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Historical Shoreline Mapping (I): Improving Techniques and Reducing Positioning Errors

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ABSTRACT

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A critical need exists among coastal researchers and policy-makers for a precise method to obtain shoreline positions from historical maps and aerial photographs. A number of methods that vary widely in approach and accuracy have been developed to meet this need. None of the existing methods, however, address the entire range of cartographic and photogrammetric techniques required for accurate coastal mapping. Thus, their application to many typical shoreline mapping problems is limited. In addition, no shoreline mapping technique provides an adequate basis for quantifying the many errors inherent in shoreline mapping using maps and air photos. As a result, current assessments of errors in air photo mapping techniques generally (and falsely) assume that errors in shoreline positions are represented by the sum of a series of worst-case assumptions about digitizer operator resolution and ground control accuracy. These assessments also ignore altogether other errors that commonly approach ground distances of 10 m.

This paper provides a conceptual and analytical framework for improved methods of extracting geographic data from maps and aerial photographs. We also present a new approach to shoreline mapping using air photos that revises and extends a number of photogrammetric techniques. These techniques include (1) developing spatially and temporally overlapping control networks for large groups of photos; (2) digitizing air photos for use in shoreline mapping; (3) preprocessing digitized photos to remove lens distortion and film deformation effects; (4) simultaneous aerotriangulation of large groups of spatially and temporally overlapping photos; and (5) using a single-ray intersection technique to determine geographic shoreline coordinates and express the horizontal and vertical error associated with a given digitized shoreline.

As long as historical maps and air photos are used in studies of shoreline change, there will be a considerable amount of error (on the order of several meters) present in shoreline position and rate-of-change calculations. The techniques presented in this paper, however, provide a means to reduce and quantify these errors so that realistic assessments of the technological noise (as opposed to geological noise) in geographic shoreline positions can be made.

ADDITIONAL INDEX WORDS: *Aerial photography, cartography, coastal erosion, photogrammetry, shoreline change.*

INTRODUCTION

This paper is the first of two in which we discuss principles of historical shoreline mapping and their application. In this paper, we present cartographic and photogrammetric techniques that can be used to determine geographic shoreline positions from maps and aerial photographs. The second paper (THIELER and DANFORTH, 1994, this volume) presents the application of a new approach to historical shoreline mapping we have developed based on the techniques presented here.

As shoreline mapping has come into increased use as both a scientific and management tool, a critical need has developed among coastal researchers and policy-makers for a widely applicable, accurate method to obtain shoreline position data. Historical shoreline erosion rates, for example, presently are used in several U.S. states to locate oceanfront building setbacks. Recently, the National Research Council (NATIONAL RESEARCH COUNCIL, 1990) and the Federal Emergency Management Agency (CROWELL *et al.*, 1991) have discussed the potential application of erosion rate-based setbacks at a national level. MAY *et al.* (1982) and DOLAN *et al.* (1985) observed that the lack of a standard method among coastal scientists for analyzing shoreline changes has resulted in the publication of data utilizing a variety

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of measurement techniques and rate-of-change calculations, which can be a significant problem when comparing coastal changes at regional to national scales. MORTON (1991) states that shoreline mapping "has progressed from an exercise of scientific curiosity to a primary basis for coastal management and planning. Mapping shoreline changes and predicting future shoreline positions are currently worldwide scientific and coastal management objectives."

A number of methods have been developed to produce shoreline change data from maps and aerial photographs (*e.g.*, STAFFORD and LANGFELDER, 1971; DOLAN *et al.*, 1978; CLOW and LEATHERMAN, 1984; MCBRIDE, 1989; MCBRIDE *et al.*, 1991; SHOSHANY and DEGANI, 1992). These methods vary widely in approach and accuracy, and none address the entire range of cartographic and photogrammetric techniques needed for precise coastal mapping, thus limiting their application. In addition, no extant method provides an adequate basis for realistically quantifying its inherent errors. Hence, current assessments of the errors in shoreline locations determined from maps and air photos typically lack a quantitative treatment of the errors contributed by each data source and the measurements made from them. In many instances, a "best guess" estimate is made, or errors are ignored altogether.

Given the expanded need to map coastal changes for scientific and planning purposes, it is crucial that coastal researchers understand the application and underlying limitations of the methods and source materials used to obtain shoreline position and rate-of-change data. This paper reviews the basic cartographic and photogrammetric relationships and techniques that provide a foundation for obtaining shoreline position data from historical maps and aerial photographs. We also present a new approach to historical shoreline mapping using air photos that extends traditional spatially-oriented photogrammetric techniques to the temporal domain. The application of these techniques to a typical shoreline mapping problem is the subject of a companion paper (THIELER and DANFORTH, 1994, this volume).

Maps and aerial photographs comprise the two basic sources of data used in historical shoreline change studies. There are inherent differences, however, between maps and aerial photographs that require them to be treated differently to obtain geographic data. This paper deals primarily with those aspects of cartography and photogrammetry that concern the accurate determination of shoreline positions from maps and aerial photographs. SNYDER (1987) and SNYDER and VOXLAND (1989) furnish a discussion of the characteristics and derivation of map projections. AMERICAN SOCIETY OF PHOTOGRAMMETRY (1980) provides a detailed discussion of the fundamentals of photogrammetry and derivations of the photogrammetric formulae and techniques discussed here.

MAPS

The electronic digitizers used by most coastal researchers to digitize historical shoreline maps, such as National Ocean Service (NOS) T-sheets, produce Cartesian coordinates (x, y) that must be converted to geographic coordinates (u, v) to be incorporated into a Geographic Information System (GIS) or cartographic database with other shoreline data. This conversion requires an inverse transformation function of the form

$$T^{-1}(x, y) \rightarrow u, v \tag{1}$$

(EVENDEN, 1991). Two bivariate polynomial functions can be used for this conversion, such that

$$u = \sum_{i=0}^{N} \sum_{j=0}^{N-i} a_{ij} P_i(x) P_j(y)$$
$$v = \sum_{i=0}^{N} \sum_{j=0}^{N-i} b_{ij} P_i(x) P_j(y) \tag{2}$$

where the function P is a monomial series or the Tchebycheff polynomial and the coefficients a_{ij} and b_{ij} are determined by least squares methods. The degree or order of the transformation, N, is limited by the number of calibration points (a set of known (u, v) coordinates and their corresponding (x, y) values). For example, three calibration points are needed for a first degree transformation; at least six points are needed for second degree equations. Thus, Equation 1 can be rewritten as

$$T_N^{-1}([u_i, v_i]_k, [x_i, y_i]_k, x, y) \rightarrow u, v \tag{3}$$

The bracketed pairs represent arrays of k calibration points where

$$k \geq (N + 1)(N + 2)/2 \tag{4}$$

Higher order polynomials can be used in formulating the coefficients of Equation 2 to model curved projections more closely and map the digitizer coordinates to a geographic system. This technique requires additional calibration points, ideally balanced in both the x and y directions, so that the requirement of Equation 4 is met.

The procedure used to obtain shoreline data from a map, using the above relationships, consists of three steps: (1) digitizing a series of known geographic coordinates such as the latitude-longitude ticks around the map graticule; (2) digitizing the shoreline shown on the map (*e.g.*, the Mean High Water Line); and (3) converting the digitizer output to geographic coordinates.

Various forms of Equation 3 are typically used either interactively during digitizing or in post-processing to establish a correspondence between digitizer coordinates and the geographic coordinates of the calibration points. The procedure is fairly straightforward; most GIS software used today employs similar methods for digitizing and converting map data to different coordinate systems (*e.g.*, McBride, 1989).

In most cases, a simple Cartesian conversion from the digitizing table's coordinate system to another rectangular coordinate system is adequate. This conversion is best suited to maps of limited extent (*e.g.*, U.S. Geological Survey 7.5-minute quadrangles), maps of low-latitude areas, and map projections with only minor curvature or scale change (*e.g.*, Universal Transverse Mercator, State Plane). To the extent that the digitized map has these qualities, application of Equation 3 produces a set of shoreline coordinates that are in the same projection (*e.g.*, UTM or Polyconic) and reflect the same ellipsoid constants (*e.g.*, the Clarke 1866 ellipsoid) and datum as the digitized map.

Where non-rectangular projections are used, as is the case with most maps, or where greater transformation accuracy is desired, the reduction of the digitized coordinates to an intermediate coordinate system, followed by projection of those coordinates into a map projection substantially reduces the error in the digitizer-to-geographic transformation. A transformation of the form

$$F^{-1}(T_N^{-1}([F(u_i, v_i)]k, [x_i, y_i]k, x, y)) \rightarrow u, v \quad (5)$$

is presented by Evenden (1991), where F and F^{-1} are the respective forward and inverse mathematical descriptions of a particular map projection. Here, the function T performs a Cartesian transformation (to remove axis offset, scaling and rotation) that reduces the digitizer coordinates to an intermediate coordinate system.

Evenden (1990) provides a UNIX based cartographic procedure similar to Equation 5 for the forward and inverse projection of geographic coordinates to various map projections using different ellipsoid constants and datums. This procedure can be used in conjunction with Equation 3 to convert digitized map data to a desired projection and datum.

When comparing shorelines digitized from maps with different projections and reference datums, they must be converted to a common projection and datum. McBride (1989) describes the importance of referencing shoreline change data to a common projection and datum. As noted by Evenden (1991), it is also critical that the cartographic characteristics of a map are known. This includes the projection, ellipsoid shape and geodetic datum used in the production of the map, and an adequate graticule. The graticule, for example, is usually the only source of calibration data that can be used in digitizing.

AERIAL PHOTOGRAPHS

The basic principles involved in the extraction of geographic data from air photos are derived from the geometric relationships between image space and object space. Image space refers to the world inside the camera (*i.e.*, the photographic image and measurements obtained from it). Object space refers to real-world geographic coordinates outside the camera.

In distortion-free space, points in image space are related projectively to points in object space. This relationship is based on the principle of collinearity: the perspective center of the camera lens, which is considered a point, an image point and its corresponding ground point all lie on the same straight line (Figure 1). Air photos, however, are subject to a number of distortions introduced at various stages in the photographic process that perturb the collinearity condition. These perturbations affect both image space and object space.

Image Space

The image space coordinate system is defined by the locations of the fiducial reference marks on a photograph, the calibrated focal length, and the geometric distortion characteristics of the lens system in an aerial camera (American Society of Photogrammetry, 1980). Image space is a three-dimensional, rectangular Cartesian coordinate system with the origin located at the principal point. The x-axis is typically positive in the direction of flight. The z-axis corresponds to the optical axis of the camera.

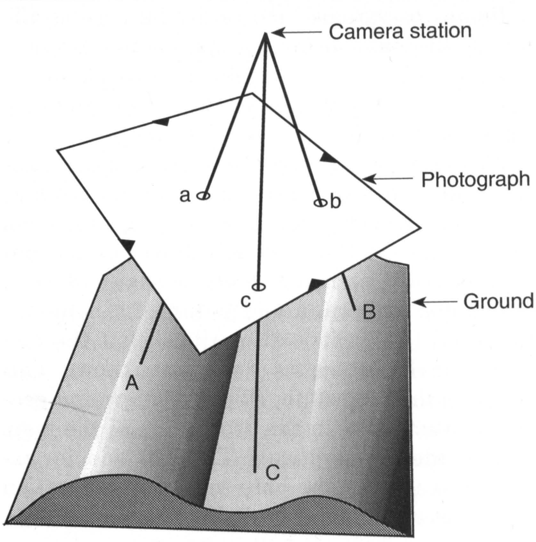

Figure 1. In distortion-free space, a projective relationship exists between image space (points on a photo) and object space (points on the ground). The camera station, an image point (a, b, c), and its corresponding ground point (A, B, C) all lie on the same straight line.

The distortions affecting image space result from lens distortion and film deformation. All aerial camera lenses have measurable distortions and optical defects that affect the representation of image points on film. Lens distortions can be radial or tangential. Radial distortion is symmetric around the principal point and is caused by optical defects in the lens. Tangential distortion is symmetric along a line through the principal point and results from the lens being slightly off-center in the camera. In a well-adjusted camera, however, only radial distortions are present.

The magnitude of lens distortion is highly variable. Some lenses used today have up to 0.110 mm radial distortion (AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980), which translates to a ground displacement of over 2 m in the position of image points for a 1:20,000 scale photograph. Similar and sometimes greater amounts of lens distortion commonly are present in photographs taken prior to World War II, which brought an increased demand for accurate photography and improved lens manufacturing techniques. In most modern camera systems, however, lens distortion is fairly small (0.010 mm or less).

Two types of film deformation exist. Deformation can be introduced in the camera during the aerial survey or in subsequent processing. Film buckling, for example, may occur during the photographic survey due to irregularities in temperature, humidity or film spool tension in the camera. Further deformation is introduced not only in the development of the original negatives, but also in each generation of prints and transparencies (typically used by coastal researchers) made from the original negatives. The end result of these deformations is a photograph that no longer represents accurately the true geometric relationships between the fiducial reference marks and image points in the photo.

In addition to deformation occurring in the camera, the amount of film deformation present in a given photograph depends upon the age and type of material (glass, film or paper); processing techniques used; and the temperature and humidity at the time measurements are made. Standard diapositive (transparency) film is generally stable within 0.005 mm (AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980). Photographic paper, however, is far less stable and may change in size up to 1 percent during processing along (AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980). We have observed shrinkage and expansion of 1–2 mm in some paper prints due to differences in age, paper quality and changes in laboratory environmental conditions. At photo scale, these are nontrivial errors and represent ground distances of 10 m.

Object Space

The characteristics of object space cause image points on film to be displaced (as opposed to distorted) from their true positions as a result of three factors: relief displacement, tilt displacement, and atmospheric refraction. Relief displacement is caused by changes in ground elevation within a photo that cause objects closer to the camera to be larger (*i.e.*, at a larger scale) than those farther away. Relief displacement takes place radially from the nadir. Objects higher than the ground elevation at the point where the nadir intersects the ground (the ground nadir; Figure 2) are displaced outward; objects lower than the ground nadir point are displaced inward.

Tilt displacement occurs due to the inability to keep the aerial camera perfectly level during photography. Some degree of tilt is always present in an aerial photograph. On a tilted photograph, the sense of displacement depends on whether the image point is on the low or high side of the isometric parallel (Figure 2). Points on the low side of the isometric parallel are displaced outward from the isocenter; on the high side, they are displaced inward. Points on the isometric parallel are not displaced. The determination and magnitude of relief and tilt displacements have been widely discussed in the context of shoreline mapping (*e.g.*, ANDERS and BYRNES, 1991; CROWELL *et al.*, 1991) and are not reproduced here.

The bending of light rays through the atmosphere (atmospheric refraction) also causes photograph image points to be displaced. The displacement occurs radially outward from the nadir. The magnitude of the displacement depends on the aircraft flight height, direction of the optical axis relative to the ground, and the focal length of the camera. The displacement of image points due to atmospheric refraction is generally less than 0.006 mm for photographs commonly used in shoreline mapping (AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980).

Analytical Methods for Air Photo Data Reduction

The "traditional" approach to analytical photogrammetry is composed of three steps (EL-ASSAL and MALHOTRA, 1987) that remove the perturbations described above, and exploit various geometric relationships between overlapping air photos to extract geographic data: (1) preprocessing; (2) triangulation; and (3) postprocessing. Preprocessing reduces measured (digitized) image coordinates to the image space coordinate system described above, as well as removes systematic errors such as lens distortion and film deformation effects. Triangulation is used to solve simultaneously for the camera position of each photograph in a large group of overlapping photographs (also called a block), as well as the coordinates of unknown ground points. Postprocessing typically involves transforming the camera position information into instrument settings used in analytical stereoplotters or other photogrammetric equipment in order to compile basemaps or generate rectified orthophotographs.

A NEW APPROACH TO SHORELINE MAPPING USING AIR PHOTOS

For historical shoreline mapping, the steps in the analytical approach are easily extended and modified so that a single person in a small laboratory can rapidly execute them using basic computer equipment, including an electronic digitizing table, a computer, GIS software, and a plotting

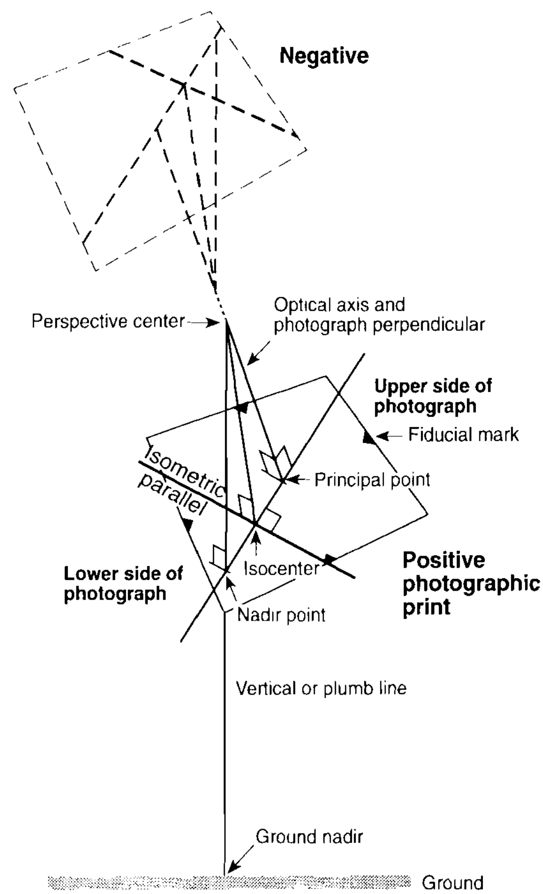

Figure 2. Definition sketch of terms used to describe the various elements of a tilted aerial photograph. Some degree of tilt is always present in an aerial photograph, which causes image points to be displaced from their true position (see text for discussion). (Modified after AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980.)

device. A shoreline mapping process using photogrammetric techniques can be defined by six steps: (1) establish a control network for a group of photos, (2) digitize features on the photos, (3) remove image space distortions from each photo, (4) establish the absolute orientation of the group, (5) calculate the geographic shoreline position for each photo, and (6) compile the shoreline positions.

Establish a Control Network

A control network, a set of points that appears in one or more photographs, provides the basic

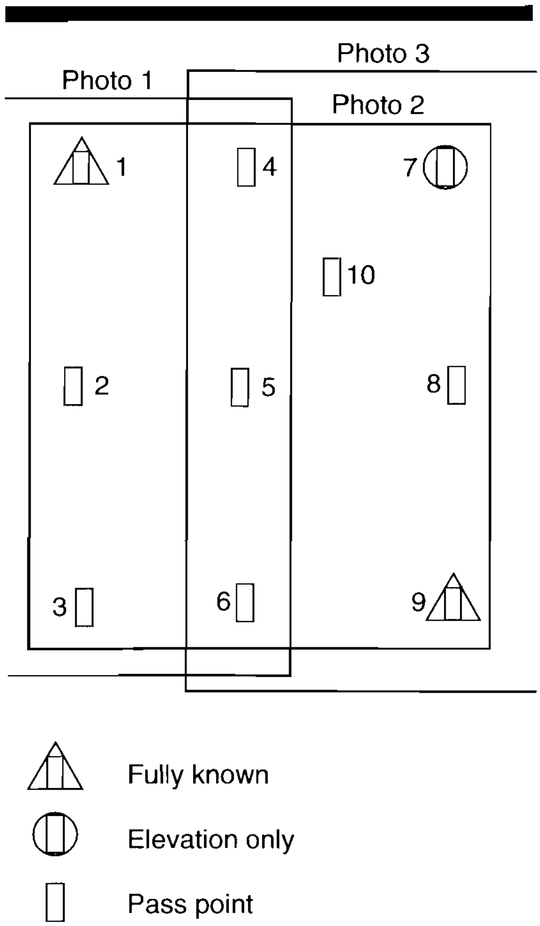

Figure 3. A properly controlled group of three overlapping air photos furnishes balanced control in all directions. Note that only three ground control points are used to orient the group with respect to the ground. The other points, known as pass points, are used to orient the photos with respect to each other. (After U.S. Department of Agriculture, 1981.)

means of establishing a correspondence between photos and the ground. In other words, a control network is used to orient the photos with respect to the ground. There are essentially two types of points used in photogrammetry: ground control points and pass points. A point appearing in one or more photos for which information about its location is known (*e.g.*, latitude, longitude, elevation) is called a control point or ground control point. The image space coordinates of ground control points and their corresponding geographic coordinates are used to establish the collinear relationship between image space and object space shown in Figure 1.

There are a number of ground control data sources, including maps, field surveys and geodetic control tables. The locations of well-defined points shown on maps can be digitized, converted to geographic coordinates and used as ground control points. These points are sometimes called "secondary control points" (Clow and Leatherman, 1984), but more properly are called "supplemental control points" because they are obtained from a map rather than by field survey. Supplemental control points, consisting primarily of buildings and road intersections, are the primary source of ground control for many historical shoreline mapping projects because other data are unavailable.

A pass point is defined as a point appearing in two or more photos, for which a corresponding ground position is not known. Pass points are used to "pass" or extend control between overlapping photos. These points are used in addition to ground control points to establish the relative orientation of photos to each other, such as done when viewing a pair of overlapping photos through a stereoscope. Pass points commonly include features such as trees, buildings and road intersections.

For most applications, at least four and preferably six to nine points are needed to provide adequate control for a given photograph. These points should be distributed throughout the photograph. Figure 3 shows a hypothetical control network with these characteristics. Only a few control points are needed to establish the geographic orientation of a group of photos in object space; most points used to control a photograph may simply be pass points. A well-controlled group of 20 photos, for example, might include 140 points, of which 10–15 are ground control points and the rest are pass points.

Ideally, the exact planimetry (latitude, longitude, elevation) is known for a large number of spatially and temporally well-distributed ground control points throughout all photographs. This is never the case, however, and ground control points of varying quantity and quality must be used when constructing the control network for a given mapping project. The issues relevant to establishing ground control in historical shoreline mapping include quantity, distribution, quality, and recoverability.

In historical shoreline mapping, it is often problematic to furnish an adequate quantity and distribution of ground control and pass points due to the nature of photography along the shoreline and changes in coastal environments over time. Most photographs that include the shoreline, for example, are typically devoid of control seaward of the shoreline. Coastal areas may also change rapidly over time, due to natural processes or human development, which reduces the number of stable points that can be used as ground control or pass points. In these situations, it is often necessary to use additional overlapping photos taken of more landward areas in order to balance the control network for the shoreline photographs.

Used here, the quality of a ground control point refers to the amount of information known about a given point (*e.g.*, latitude, longitude, and elevation; horizontal only; elevation only; *etcetera*), as well as the accuracy of the survey that determined the point's location. For a location digitized from a map, control point quality is also affected by its representation on the map and the map scale. The scale of the photos used also affects the accuracy with which ground points (and the shoreline) can be identified. ELLIS (1978) discusses the resolution obtainable from maps and photos at scales that commonly are used in coastal mapping. Most coastal researchers consider 1:20,000 air photos the usable upper limit for shoreline change studies (TANNER, 1978; BYRNES *et al.*, 1991; CROWELL, *et al.*, 1991).

Recoverability refers not only to the ability of the photo interpreter to identify accurately a given point, but also how well an image point shows up in one or more sets of photographs. It is fairly common, for example, for buildings and roads used as ground control points to be destroyed or relocated between aerial surveys. Loss of control points can be a significant problem in historical studies and mapping after large storms.

Adequate ground control is a fundamental requirement in photogrammetric mapping, and thus has received much attention in terms of the development of graphical and analytical solutions for extending ground control for map production. The process of extending geographic control among a group of air photos is generally referred to as aerotriangulation. Aerotriangulation extends ground control by using measurements derived from the spatial relationships between several overlapping aerial photographs. Control extension usually entails digitizing a pass point on two or more photos and determining the point's planimetry based on the intersection of rays from each camera station through the pass point. EBNER (1972) provides a discussion of the theoretical accuracy of control extension by analytical methods. Control derived by aerotriangulation is generally accurate to within 5–10 m (AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980).

Fully analytical methods of aerotriangulation have been employed since the 1960's, when the advent of digital computing made it possible to compute rapidly and economically the calculations required to process many photographs simultaneously. The primary attributes of this approach are the ability to input initial approximations of various parameters, and enforce control of camera positions, image coordinates and ground control to reflect prior knowledge of their precision. Several aerotriangulation computer programs (*e.g.*, ELASSAL and MALHOTRA, 1987) permit a variety of ground control data to be used in constructing and extending a control network.

In historical shoreline change studies, when several sets of photos spanning several years of the same geographic area are used, the common points of both types (ground control and pass points) form a relatively oriented model that effectively "ties" the photos together (Figure 4). This establishes correlations between images, in space and through time, that are readily exploited in aerotriangulation and error analysis.

The approach shown in Figure 4 also provides an important feature when working with several sets of photography. Specifically, it is possible to use one set of photography as the primary source of ground control and tie other sets of photography to the primary set using pass points. This attribute can be particularly useful when using supplemental ground control points derived from a historical map and a set of photographs that correspond closely in time to the survey date of the map. For example, supplemental control features shown on a U.S. Geological Survey 7.5-minute quadrangle surveyed in 1950 may provide excellent control for a set of photographs taken in 1951, but only a few points shown on the map are suitable for controlling earlier or later photography. It is then possible to tie other sets of photography (for example, from 1940, 1960, 1970 and 1980) to the 1951 photos predominantly using pass points.

Digitize Photo Data

Three pieces of information must be measured (digitized) on an aerial photograph for use in shoreline mapping: the locations of (1) the fiducial

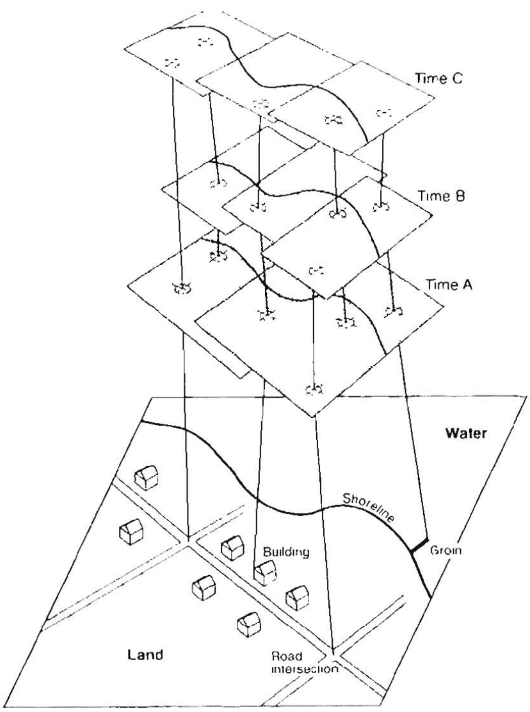

Figure 4. When several sets of photos of the same area are used to determine historical shoreline changes, the common image points on the overlapping photos form a relatively oriented model that effectively "ties" the photos together in space and time. The eight-photo group shown here represents three different sets of photography taken of the same coastal area at different times A through C (e.g., months to decades apart) and consists of only six common points. In practice, it is common to have a dense network of 60 or more points for the same number of photographs. This model establishes correlations between photos in space and through time that are readily exploited in later stages of data reduction and error analysis.

reference marks, (2) image points (ground control and pass points), and (3) points along the shoreline. The fiducials are usually indicated by marks around the perimeter of the photo. Ground control and pass points should be readily identifiable in each photo.

The wet/dry line on the beach, which is generally assumed to approximate the High Water Line (DOLAN *et al.*, 1980; CROWELL *et al.*, 1991), is the most frequently used shoreline for digitizing because it is easily identified by the tonal difference between wet and dry sand. However, from a geological standpoint, the wet/dry line may not be the best shoreline indicator for determining shoreline positions or rates-of-change. On low-sloping beaches, for example, the displacement of the wet/dry line due to wave, tide or wind effects can easily approach several tens of meters. Variations in wet/dry line position due to seasonal beach erosion/accretion patterns (*e.g.*, SMITH and ZARILLO, 1990) or the timing of aerial surveys relative to changes in the trend of shoreline behavior (*e.g.*, DOLAN *et al.*, 1991) may also affect the geological significance of the shoreline position at a given moment in time. Clearly, other shoreline indicators such as the vegetation line or bluff line may be more useful, depending on the nature of the coastal system under investigation.

Remove Image Space Distortions

The errors in image point locations introduced by film deformation and lens distortion are typically addressed by preprocessing. The goal of preprocessing is to transform measured image data from a photograph to an idealized image space in which distortions do not exist. In other words, the digitized coordinates are refined to remove image space distortions before the data are used in subsequent processing.

Camera calibration data are used to remove errors in the image space coordinate system. Most aerial cameras are frequently tested; calibration reports produced according to accepted standards (*e.g.*, U.S. Geological Survey) provide camera system data that are used in preprocessing. This information includes the calibrated focal length, radial lens distortion characteristics, and the locations and distances between fiducial marks. For older photography dating from the 1930's to about 1960, calibration data are frequently unavailable. In this situation, however, it is often possible to make realistic simplifying assumptions to reconstruct the image space coordinate system.

To correct for film deformation, the coordinates of the calibrated fiducial system and the fiducial system measured (digitized) on each photo are compared. This gives the image deformation at each reference mark; the deformation pattern is then used to correct the measured image points for film deformation. If four fiducials are digitized, a first degree transformation such as Equation 2 can be used to map the digitized coordinates into the calibrated coordinate system.

Equation 2 can also be used to minimize and examine errors made in digitizing aerial photographs, as well as normalize the image space coordinate system for all photos from a given aerial survey. For example, calibrated or estimated fiducial coordinate locations can be used to map all photos using the same camera system to a common image space coordinate system, and provide residual errors that measure the accuracy and consistency of the digitizer operator in image space units. This transformation also permits the use of full-frame photo enlargements in mapping projects, by furnishing a means to scale digitized coordinates to the desired image space system.

The lens distortion characteristics of a camera system typically are provided in two forms that provide a sufficient basis for removing radial lens distortion by analytical methods: a table of radial displacements at intervals of a given distance from the point of lens symmetry across the image area, or an odd-powered polynomial of the form

$$\Delta r = K_1 r^3 + K_2 r^5 + K_3 r^7 + \ldots \qquad (6)$$

Because the instruments used to make measurements on aerial photographs are never error-free, some machine error is introduced in the digitizing process. Frequent testing and adjustment of this equipment, however, can generally keep these errors to a minimum. A digitizer with an accuracy of 0.025 mm is generally considered adequate for mapping shoreline positions on aerial photographs (ANDERS and BYRNES, 1991; CROWELL *et al.*, 1991). Machine errors are normally very small, particularly compared to errors introduced by human misinterpretation of features in a photograph. Hence, they are frequently disregarded in preprocessing.

Establish Absolute Orientation

As discussed above, a well-assembled control network establishes the relative orientation of a group such that the photos are tied to each other by the ground control and pass points (see Figure 4). In an aerotriangulation adjustment, the absolute orientation of the group is performed. That is, the coordinates of ground control points are used to orient the group in object space. Typically, aerotriangulation involves solving for the position and attitude of the aerial camera, known as the camera station, at the instant of exposure for each photo in the group, as well as the ground coordinates of unknown points such as pass points. The widely used General Integrated Analytical Triangulation (*GIANT*) program (ELASSAL and MALHOTRA, 1987), for example, furnishes these capabilities. Simultaneous adjustment programs do not treat the relative and absolute orientations separately. Thus, the relative orientation of the group exists primarily as a conceptual framework for developing a good control network.

The absolute orientation performed in a simultaneous adjustment is basically an extension of the technique of space resection, which determines the six elements of exterior orientation for a photograph, including the position (latitude, longitude and elevation), and attitude (roll, pitch, and yaw; designated ω, ϕ, and κ, respectively) of the aerial camera. Atmospheric refraction effects can also be removed during space resection or aerotriangulation by applying a correction function each time the orientation of the camera is updated in the solution process.

Calculate Shoreline Positions

Once the absolute orientation of a group of photos has been established and the camera station for each photo is known, the geographic coordinates of the shoreline in each photo can be calculated on photo-by-photo basis using the computed camera stations and the image space coordinates of the digitized shoreline. A procedure known as single-ray intersection can be used to determine the intersection of a ray from a camera station through shoreline image point with the ground.

As shown in Figure 5, a ray from the camera lens through a shoreline image coordinate intersects with the ground at a specific latitude and longitude, for which there exists a corresponding elevation. In most instances, the desired shoreline elevation is at or near mean sea level, but any shoreline indicator with a known, relatively uniform elevation, such as a lake or reservoir shoreline, or bluff edge can be used if an appropriate elevation is estimated. A single-ray intersection is performed iteratively for each digitized shoreline point in each photograph to produce a series of geographic shoreline coordinates, based on a geocentric (x, y, z) coordinate system. The solution is extended easily to account for earth curvature, and to use different geodetic datums and ellipsoid shapes to determine geographic shoreline coordinates (DANFORTH and THIELER, 1992b).

The error in determining the geographic location of any point in a photograph depends almost entirely on the camera station; essentially, the computed camera position reflects all of the errors described above. To quantify the horizontal and

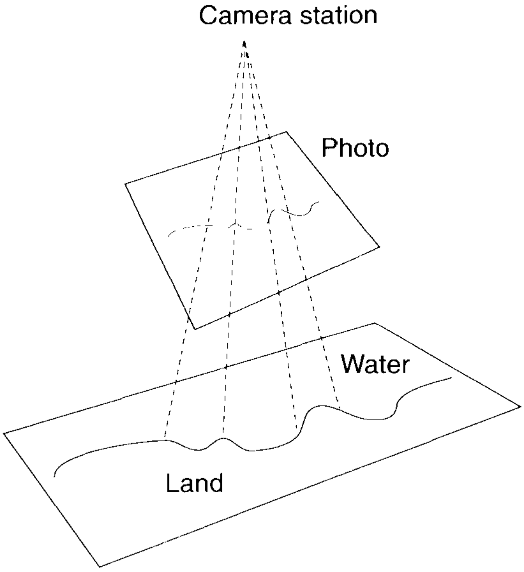

Figure 5. Once the exterior orientation of a photograph has been established, a single-ray intersection technique can be used to calculate the geographic coordinates of digitized shoreline points. This technique uses the collinear relationship between the camera station, photo and ground shown in Figure 1 to determine the intersection with the ground of a ray from a camera station through a digitized shoreline point. A single ray intersection solution is performed iteratively for each digitized shoreline point in each photograph. Four rays are shown here. In practice, a digitized shoreline may consist of several hundred points.

vertical error of any digitized shoreline point, the error in camera orientation can be propagated back to the ground when performing the single-ray intersection. The standard deviations for a given camera station, for example, can be calculated from the diagonal terms in its covariance matrices (ELASSAL and MALHOTRA, 1987). Two 3 × 3 covariance matrices per camera station are used: one for position and one for attitude. These matrices can be incorporated into the single-ray intersection solution to express the horizontal and vertical error in any shoreline point as a single 3 × 3 matrix of partial derivatives for each component of latitude, longitude and elevation.

An alternative approach to single-ray intersection is presented by KELLER and TEWINKEL (1966), in which an orthogonal rotation matrix is calculated based on the camera position and attitude for each photo. The matrix is composed of direction cosines that can be used to map image points to a rectangular coordinate system. The application of this technique to shoreline mapping is described by CLOW and LEATHERMAN (1984). The implementation described by CLOW and LEATHERMAN (1984), however, apparently does not account for earth curvature or different datums, is limited to a rectangular (State Plane) coordinate system, and does not propagate camera station errors back to the ground.

Compile Shoreline Positions

The final step in the air photo mapping process is the compilation, editing and presentation of the shoreline data. Typically, the shoreline position data are imported into a GIS, where several photos from a single date are overlaid to form a continuous shoreline. A shoreline for a given date is usually assigned to a specific overlay or coverage in the GIS (*e.g.*, MCBRIDE, 1989; WILLIAMS *et al.*, 1992). When all shorelines have been compiled from both maps and photos, they are usually output as composite shoreline change maps on a plotting device using different line colors or symbols.

Once the shoreline positions have been compiled into a digital GIS database, shoreline rates-of-change can be calculated. DANFORTH and THIELER (1992a), for example, present an automated method whereby a number of different measures of shoreline change can be calculated using a time series of historical shoreline positions residing in a GIS. Other techniques are presented by CLOW and LEATHERMAN (1984) and MCBRIDE *et al.* (1991).

DISCUSSION

In this paper, we are concerned only with the accuracy of the calculated shoreline positions relative to their actual positions on the date of photography. Thus, we do not address errors in shoreline position or rates-of-change due to the timing of aerial surveys relative to seasonal or storm-induced changes in shoreline position. For example, the shoreline shown in an aerial photograph is commonly assumed to represent the average seasonal shoreline position. MORTON (1979, 1991), DOLAN *et al.* (1980) and SMITH and ZARILLO (1990), however, point out that this is not always a reasonable assumption. CROWELL *et al.* (1991) furnish an extensive discussion of the accuracy of shorelines digitized from maps.

An important question in shoreline change mapping is the accuracy of the maps produced and the resolution of the rates-of-change determined from them. Until this question is answered, we cannot ask geologically important questions about what exactly we are measuring when we measure shoreline change. It is first necessary to reduce the technological noise to a reasonable level; in other words, to increase the signal/noise ratio. Shoreline mapping accuracy is limited fundamentally by the techniques and materials used to obtain shoreline position data. Thus, techniques must be used to reduce and quantify errors, and source materials must be checked as to their accuracy and suitability for a particular project. Only then can realistic accuracy assessments be made of shoreline change maps and rate-of-change measurements.

Limitations of Existing Techniques

The methods developed by coastal researchers to obtain geographic data from aerial photographs vary widely, ranging from manual point measurements to electronic raster scanning. Some methods crudely employ some photogrammetric techniques (*e.g.*, CLOW and LEATHERMAN, 1984). Others manipulate aerial photographs in ways that are inappropriate (*e.g.*, SHOSHANY and DEGANI, 1992) because the photos are treated as map projections, which they are not. An aerial photograph is produced by central projection through a lens onto a plane, while a map is constructed by radial projection onto a sphere (or ellipse) and then transferred to a plane. This characteristic introduces the distortions described above and precludes the direct compilation of accurate maps from aerial photography.

Most historical shoreline mapping techniques have been developed under ideal circumstances, such as those found along the developed shorelines of the U.S. East Coast. This is a data-rich environment for shoreline mapping: large-scale photos bracketing long time frames are available, photographs are taken by several government agencies on a fairly regular basis, there is a plethora of ground control, several accurate historical maps for an area often are available, and historical shoreline changes are either qualitatively understood or easily verified by supplementary field evidence and cultural records.

Some shoreline mapping methods may produce acceptable results under these ideal circumstances, such as when applied to at least qualitatively-understood and generally well-documented systems. These methods, however, typically require more information than is actually necessary to achieve comparable or better accuracy, and fail to take advantage of techniques that can reduce and quantify errors. This problem precludes their broad application to shoreline mapping in the typically data-poor world outside of the U.S. East Coast. The following example illustrates this point.

One of the methods presently used to determine shoreline positions from air photos is the single-frame space resection approach (CLOW and LEATHERMAN, 1984). This method requires at least three fully known control points be used for each photograph to determine the position of the aerial camera. Not only does this method require a substantial amount of control for a given group of photographs, but also introduces an unnecessarily large and complex series of errors resulting from the independent orientation of each photo to fit a given set of ground control points. The requirements of this technique have several important implications.

First, when using a single-frame space resection approach, photos lacking a sufficient number or distribution of fully known control points cannot be used. This significantly limits the application of the technique for mapping undeveloped coastlines. Second, errors are introduced that result from the independent orientation of each photo to fit a set of ground control points. This can result in shorelines not matching properly in adjacent photos from the same strip, and may introduce serious errors in rate-of-change-calculations when comparing photos from two or more dates. The error inherent in using supplemental control points digitized from a map compounds these problems because they commonly have a low accuracy. Third, the amount of information contained in a photograph is drastically underutilized. For example, correlations between overlapping images in space and through time are ignored.

A rigorous analysis of the errors inherent in shoreline positions obtained using the single-frame resection method is difficult to perform, not only because each photo is independent of the others, but also, and perhaps more importantly, because the error in the space resection solution for each photo is not propagated to the ground and thus is not known. For example, current error assessments (*e.g.*, CROWELL *et al.*, 1991; ANDERS and BYRNES, 1991) of the space resection technique assume that errors in shoreline positions are represented by the sum of a series of worst-case assumptions about digitizer operator resolution and ground control accuracy. This is only partly true.

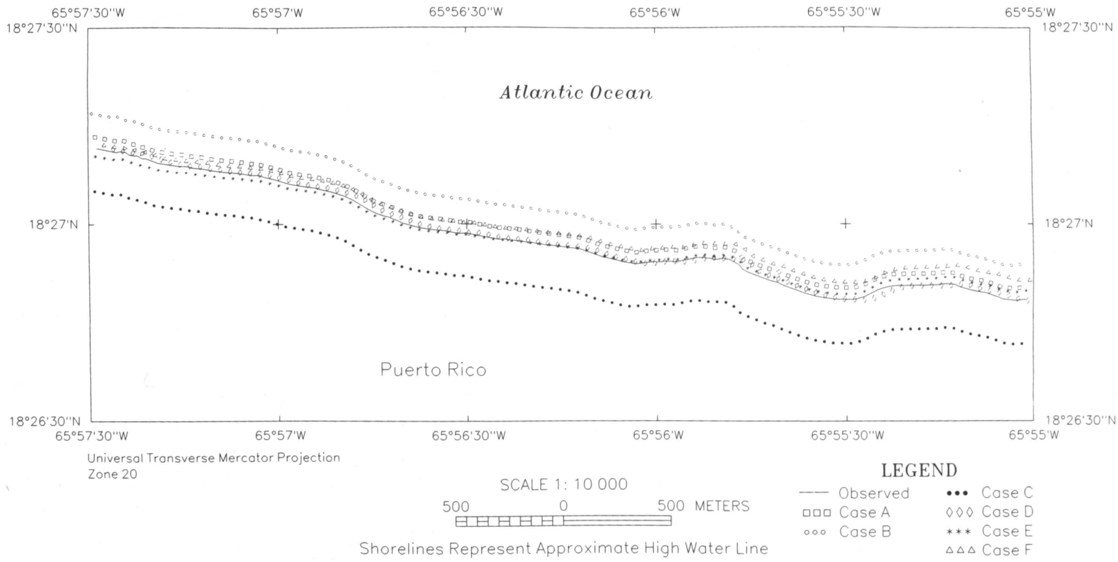

Figure 6. This shoreline map shows the distribution of shoreline positions resulting from a single-ray intersection solution for one photograph using the camera parameters in Table 1. The distribution of hypothetical shorelines represents differences in calculated shoreline position, for one photo, that encompass nearly 350 m. Clearly, the effects of even small errors in camera orientation have dramatic implications for shoreline mapping accuracy, as well as shoreline rate-of-change and shoreline orientation studies.

As discussed above, the error in a given shoreline position is almost entirely dependent on the camera station used to compute the shoreline position. To the extent that errors in photo digitizing and ground control point accuracy affect the camera station solution, shoreline positions are affected by ground control. But the interaction of these errors often profoundly affects the camera station, which can result in incorrect height, position and roll, pitch and yaw parameters. For both the single-ray intersection and direction cosine methods described above, the error in camera orientation is propagated back to the ground when solving for the shoreline position. Expressing the magnitude of the error, then, is extremely important.

The importance of accurately determining the camera stations for aerial photographs cannot be overstated. The effects of even small errors in camera orientation have dramatic implications for shoreline mapping accuracy, as well as rate-of-change and shoreline orientation studies. Figure 6 shows the range of shoreline positions that can result from a poor or incorrect camera station for a typical 1:20,000 photograph. The camera stations used to compute the position of each shoreline are shown in Table 1. Except for the observed solution, the parameters shown in Table 1 are arbitrary, but within realistic values for roll, pitch and yaw. The same digitized data are used in each case, only the camera attitude used to compute the single-ray intersection is varied. The distribution of hypothetical shorelines in Figure 6 represents differences in calculated shoreline position, for one photo, that encompass nearly 350 m. Clearly, errors of this magnitude are unacceptable for most shoreline mapping applications.

There are significant advantages to using a group of relatively oriented photos rather than the single-photo approach when calculating shoreline positions. It is far easier, for example, to develop a network ground control and pass points for a group of photos than to provide full ground control for many single frames. Ground control requirements are significantly reduced for a well-controlled, relatively oriented group, and more of the information in the photos is utilized. In fact, far greater accuracy in shoreline rate-of-change calculations can be achieved if only a relatively oriented model is used to calculate distances between shorelines. The additional errors inherent in the absolute orientation of the model to the

Table 1. *Input camera station parameters for one aerial photograph used to determine the shoreline positions shown in Figure 6.*

Case	Roll	Pitch	Yaw
Observed	0°20′48.64″	0°17′25.49″	1°49′42.50″
A (−1° roll)	1°20′48.64″	0°17′25.49″	1°49′42.50″
B (−3° roll)	3°20′48.64″	0°17′25.49″	1°49′42.50″
C (+3° roll)	3°20′48.64″	0°17′25.49″	1°49′42.50″
D (+1° pitch)	0°20′48.64″	1°17′25.49″	1°49′42.50″
E (−1° yaw)	0°20′48.64″	0°17′25.49″	2°49′42.50″
F (A, D and E combined)	1°20′48.64″	1°17′25.49″	2°49′42.50″

Note: Only the camera attitude is varied; position (latitude, longitude, elevation) is constant in each case. The observed camera position for the photo is: longitude 65°56′17.58″ latitude 18°27′00.54″, elevation 3,088.45 m.

ground are excluded, and the error in calculating a rate of change between two shorelines reduces to the error in photo measurements. When a high accuracy stereo- or mono-comparator is used to digitize the photos, this error may approach 1×10^{-5} of the flight height (ELASSAL, *personal communication*), or about 0.3 m for 1:20,000 photography using a 153 mm lens. While this level of accuracy may be attractive from a theoretical standpoint, the application and interpretation of shoreline changes typically requires the model to be oriented in object space so that geographic data can be extracted.

Limitations of Maps and Photos

There are several areas in which improvements in the accuracy of shoreline map data can be realized. For example, careful selection and testing of maps for use in shoreline change studies, such as those reported by CROWELL *et al.* (1991) in the development of historical map databases for parts of the U.S. East Coast, can keep potential errors to a minimum.

The accuracy of air photo data can be improved by increasing the fidelity of measurements made on air photos, using more precise ground control point locations, and using photos with a large scale. Recent technological advances in aerial surveying techniques can also be used to improve accuracy.

Increasing the fidelity of the measurements made on aerial photographs can significantly improve the accuracy of shoreline positions. Preprocessing digitized photos to remove film deformation and lens distortion, for example, reduces the residual errors caused by bad image coordinates. This results in more accurate relative orientations and has a positive effect on the adjustments made during aerotriangulation to ground control points and camera stations to fit them to image coordinate data.

To decrease the potential error in absolute oriention of a group of photos, the accuracy of ground control point positions can be increased. For example, rod-and-transit or differential Global Positioning System (GPS) surveys can be used to locate accurately control points used in a mapping project. Control coordinates determined by these methods are typically more accurate than ground control points digitized from a map. Geodetic control tables are also available for most basemaps used as sources of ground control for photogrammetric mapping (*e.g.*, NOS and USGS maps). Such tables provide descriptions of control points and their surveyed coordinates that can be used in digitizing air photos. These control points, however, are commonly not recoverable (identifiable) at the scale of photos used in shoreline mapping. For areas that are inaccessible, have been developed only recently, or that periodically have control points destroyed by new construction or coastal storms, surveyed positions may not be available. In that case, historical maps and analytical triangulation must be used to determine ground control coordinates, with a corresponding decrease in accuracy.

Using photos with a scale of 1:20,000 or larger increases the ability of the photo interpreter to accurately identify image points, including the shoreline. While there is a gain made in the precision of measurements from large-scale photos, it must be balanced against the following factors. First, a large-scale photo covers less geographic area per frame than one at a smaller scale. This reduces the potential number of ground control and pass points appearing in a photo, which may prohibit identification of an adequate number and distribution of points. Large scale photography also requires that more photographs be used to cover a given length of coast. Second, if control is determined using a small-scale map, the error associated with the ground control point coordinates has a relatively high standard deviation of measurement compared to the precision obtainable in photo measurements. The magnitude of the error can significantly reduce the quality of the aerotriangulation solution. In an extreme case, it causes the solution to fail or diverge. Thus, a mapping project requires that a balance be struck between the scale of the photos used, and the density and quality of image points.

Recent advances in aerial surveying techniques may improve future shoreline mapping accuracy by significantly reducing the ground control requirements for individual photographs. For example, high-resolution (centimeter-scale) kinematic GPS receivers are presently being used to determine camera station parameters during photo surveys. This approach permits the direct use of the single-ray intersection solution for ground coordinates based only on the camera station (LUCAS and MADER, 1989). Preprocessing is still required, however, to remove film deformation and lens distortion effects from the digitized photos.

CONCLUSIONS

The increasing need to quantify historical shoreline changes requires more accurate methods than are currently available. Existing shoreline mapping techniques suffer from incomplete implementation or apply inappropriate methods to determine shoreline positions. They also fail to exploit fully the large amount of geometric information provided by spatially and temporally overlapping imagery or employ the range of techniques that can be used to obtain accurate shoreline position information. Thus, their application is limited to data-rich environments such as the U.S. East Coast.

The application of existing methods in their present form also contributes to the inability to quantify the many errors inherent in shoreline mapping using historical maps and aerial photographs. For example, current assessments by coastal researchers of the errors in historical shoreline locations lack a quantitative treatment. In many instances, a "best guess" estimate is made; other errors are ignored altogether. Frequently ignored sources of error commonly represent ground distances of 10 m at photo scale.

This paper provides a conceptual and analytical framework based on standard cartographic and photogrammetric techniques for improved methods of extracting geographic data from maps and aerial photographs. These methods provide a means to reduce errors, and quantify the remaining error so that realistic assessments of the shoreline position data can be made. The design and implementation of a new shoreline mapping system that uses these methods is described in a companion paper (THIELER and DANFORTH, 1994, this volume).

Historical shoreline change studies are fundamentally limited by the accuracy of the techniques and materials used to acquire geographic shoreline position data. The magnitude of potential errors in shoreline mapping depends upon the fidelity, accuracy, scale and temporal distribution of the map, photo and ground control data used, as well as the accuracy of the equipment and the human operator used to make measurements. The degree to which techniques are applied to reduce these errors also affects the accuracy of shoreline positions and rate-of-change calculations. As long as historical maps and aerial photographs are used in shoreline change studies, there will be a considerable amount of technological error (as opposed to geologic or oceanographic error), on the order of several meters, present in shoreline position and rate-of-change calculations.

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LITERATURE CITED

AMERICAN SOCIETY OF PHOTOGRAMMETRY, 1980. *Manual of Photogrammetry*. Falls Church, Virginia: American Society of Photogrammetry, 4th ed., 1056p.

ANDERS, F.J. and BYRNES, M.R., 1991. Accuracy of shoreline change rates as determined from maps and aerial photographs. *Shore and Beach*, 59(1), 17–26.

BYRNES, M.R.; MCBRIDE, R.A., and HILAND, M.W., 1991. Accuracy standards and development of a national shoreline change data base. *In*: KRAUS, N.C. (ed.), *Coastal Sediments '91* (New York: ASCE) pp. 1027–1042.

CLOW, J.B. and LEATHERMAN, S.P., 1984. Metric mapping: An automated technique of shoreline mapping. *Proceedings, 44th American Congress on Surveying and Mapping*. Falls Church, Virginia: American Society of Photogrammetry, pp. 309–318.

CROWELL, M.; LEATHERMAN, S.P., and BUCKLEY, M.K., 1991. Historical shoreline change: Error analysis and mapping accuracy. *Journal of Coastal Research*, 7(3), 839–852.

DANFORTH, W.W. and THIELER, E.R., 1992a. *Digital Shoreline Analysis System (DSAS) User's Guide, Version 1.0.* Reston, Virginia: U.S. Geological Survey Open-File Report No. 92-355, 42p.

DANFORTH, W.W. and THIELER, E.R., 1992b. *Digital Shoreline Mapping System (DSMS) User's Guide, Version 1.0* Reston, Virginia: U.S. Geological Survey Open-File Report No. 92-240, 33p.

DOLAN, R.; ANDERS, F., and KIMBALL, S., 1985. *Coastal Erosion and Accretion. National Atlas of the United States of America.* Reston, Virginia: Department of the Interior, U.S. Geological Survey, 1 sheet.

DOLAN, R.; FENSTER, M.S., and HOLME, S.J., 1991. Temporal analysis of shoreline recession and accretion. *Journal of Coastal Research*, 7(3), 723–744.

DOLAN, R.; HAYDEN, B., and HEYWOOD, J., 1978. A new photogrammetric method for determining shoreline erosion. *Coastal Engineering*, 2, 21–39.

DOLAN, R.; HAYDEN, B.P.; MAY, P., and MAY, S., 1980. The reliability of shoreline change measurements from aerial photographs. *Shore and Beach*, 48(4), 22–29.

EBNER, H., 1972. Theoretical accuracy models for block triangulation. *Bildmessung und Luftbildwesen*, 40, 214–221.

ELASSAL, A.A. and MALHOTRA, R.C., 1987. *General Integrated Analytical Triangulation Program (GIANT) User's Guide.* Rockville, Maryland: National Oceanic and Atmospheric Administration Technical Report NOS 126 CGS 11, 66p.

ELLIS, M.Y., (ed.), 1978. *Coastal Mapping Handbook.* Washington: U.S. Department of Commerce, U.S. Department of the Interior, 199p.

EVENDEN, G.I., 1990. *Cartographic Projection Procedures for the Unix Environment—A User's Manual.* Woods Hole, Massachusetts: U.S. Geological Survey Open-File Report No. 90-284, 63p.

EVENDEN, G.I., 1991 *Notes on a Method to Transform Digitized Coordinates to Geographic Coordinates.* Woods Hole, Massachusetts: U.S. Geological Survey Open-File Report No. 91-17, 5p.

KELLER, M. and TEWINKEL, G.C., 1966. *Space Resection in Photogrammetry.* Washington, D.C.: Coast and Geodetic Survey Technical Bulletin No. 32, 10p.

LEATHERMAN, S.P., 1983. Shoreline mapping: A comparison of techniques. *Shore and Beach*, 51, 28–33.

LUCAS, J.R. and MADER, G. L., 1989. Recent advances in kinematic GPS photogrammetry. *Journal of Surveying Engineering*, 115(1), 78–92.

MAY, S.K.; KIMBALL, W.H.; GRADY, N.; and DOLAN, R., 1982. CEIS: The Coastal Erosion Information System. *Shore and Beach*, 50, 19–26.

MCBRIDE, R.A., 1989. Accurate computer mapping of coastal change: Bayou Lafourche shoreline, Louisiana, USA. *In:* MAGOON, O.T. (ed.), *Coastal Zone '89*, (New York: ASCE) pp. 707–719.

MCBRIDE, R.A.; HILAND, M.W.; PENLAND, S.; WILLIAMS, S.J.; BYRNES, M.R.; WESTPHAL, K.A.; JAFFE, B.E., and SALLENGER, A.H., 1991. Mapping barrier island changes in Louisiana: Techniques, accuracy and results. *In:* KRAUS, N.C. (ed.), *Coastal Sediments '91*, (New York: ASCE) pp. 1011–1026.

MORTON, R.A., 1979. Temporal and spatial variations in shoreline changes and their implications, examples from the Texas Gulf Coast. *Journal of Sedimentary Petrology*, 49, 1101–1111.

MORTON, R.A., 1991. Accurate shoreline mapping: Past, present and future. *In.* KRAUS, N.C. (ed.), *Coastal Sediments '91*, (New York: ASCE) pp. 997–1010.

NATIONAL RESEARCH COUNCIL, 1990. *Managing Coastal Erosion.* Washington: National Academy Press, 182p.

SHOSHANY, M. and DEGANI, A., 1992. Shoreline detection by digital image processing of aerial photography. *Journal of Coastal Research*, 8(1), 29–34.

SMITH, G.L. and ZARILLO, G.A., 1990. Calculating long-term shoreline recession rates using aerial photographic and beach profiling techniques. *Journal of Coastal Research*, 6(1), 111–120.

SNYDER, J.P., 1987. *Map Projections—A Working Manual.* Washington, D.C.: U.S. Geological Survey Professional Paper 1395, 383p.

SNYDER, J.P. and VOXLAND, R.M., 1989. *An Album of Map Projections* Washington, D.C.: U.S. Geological Survey Professional Paper 1453, 249p.

STAFFORD, D.B. and LANGFELDER, J., 1971. Air photo survey of coastal erosion. *Photogrammetric Engineering*, 37, 565–575.

TANNER, W.F., (ed.), 1978. *Standards for Measuring Shoreline Changes. A Study of the Precision Obtainable, and Needed, in Making Measurements of Changes (Erosion and Accretion).* Tallahassee, Florida: Coastal Research (Florida State University), 87p.

THIELER, E.R. and DANFORTH, W.W., 1994. Historical shoreline mapping (II): Application of the Digital Shoreline Mapping and Analysis Systems (DSMS/DSAS) to shoreline change mapping in Puerto Rico. *Journal of Coastal Research*, 10(3).

U.S. DEPARTMENT OF AGRICULTURE, 1981. *Analytical Mapping System (AMS) User's Guide—Volume III Analytical Triangulation.* Washington: U.S. Forest Service EM-7140-11, 341p.

WILLIAMS, S.J.; PENLAND, S., and SALLENGER, A.H., (eds.), 1992. *Atlas of Shoreline Changes in Louisiana from 1853 to 1989* Washington, D.C.: U.S. Geological Survey Miscellaneous Investigations Series I-2150-A, 103p.