy of the ice, and the apparent "foundation stiffness" is equal to the specific gravity of the seawater. For long beams, the dimensionless constant c varies between one and two depending on the boundary conditions of the beam. For small values of ice thickness, a moderate critical buckling pressure is obtained, corresponding to a modest height of the ice pile-up. PALOSUO (1971) noted an "almost linear" relationship between ice thickness and the wind velocity needed to break up stationary sheet ice. His observations covered ice thicknesses in the range 0.07 m to 0.25 m, where buckling is a dominating mode of failure. Since the *plate* buckling load is proportional to the thickness to the power of 9/4 (beam buckling is 6/4) and the wind shear is proportional to the wind velocity squared, the wind velocity required to cause break-up of stationary ice through plate buckling becomes proportional to the thickness to the power 9/8. The curvature of a best-fit line to PALOSUO's (1971) data actually suggests a power slightly larger than one. It appears realistic that the buckling pressure constitutes a limit pressure in this range. For large ice thicknesses, a high critical failure pressure is obtained, and the pile-up height becomes limited by the magnitude of the driving forces instead.

In order to determine the pressure which is transferred to the ice pile-up, it is theoretically necessary to calculate both the critical failure pressure, whether buckling, crushing or fracture, and the pressure offered by the driving forces. The

Table 1. *Largest possible ice pile-up heights based on* ALLEN*'s (1970) formula and on expressions by* KOVACS *and* SODHI *(1980), with the critical buckling pressure applied as the pressure induced by the driving forces, and with* $E = 2$ *Gpa,* $\nu = 0.33$, $k = 10$ KN/m^3, $\theta_i = 35°$, $f = 0.25$, *and* $\gamma_p = 6$ KN/m^3. *In Kovacs and Sodhi's expressions,* $\alpha = 11.3°$ *(1:5),* $\theta_2 = 45°$ *and* $\mu = 0.1$, *have furthermore been applied. Note that for thick ice, buckling is not necessarily the dominating failure mode.*

Ice Thickness (m)	Characteristic Length (m)	Buckling Pressure (MN/m)	Ice Pile-Up Height (m)	
			Allen	Kovacs and Sodhi
0.1	2.1	0.043	3.2	(57.8)
0.3	4.7	0.225	7.4	(100.9)
0.5	7.0	0.484	10.9	(130.2)
0.7	8.9	0.801	14.0	(153.9)
0.9	10.8	1.168	16.9	(174.5)

lowest of the two is the maximum pressure transmitted to the pile-up. In many cases it might be reasonable to use only the failure pressure limitation. This corresponds to assuming that there is always sufficient driving force to form the maximum limiting pile-up height. The reason for this recommendation is that long stretches of shoreline directly exposed to ice attack combined with ice floe geometries may engender focusing of the driving forces. A single floe of say more than 1 kilometre diameter may interact with a revetment over a width of say less than 100 metres. The conservative ice strength limitation for pile-up heights is recommended as a first estimate. In cases where it can be documented that floes cannot attain sufficient kinetic energy, the driving force limitation for a floe at rest may be used instead. In example calculations, a modulus of elasticity of 2 GPa for the ice, a density of the water of 10 kN/m^3, and a Poisson's ratio of 0.33 are applied. Based on this, the maximum *possible* ice pile-up height can be calculated for various ice thicknesses by application of the critical buckling pressure in Allen's formula. The result of this calculation is shown in Table 1. Obviously, the heights predicted on the basis of the model proposed by KOVACS and SODHI (1980) are quite unrealistic. The heights based on ALLEN's (1970) formula are of a more reasonable magnitude. The high values resulting from Kovacs and Sodhi's expressions will not occur in nature where other limitations *e.g.*, sliding stability of the entire pile-up, come into play. The larger Allen-based heights are only quite rarely obtained. These pile-ups require simultaneous occurrence of several conditions *i.e.*, ice thickness, floe size and geometry, direction of driving forces, *etc.* Pressures of this relatively large magnitude are attainable, however, provided the ice sheet has sufficient strength. KOVACS and SODHI (1981) estimated pressures up to 3 MPa in 1.5 m thick sea ice at Fairway Rock in the Bering Strait. KOVACS *et al.* (1982) also analyzed five different failure modes of the ice for the Fairway Rock icefoot: creep, crushing, flexure, rubble formation and buckling. For the 1.5 m thick ice, they arrived at pressures ranging from 0.4 MPa to 2.6 MPa for the different failure modes.

In the same manner, actual pile-up heights can be calculated based on the driving forces pertaining to *e.g.*, wind velocities when it is assumed that the ice strength does not impose any limitations on the height or, rather, that other limitations come into play first. Typical pressures in plane models would be in the order of 8 kN/m, using a 1 by 5,000 m ice strip in equation (7) with V_{10} = 20 m/sec and ρ_a = 1.29 kg/m^3. Even with different values of the involved parameters, pressures in the order of magnitude of 100–1,000 kN/m such as the buckling pressures in Table 1 cannot be obtained clearly. For perfect contact situations, *i.e.*, nearly plane stress states, it is thus likely that the driving forces constitute the limiting factor.

There are thus two different models predicting the ice pressure, *viz.*:

(1) the limit-force model, *i.e.*, plane accumulation of driving shear, or
(2) the limit-strength model, where focusing takes place to obtain the failure pressure.

Similarly, there are two different models of the piling-up process where the essential difference is whether to overcome

(1) gravity of the entire pile-up (ALLEN), or
(2) only gravity of the advancing ice itself (KOVACS and SODHI).

The two piling-up models furthermore have different approaches to the overcoming of friction. Allen's formula is based on a coefficient of internal friction, as the ice sheet is pushed into the pile-up. Kovacs and Sodhi's expressions consider simple friction at the ice underside in a ride-up type process. Allen's formula works well with the failure pressure inserted for the unit driving force but would grossly underestimate pile-up heights if the average unit driving force was used. Kovacs and Sodhi's expressions work well with average unit driving force inserted, see *e.g.*, CAMMAERT and MUGGERIDGE (1988) but grossly overesti-

Table 2. *Largest ice pile-up heights based on* ALLEN'S *(1970) formula and on expressions by* KOVACS *and* SODHI *(1980), with the unit pressure induced by wind shear 5,000 m upstream of the shore as the unit driving force, and with $\theta_1 = 35°$, $f = 0.25$, and $\gamma_p = 6$ kN/m^3 in Allen's formula. In Kovacs and Sodhi's expressions, $\alpha = 11.3°$ (1:5), $\theta_2 = 45°$ and $\mu = 0.1$, have furthermore been applied.*

Ice Thickness (m)	Wind Speed (m/sec)	Unit driving force (kN/m)	Ice Pile-up Height (m)	
			Allen	Kovacs and Sodhi
0.3	10	1.94	(0.69)	0.87
0.3	15	4.35	(1.03)	1.95
0.3	20	7.74	(1.38)	3.47
0.3	25	12.1	(1.72)	5.42
0.3	30	17.4	(2.07)	7.80

mates pile-up heights if the horizontal failure pressure is used. Kovacs and Sodhi claimed that the high pressures suggested by Allen pertain to freshwater ice.

There are no simple arguments that connect the limit pressure models with the pile-up formation models so that reasonable heights can be predicted. The combinations that give under- and over-estimation, *i.e.*, those in parentheses in Tables 1 and 2, are not less likely to occur than those that work well. The explanation most likely is that all four combinations of pressure and pile-up process models do occur, but in the case of near-failure pressures and Kovacs and Sodhi's pile-up process, another limit applies to maximum pile-up height. This other limitation could be the sliding stability of the entire ice pile-up, or it could be a buckling of the series of ice pieces riding up the front face of the pile-up. These pre-broken pieces will have end faces at different orientations, and weak points will occur. To observe the piling-up process in detail, a model test program would be very helpful. A broad base of simultaneous full-scale observations of the many parameters included in the theoretical models would be of interest as well. A limited base of field observations is examined below.

SAYED and FREDERKING (1986) have analyzed the formation of floating ice ridges and found similarly that a potential energy method alone underestimates the involved forces. It would thus, overestimate the ridge size parallel to what was found for Kovacs and Sodhi's expressions with the failure pressure inserted. TIMCO and SAYED (1986) further conducted physical model tests of the ridge building process. They concluded that the external forcing, "appear to be proportional to the keel depth and sail height raised to a power greater than one". Inverting this relationship to express heights in terms of external forcing, they found powers less than one. For ice pile-ups, Kovacs and Sodhi's expressions correspond to a power of one, and Allen's formula corresponds to a power of 0.5. The study by TIMCO and SAYED (1986) thus supports the notion that Kovacs and Sodhi's pile-up formation model leads to conservative height estimates for high driving pressures. It should be noted that sheet ice is more likely to move *into* a floating ice ridge, whereas it may move both *into* a pile-up or *ride up* the front face of a pile-up (which by definition is grounded).

COMPARISON WITH FIELD OBSERVATIONS

The limiting pile-up heights in Tables 1 and 2 should preferably be substantiated by observations. As an example, look at the Danish domestic waters where two major tunnel and bridge projects are under way. The "Great Belt" crossing (CHRISTENSEN and SKOURUP, 1991) is nearly completed, and the "Sound" crossing is in the design phase. The latter will connect Copenhagen in Denmark with Malmö in Sweden. For these projects, the author has collected observations of ice conditions and ice pile-ups. Both local sources, museums and local residents, and more general sources, scientific and engineering literature, have been consulted. This material contains information on the largest ice pile-ups that have occurred in the respective areas in this century. It is assumed that these results are representative of the area.

BRUUN and JOHANNESSON (1971) describe Baltic experiences of interaction between ice and coastal structures. They show pictures of the Nordre Røse Lighthouse a few kilometres to the north of the planned Sound Link with ice pile-ups more than 10 m high in 1892 and also note pile-ups exceeding 8 m in 1956. In 1956 STATENS ISTJENESTE (1907–1992) reported the amount of cold as 226 °C-days. This leads to a maximum ice thickness estimated at 0.42 m. The piling most likely took place somewhat before the end of the winter. The thickness may have been around say 0.3 m. Table 1 suggests a limiting pile-up height of 8.1 m in this case. It appears that the Nordre Røse ice pile-up may have reached its limiting height in 1956. The "1892" observation by BRUUN and JOHANNESSON (1971) is assumed to actually be from 1892–1893, a severe ice winter, whereas 1891–1892 was rather mild. With 257 °C-days the

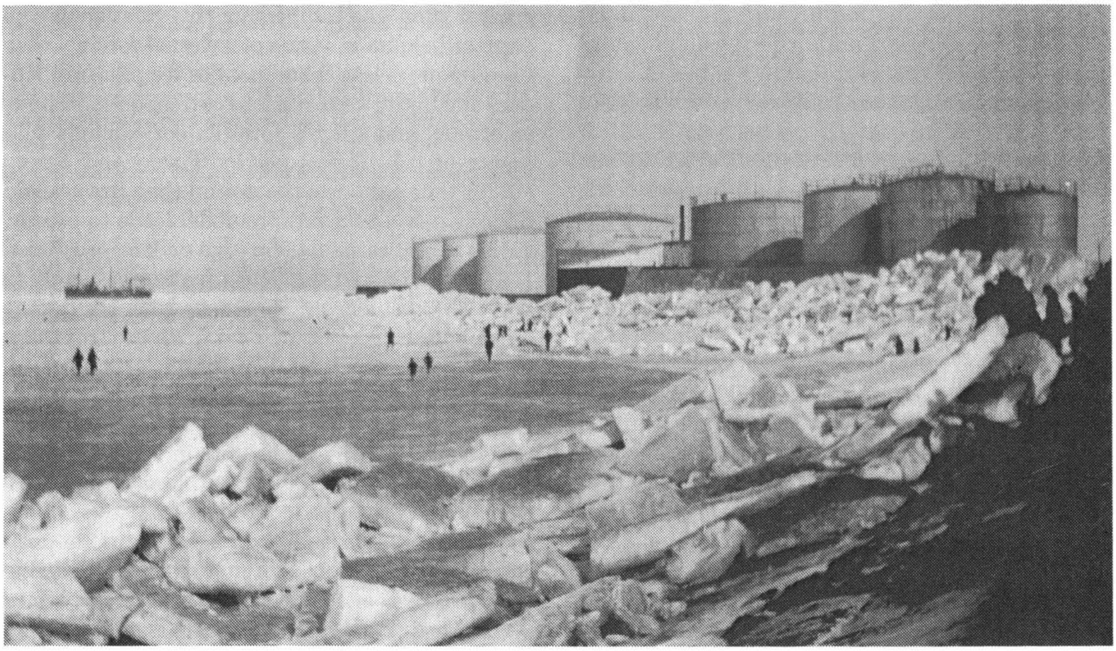

Figure 11. Ice pile-ups about 9–10 m high in front of Kockum's shipyard in Malmö, Sweden, allegedly during World War I and then probably 1917. Note the persons on the ice. Photo courtesy of Limhamn's Museum.

estimated maximum ice thickness becomes 0.46 m. With 0.40 m at the time of piling, the limiting height would be 10.0 m. Again, it appears that the piling reached its limiting height, given the assumed values of thickness, Young's modulus, *etc.* Bruun and Johannesson included a photo of the "1892" pile-up, but due to a very poor photographic quality, it is not reproduced here. The winters of 1893 and 1956 are far from the only winters with high ice pile-ups in the region.

Large ice pile-ups also occurred in the Sound region in at least 1823, 1881, 1888, 1893, 1895 and in 1901, 1912, 1917, 1922, 1929, 1936, 1940, 1941, 1942, 1947, 1956, and 1963. Large pile-ups have been observed on shores facing both east, south, and west near the future Link. Pictures from Saltholm (Danish Hydraulic Institute, 1974) indicate that 10 m high ice pile-ups occurred near Svaneklapperne in 1940. With a freezing degree-day index of 368.5 °C-days that winter, h_{max} becomes 0.57 m and the thickness at the time of piling-up say 0.40 m corresponding to a limit height around 10 m.

Limhamm's Museum near the planned Swedish landfall has located a photo in their files showing ice pile-ups about 9–10 m high in front of Kockum's Shipyard in Malmö, *cf.* Figure 11. The photo is allegedly from World War I, in which case 1917 is the only possibility. The theoretical h_{max} for 1917 is 0.35 m, and the pile-up must then have formed at the end of winter to reach that height, *cf.* Table 1. It should also be noted that the highest portions of the pile-up in the photo appear to be supported by a sloping sea wall. The upper ice pieces might actually have been pushed up in a ride-up mode, rather than by ice piling in which significant amounts of ice rubble must be pushed upward as well. This would explain that the observed height (9–10 m) exceeds the limiting height (8.3 m).

Figure 12 is a photo of ice pile-ups outside the city of Malmö, immediately north of the eastern landfall of the planned Link. The height appears to be around 9 m again, and the photo is allegedly "from World War II". The thickness appears to be in the range of 0.35–0.40 m in the photo, and the pile-up may have been close to its limiting height.

Danish Hydraulic Institute (1974) reported pile-ups of 11 m height off Kastrup in 1881, and

15 m height on long stretches of the Danish Sound coast in 1823. For the latter, the freezing degree-day index is not readily available, but for the former, it is 196 °C-days and the h_{max} thus 0.39 m. The limiting pile-up height would then be about 10 m, reasonably close to the observed 11 m.

A relatively minor ice pile-up off the harbour of Helsingborg, on the Swedish coast some 50 km north of the planned Sound Link, is shown in Figure 13. The photo was taken in 1922. The 1921–1922 winter had 165.4 °C-days and the maximum thickness might have exceeded 30 cm. From the photo, however, it appears that the pile-up formed when the ice was about 10 cm thick. Very interestingly, it appears that the pile-up has begun to grow seawards in some places indicating that the limiting height might have been reached. Table 1 suggests a 3.2 m limiting height for this ice thickness. The height in the photo is estimated to be just under 3.0 m especially in the background. This is quite close to the limiting height suggested in Table 1, and Figure 13 thus provides an interesting verification for small ice thicknesses.

In the Great Belt region, ice pile-up conditions are quite similar to The Sound. Figure 14 shows an ice pile-up in 1946–1947 off the town of Korsør immediately south of the present-day eastern landfall of the Great Belt Link. From the photo, the ice thickness and pile-up height are estimated to be 0.35 m and 8 m, respectively. This is close to the limiting height of 8.3 m. Figure 15 shows a pile-up north of the western landfall, allegedly from early 1929. From the photo, an ice thickness of 0.4 m and a pile-up height of 9 m are estimated. Again, the pile-up seems to have been close to the Allen-based limiting height.

The largest ice pile-up ever reported in the Sound area is a 20 m high pile-up on the "Bredegrund" shoal some 12–15 km south of the planned Link. The observation was made by fishermen from Dragør in 1893. Investigations and interviews by the DANISH HYDRAULIC INSTITUTE (1974) did not reveal reasons to doubt the observation as such. From a theoretical point of view, it appears exaggerated since the 1893 h_{max} of 0.46 m would only be capable of building an 11 m high pile-up according to Table 1. The exposed location of Bredegrund might account for larger heights.

This observation means that the "100-year" pile-up in the area is most likely larger than 10 m, probably in a range of 12–18 m depending on location. This relates well with the ice thickness of 0.67 m determined for an average recurrence period of 100 years by CHRISTENSEN and SKOURUP (1991). The Allen-based limit height corresponding to this thickness is 13.5 m. It appears that for exposed coasts the ice pile-up heights may be determined from the ice thicknesses. This is interesting because ice thicknesses can be determined from air temperature records combined with just a few observations for calibration. Air temperature records are far more common than ice pile-up observations and reservations are necessary. This analysis has not covered all the possible limitations to pile-up heights, and the framework is not yet complete.

Figure 12. Ice pile-ups about 9 m in height at the Malmö coast during World War II. From LYBECK (1943).

CROASDALE *et al.* (1988, p. 227) reports the rubble height contours around the caisson retained island "Amerk O-09" in the Canadian Beaufort Sea on 30 January 1985. The highest peak is 15 m, and the rubble reaches 12–14 m height in several places around the island. The rubble field has apparently grown seaward after formation of at least one of the 14 m peaks so 14–15 m is most likely the limiting height. The sea ice thickness is not reported, but typical values of h_{max} in this area are 1.8–2.0 m. In late January, if half the degree-days of the winter have passed, the ice thickness would then be about 1.35 m, the buckling pressure 2.1 MN/m and the Allen-based limit height about 23 m. This apparent overprediction of limit height is probably due to an unrealistic buckling pressure. The average contact stress is 1.6 MPa, common for limited contact areas, but

Figure 13. Minor ice pile-up at the Helsingborg, Sweden, breakwater in 1922. Photo courtesy of Swedish Meteorological and Hydrological Institute.

unlikely to occur simultaneously over a rigid contact area of 50 m by 1.35 m. The ice sheet may have crushed at lower global loads, or buckling may have occurred along pre-existing cracks in the ice and at lower global loads. Another explanation might be that the rubble itself was not stable. Finally, the uncertainty of the ice thickness estimate must be borne in mind. This example illustrates clearly that a full explanation and understanding of ice piling-up processes and the associated forces and limitations has yet to be formulated.

WRIGHT *et al.* (1978) describe ridges in the Beaufort Sea. They report observations of

Figure 14. Ice pile-up formed off Lütgensvej in the town of Korsør, Denmark, in the 1946–1947 winter. Photo courtesy of Danish Hydraulic Institute.

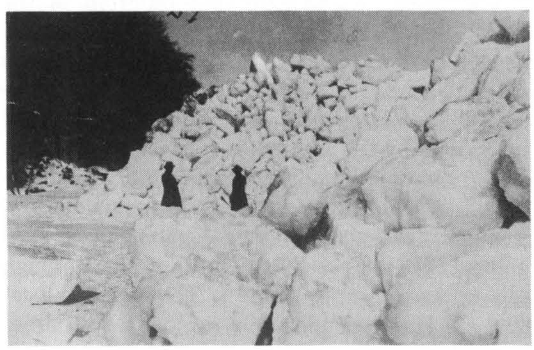

Figure 15. Ice pile-up formed off Teglværksskov on the island of Fyn, Denmark, allegedly in (early) 1929. Photo courtesy of Danish Hydraulic Institute.

grounded ridges where ice blocks have been pushed up to increase the sail height after the grounding. The feature becomes an ice pile-up. They report observations of heights up to 17 m off the east coast of Herschel Island and up to 18 m off the northwest coast of Banks Island. These observations confirm that the mid-winter limit height observed by CROASDALE *et al.* (1988) at Amerk O-09 is of a correct order of magnitude.

It must be recognized that a fairly substantial observation base is necessary to obtain reliable design values from field observations. SPEERSCHNEIDER (1915, 1927) has collected historical ice information on Danish waters covering the years 690–1906, a period of 1,216 years. He concluded that ice conditions in Danish waters do not differ significantly over the centuries and that 3–5 winters per century have particularly severe ice conditions. Dedicated scientific observation programs are, therefore, difficult to plan, and designs typically have to be based on whatever information is available. Furthermore, the industrial development and its potential impact on climate and thus ice conditions occurred after the completion of Speerschneider's analyses. It remains an open question whether ice conditions are changing due to a greenhouse effect caused by human activity.

Although the many observations described cover only a fairly narrow range of thicknesses and heights, they do support the central result well. None of the described pile-ups appeared to be limited by fetches or local geometrical conditions. Naturally, it is desirable to substantiate the result by comparing with observations in other areas as well. This suggests that one should always search for local observations and attempt to calibrate an ice pile-up model for the area in which a design is to be used. Nevertheless, for first estimates the use of Allen's formula with the failure pressure inserted appears to give results that are consistent with observations. The presented base of observations covers most of this century which is important when probabilistic aspects have not been addressed. The average recurrence period for the joint occurrence of ice and forcing conditions to build large pile-ups reaching their limit height might be in the order of decades. ALLEN (1970) found that his formula overpredicted observations, but his observation base may just have been too small.

The question of a limiting pressure for ride-up of pre-broken ice pieces on the seaward face of an ice pile-up was left open, and a full investigation of this point is beyond the scope of this paper. A rough estimate is possible if the Allen-height is accepted as a limit height. In that case when the pile-up heights from equations (46) and (48) are set equal, one finds:

$$\frac{F_{buckl}{}^{pre\text{-}broken}}{\gamma_i bh[\mu \cot\theta_1 + \frac{1}{2}\{Q(\alpha,\theta_1,\theta_2)/G(\alpha,\theta_1,\theta_2)\}]} = \left(\frac{2F_{buckl}{}^{intact}}{\gamma_p b(1 + (f/\tan\theta_1))}\right)^{0.5} \tag{49}$$

where the driving force F_p has been replaced by the buckling loads for pre-broken and intact (frictionless, *i.e.*, c = 1) ice beams, respectively. With $F_{buckl}{}^{intact}/b = k\ell^2$ and $3\gamma_p = 2\gamma_i$, the limit unit buckling load for the pre-broken pieces becomes:

$$F_{buckl}{}^{pre\text{-}broken}/b = (\mu \cot\theta_1 + \tfrac{1}{2}(Q/G)) \cdot (1 + (f/\tan\theta_1))^{0.5}(3\gamma_i k)^{0.5}h\ell \tag{50}$$

where Q and G were defined in equations (41) and (37), respectively. With an example of μ = 0.2, θ_1 = 35°, Q/G = 1.5, f = 0.2, and γ_i = 0.9k, equation (50) reduces to the simpler expression $F_{buckl}{}^{pre\text{-}broken}/b = 1.50\ kh\ell$. For thick ice in Table 1, ℓ can be approximated by 12 h leading to a limit pressure in the order of $(1/8)k\ell^2$ as compared with $k\ell^2$ for the intact ice. For thin ice in Table 1, ℓ can be approximated by 21 h leading to a limit pressure of $(1/14)k\ell^2$ as compared with $k\ell^2$ for intact ice. The ratios 1/8 and 1/14 are naturally the approximate ratios between the two different heights estimated in Table 1. It must be emphasized that a range of values is possible. Nevertheless, ratios of eight to fourteen between buckling loads for intact frictionless ice beams and for pre-broken ice pieces appear plausible. Finally, it should be noted that the buckling load for long beams with one frictionless end, $k\ell^2$, has been used throughout this paper. Under conditions where only hinged and fixed boundary conditions apply, the buckling load would rise to $2k\ell^2$, *cf. e.g.*, ASHTON (1986), and the Allen-based pile-up height would then increase by the square root of two. Another possibility is a plate buckling failure with a failure load in the order of $5k\ell^3$, *cf.* ASHTON (1986), leading to even higher loads. As noted by Allen, however, local weaknesses in the ice sheet precludes the attainment of the theoretical maximum pressures.

CONCLUSIONS

Techniques for predicting ride-up and pile-up of ice against shores and coastal structures have been outlined. Parameters of special interest for design purposes include the maximum landward extent of ride-ups and pile-ups, as well as the maximum height of ice pile-ups. Methods for determining design values of all these parameters were described. For protected shores or coastal structures, where *e.g.*, a limited fetch hampers the accumulation of driving forces and the attainment of significant velocities, a probabilistic approach may better describe the resulting ice pressure.

For ride-ups, equations were given for determining the maximum landward extend on a continuous slope. Dynamic effects have been neglected.

For determination of limiting ice pile-up heights, two ice pressure models and two ice piling-up process models can be combined in four different ways. It was demonstrated through comparisons with field observations that reasonable estimates are obtained using ALLEN's (1970) formula with the driving force limited to the horizontal failure load of the advancing ice sheet in front of the pile-up. Allen's formula assumes that the advancing ice sheet must overcome gravity of the entire pile-up pushing it upwards. KOVACS and SODHI's (1980) expressions include gravity of only the advancing ice itself as it rides up a rubble pile. This latter model works reasonably well when an ice pressure corresponding to the driving forces on a floe at rest is inserted. There is a lack of a logical upper bound to the area in which to accumulate driving stress. From a logical point of view, both ice pressure scenarios and both ice piling-up process models appear likely and probably occur regularly. Regardless of which model is used, sliding stability of the pile-up should also be checked. This may limit the extreme heights predicted in Table 1 with Kovacs and Sodhi's model to a reasonable magnitude. Another possible limitation is the maximum compressive load that a series of riding-up ice pieces can sustain without buckling. This limit load should be determined in a research program. As the geometry of the failure surfaces will greatly influence the limit load, sea ice of a natural thickness, as opposed to a scale model, should be used in the experiments. A rough estimate resulted in values in the order of magnitude of 10% of the buckling load for an intact beam with one frictionless or free end.

An important lesson from this investigation is that there is a lack of consensus on how to predict ice pile-up heights. Different models work best under different assumptions. It is therefore recommended to *always* establish a model, whether theoretical or numerical, to explain prevailing ice pile-up conditions when a height has to be selected for design. It is furthermore recommended that the world community of ice researchers attempt to establish a sufficiently broad base of observations and model test results to describe and understand the piling-up process clearer. KOVACS and SODHI (1988) gave a similar recommendation.

PHOTO QUALITY

Many of the photos included in this paper are of a quality inferior to present day standards. They illustrate well the standard of materials a designer has to work with when faced with having to determine design ice pile-up heights in a mild seasonal ice zone based on historical records. Numerous investigated pictures were of an even poorer quality.

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NOTATION

b = unit width
B = width of ice floe
C_m = added mass coefficient
D = diameter of a circular ice floe
e = vertical eccentricity of horizontal edge load
e = height of bump
E = Young's modulus of ice
E_k = kinetic energy of ice floe
E_p = potential energy of ice piling
f = dimensionless eccentricity factor, $f = e/h$
f = coefficient of internal friction in Allen's pile-up model
F_{fr} = force necessary to overcome friction
F_{gr} = force necessary to overcome gravity
F_p = force necessary to increase height of piling
F_μ = friction force at contact face
$\bar{F}$ = horizontal driving force vector
$\bar{F}_{water}$ = current drag vector
$\bar{F}_{wind}$ = wind shear vector
g = acceleration of gravity
G = dimensionless function
h = ice floe thickness
h_p = height of ice piling above water level
h_{cr} = beach elevation increase under ice piling
H = horizontal reaction from slope
H_0 = horizontal reaction from frictionless slope
ℓ = characteristic length of continuous ice sheet
ℓ_p = distance from floe edge to critical cross-section
L = length of slope covered by ice pieces (independent variable)
L_p = limiting length of slope covered by ice (dependent variable)
P = compressive force in ice during ride-up
Q = dimensionless function
R = resistance parallel to slope
S = dimensionless parameter
u = current velocity 1 metre below ice floe
v = velocity of ice floe
V = vertical reaction from slope
V = vertical edge load on ice floe
V_{cr} = critical vertical edge load on ice floe
V_p = volume of ice piling
V_0 = vertical reaction from frictionless slope
V_{10} = wind velocity 10 metres above ice floe
W = work done during ride-up
z_h = horizontal extension of crushed zone

z_v	= vertical extension of crushed zone	μ_s	= coefficient of friction (ice-to-slope)
Z	= vertical coordinate of uppermost ice front	ν	= Poisson's ratio
α	= slope angle with horizontal	ρ_p	= density of pile-up (ice and air)
γ_i	= specific weight of ice	ρ_w	= density of sea water
γ_p	= specific weight of pile-up including voids	σ_{bc}	= combined bending-compression strength
γ_w	= specific weight of water	σ_{cr}	= stress in critical cross-section from the combined effects of compression and bending
θ_1	= slope of seaward face of ice piling		
θ_2	= slope of landward face of ice piling		
ρ_a	= density of air (1.29 kg/m^3 at 0 °C)	σ_f	= flexural ice strength
ρ_i	= density of ice	σ_u	= uniaxial compressive ice strength
μ_i	= coefficient of friction (ice-to-ice)	ϕ	= inclination angle of bump