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Efficiency of the Log-Hyperbolic Distribution—A Case Study: Pattern of Sediment Sorting in a Small Tidal-Inlet—Het Zwin, The Netherlands

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ABSTRACT

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Sediment sorting dispersal pattern in The Zwin tidal-inlet, The Netherlands, was studied using grain size data. The descriptive power of the log-hyperbolic distribution model, the hyperbolic shape triangle, current meters recordings and bedform analysis were applied. The process-oriented discriminating approach suggests four distinct log-hyperbolic grain size curves, related to four morphodynamic sub-environments along The Zwin Inlet. The log-hyperbolic shape indicates, in response to the governing dynamics, a gradual change from erosive, at the inlet front, to a depositional sedimentary facies inland. Applying the population concept, the Zwin supersample, the combined total sampled grain size population, falls into the right wing of the hyperbolic shape triangle, indicating a winnowed-erosional facies (ρ = 0.44) which is the main sedimentary fingerprint of The Zwin Inlet. It indicates winnowing-out of the suspended fraction, suppposedly complementary to a similar sedimentary heritage from the longshore drifted sediments.

ADDITIONAL INDEX WORDS: *Grain size analysis, log-hyperbolic distribution, hyperbolic shape triangle, erosion-deposition model, population concept, tidal-inlet.*

INTRODUCTION

Sediment transport patterns in tidal inlets and their associated morphology, bedforms, sedimentary sequences and waves and tidal currents control have been studied by numerous sedimentologists and coastal engineers (Boersma and Terwindt, 1981; Boothroyd and Hubbard, 1975; Bruun *et al.*, 1978; Fitzgerald *et al.*, 1976, 1984; Hayes, 1980; Hubbard *et al.*, 1979; Luck, 1976; Sha, 1989; Terwindt, 1981). However, less has been done regarding the relationships between grain size characteristics and inlet hydrodynamics.

Bruun *et al.* (1978) pointed out the importance of the mode of transport on the behavior of sediments and on their composition. As grain size data reflect the sum of the processes acting upon the sediment, a clearer dispersal pattern of sediment transport, deposition and erosion should be available (Barndorff-Nielsen and Christiansen, 1988; Hartmann, 1988a, 1990; Hartmann and Christiansen, 1988; McLaren and Bowles, 1985). Hartmann (1988a, 1990, 1991) demonstrated that the various modes of transport and suspension winnowing are responsible for fine and coarse tails cutoffs of sampled populations and for their typical grain size. He concluded that the specific ratio between the modes of transport dictates the typical grain size and the shape of the sediment distributions.

This paper aims to demonstrate the sediment response to the morphodynamics in a small tidal-inlet. The study focuses on the sediment size-sorting pattern in intertidal sub-environments. The controlling processes are tracked by the grain size data. We investigate the main textural trends along the inlet berms and along the main Het Zwin channel as reflected by selective sorting processes and dispersal paths. The approach uses the log-hyperbolic distributional model (Bagnold and Barndorff-Nielsen, 1980) and the domain of the hyperbolic shape triangle to characterize the grain size data and to study their evolution (Barndorff-Nielsen and Christiansen, 1988). The final stage is the examination of the data when related to the erosion-deposition model introduced by Barndorff-Nielsen and Christiansen (1988).

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THE APPROACH TOWARDS A DISTRIBUTIONAL MODEL

The choice of a probabilistic model for characterizing a physical system is generally motivated by some initial understanding of the nature of the underlying phenomenon and should be verified by the available data (Hahn and Shapiro, 1967; Hartmann, 1988b). After a model has been chosen, its parameters are estimated. Tests of the adequacy of the chosen distributional models and the degree of goodness-of-fit which exists between the observed data and the estimated values help to determine whether or not the data contradict the assumed model (Christiansen and Hartmann, 1988a; Hartmann, 1988b). In this study, we investigated the data in two forms: the probabilistic form of the single grain-size samples, and the combined environmental "supersample" form (Hartmann, 1988a, 1991; Hartmann and Christiansen, 1992), which consists of a combination of all the grain size samples in a given environment. The working hypothesis was that the descriptive power of the log-hyperbolic distributional model is most adequate for environmental characterization.

The log-hyperbolic probability distribution was originally introduced to describe mass-size distributions of aeolian sands (Bagnold, 1941; Barndorff-Nielsen, 1977; Christiansen and Hartmann, 1991). Christiansen (1984) compared between moments statistics and parameters of the hyperbolic distribution. Barndorff-Nielsen *et al.* (1982) studied the variability of the log-hyperbolic parameters and their derivatives. Hartmann (1988a, 1991) investigated the four first moments, the log-hyperbolic parameters and some percentile parameters with efforts towards understanding the different shape parameters.

The location-scale invariant parameters χ and ξ of the log-hyperbolic distribution express respectively the skewness and the kurtosis of that distribution (Barndorff-Nielsen and Blæsild, 1981). The domain of variation of these two parameters (Figure 1) is referred to as the hyperbolic shape triangle (Barndorff-Nielsen *et al.*, 1985; Barndorff-Nielsen and Christiansen, 1985, 1988). Barndorff-Nielsen and Christiansen (1988) presented a mathematical-physical erosion-deposition model and showed the net effects of these processes in the hyperbolic shape triangle. According to their model, predominantly erosion will move the shape of a grain size distribution towards the right hand part of the triangle, and predominantly deposition towards the left, along specific sets of curves and lines in the nonlinear (ϵ, κ) space of the hyperbolic shape triangle. The applicability of this process-oriented environmental discrimination has been confirmed in a variety of natural environments (Barndorff-Nielsen and Christiansen, 1988; Hartmann, 1988a, 1991; Hartmann and Christiansen, 1988, 1992). Barndorff-Nielsen *et al.* (1991) concluded that often in sedimentological studies the parameter ξ underlies a highly stochastical tendency, and therefore the ratio $\rho = \chi/\xi$ contains the best information about the shape of a single grain size distribution and was defined by them as a tilt parameter.

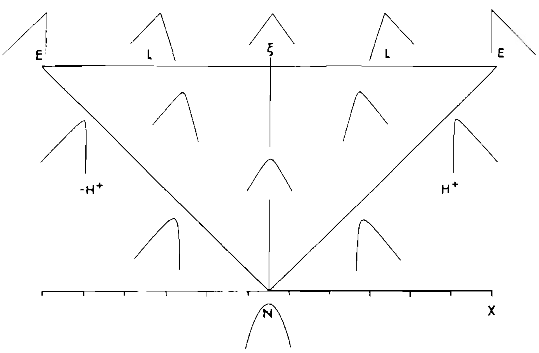

Figure 1. The hyperbolic shape triangle; *i.e.*, the domain of variation of the invariant parameters χ and ξ of the hyperbolic distribution. Representative logarithmic probability functions corresponding to selected (χ, ξ) values, including limiting forms of the hyperbolic distribution are shown. The distributions have been selected so as to have variance equal to unity. The letters at the boundaries indicate how the normal distribution (N), the positive and negative hyperbolic distributions (H^+ and $-H^+$), the Laplace distribution (symmetrical or skewed, L), and the exponential distribution (E) are limits of the hyperbolic distribution. From Barndorff-Nielsen and Christiansen (1988).

Most of the statistical multivariate data analyses use concepts, *e.g.*, the Gaussian distribution model, which are not always verified by the empirical data (Hartmann, 1988a; Vincent, 1986). It was further demonstrated that independent variables used in sedimentary environmental discrimination procedures often follow the beta distribution model (Hartmann, 1988a; Barndorff-Nielsen *et al.*, 1991). As none of the grain size parameters nor their derivatives fulfill the necessary conditions for most types of parametric multivariate analysis, *e.g.*, the independent vari-

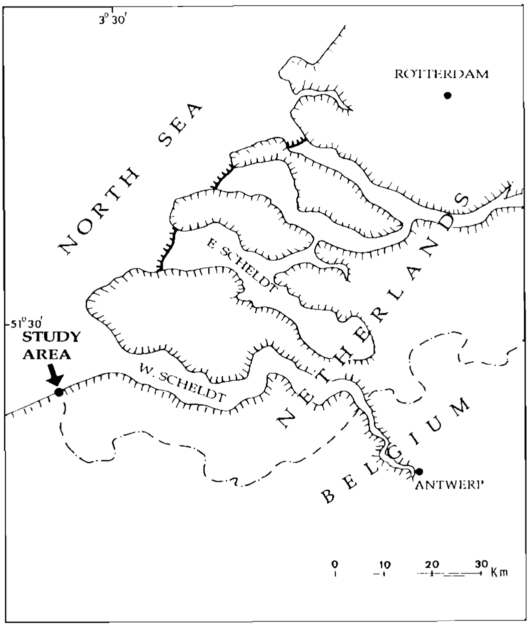

Figure 2. Location map of the study area.

ables follow a multivariate normal distribution with equal variance-covariance matrices in the populations, the use of the traditional "standard" statistical technique was rejected in this study.

METHODS

The Zwin study area is a nature reserve located on the Dutch-Belgian borderline in Zeeland (Figure 2). It is an inland basin embanked by dykes exhibiting a wide range of diverse bedforms, orientations and superpositions in well-organized bedform fields (Bowman, 1993). The semidiurnal mesotidal range was during the study period June–July, 1989, 3.32–4.24 m with a 90–100 cm/sec peak current velocity and showed a time velocity asymmetry pattern. The sedimentary trends in The Zwin were studied by sampling the upper 2 cm sand layer at discrete bedform fields. Fifty surface sand samples were dry sieved at 0.5 phi intervals using standard techniques. The bidirectional flow pattern was recorded by mechanical Ott current meters, 10–20 cm above bed. One minute averaged velocity was recorded at five minute intervals over single tidal cycles.

Sediments sampled from the active layer often have a mass-size distribution of the log-hyperbolic type (Bagnold, 1941; Bagnold and Barndorff-Nielsen, 1980; Barndorff-Nielsen *et al.*, 1982; Christiansen, 1984; Christiansen and Hartmann, 1988a; Hartmann, 1988b; McArthur, 1987; Vincent, 1986). The raw data of the 50 single sieved samples were statistically analyzed by using the SAHARA program (Christiansen and Hartmann, 1988b). Samples within four defined depositional sub-environments were grouped together to create four environmental grain size supersamples. Finally, all the 50 samples were combined into one overall grain size supersample which represents the entire sediment population of The Zwin Inlet. All the single grain-size distributions and the second-order grouped data, *e.g.*, supersamples, were estimated according to the log-hyperbolic and the log-normal probabilistic models. The estimated log-hyperbolic and moments parameters of all grain size data were rigorously studied. The log-normal model was rejected for all the investigated samples.

We regard the various local data as bearing a combination of information and noise. Depending on the variability within an environment compared to the variability between environments, and assuming that the sampled population is big enough, the probabilistic nature of the grain size selective sorting processes in the various sub-environments is expected to emerge.

RESULTS

Figure 3 demonstrates the distinct form of the observed single grain size samples and their estimated log-hyperbolic curves. The estimated distributions have a relative high goodness-of-fit to the observed ones and can be satisfactorily described as log-hyperbolic. The supersamples I–IV (Figure 4) and the total Zwin supersample (Figure 5) are log-hyperbolic distributed as well. The distributions have no fixed, symmetrical shape; and therefore, one should not use parameters as mean and standard deviation to characterize these populations. When examining the deviations of the observed data points at the tails, from the esti-

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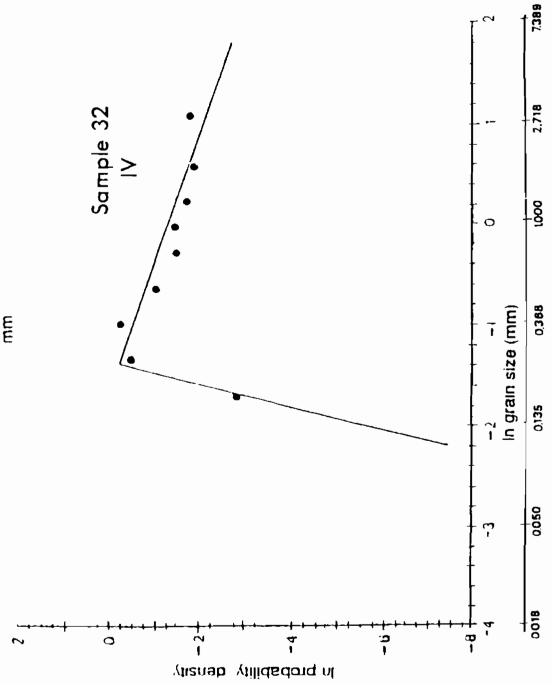
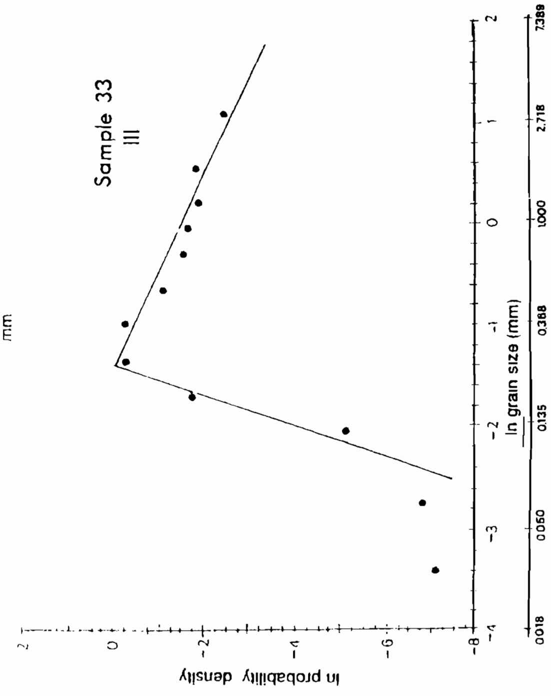
Figure 3. Observed and estimated log-hyperbolic distributions for single Zwin grain size samples. The figure labels I–IV refer to the facies sub-environments.


Sample 32
IV
ln probability density
2
0
-2
-4
-6
-8
-4
-3
-2
-1
0
1
2
ln grain size (mm)
0.018
0.050
0.135
0.368
1.000
2.718
7.389
mm


Sample 33
III
ln probability density
2
0
-2
-4
-6
-8
-4
-3
-2
-1
0
1
2
ln grain size (mm)
0.018
0.050
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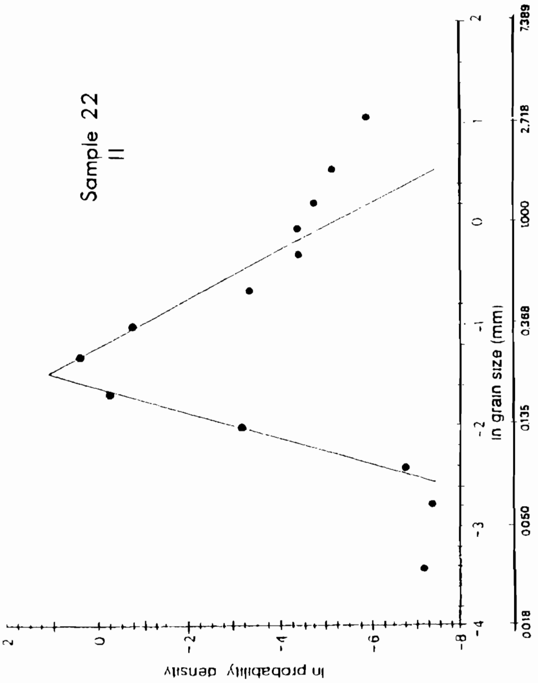
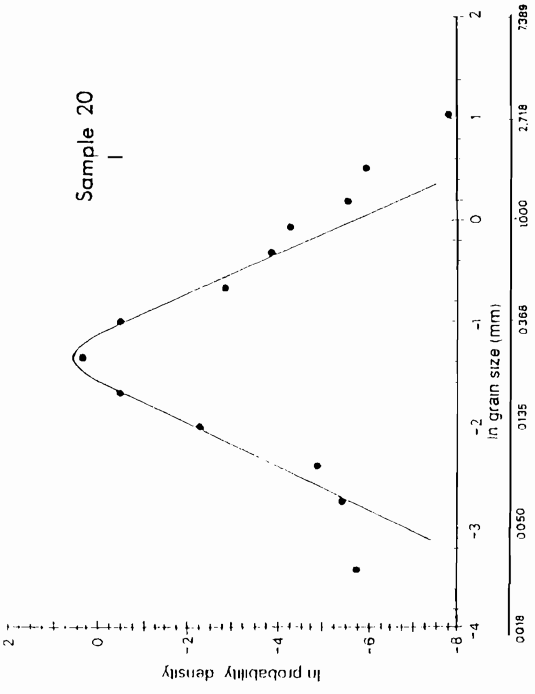

Sample 22
II
ln probability density
2
0
-2
-4
-6
-8
-4
-3
-2
-1
0
1
2
ln grain size (mm)
0.018
0.050
0.135
0.368
1.000
2.718
7.389
mm


Sample 20
I
ln probability density
2
0
-2
-4
-6
-8
-4
-3
-2
-1
0
1
2
ln grain size (mm)
0.018
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0.135
0.368
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mm

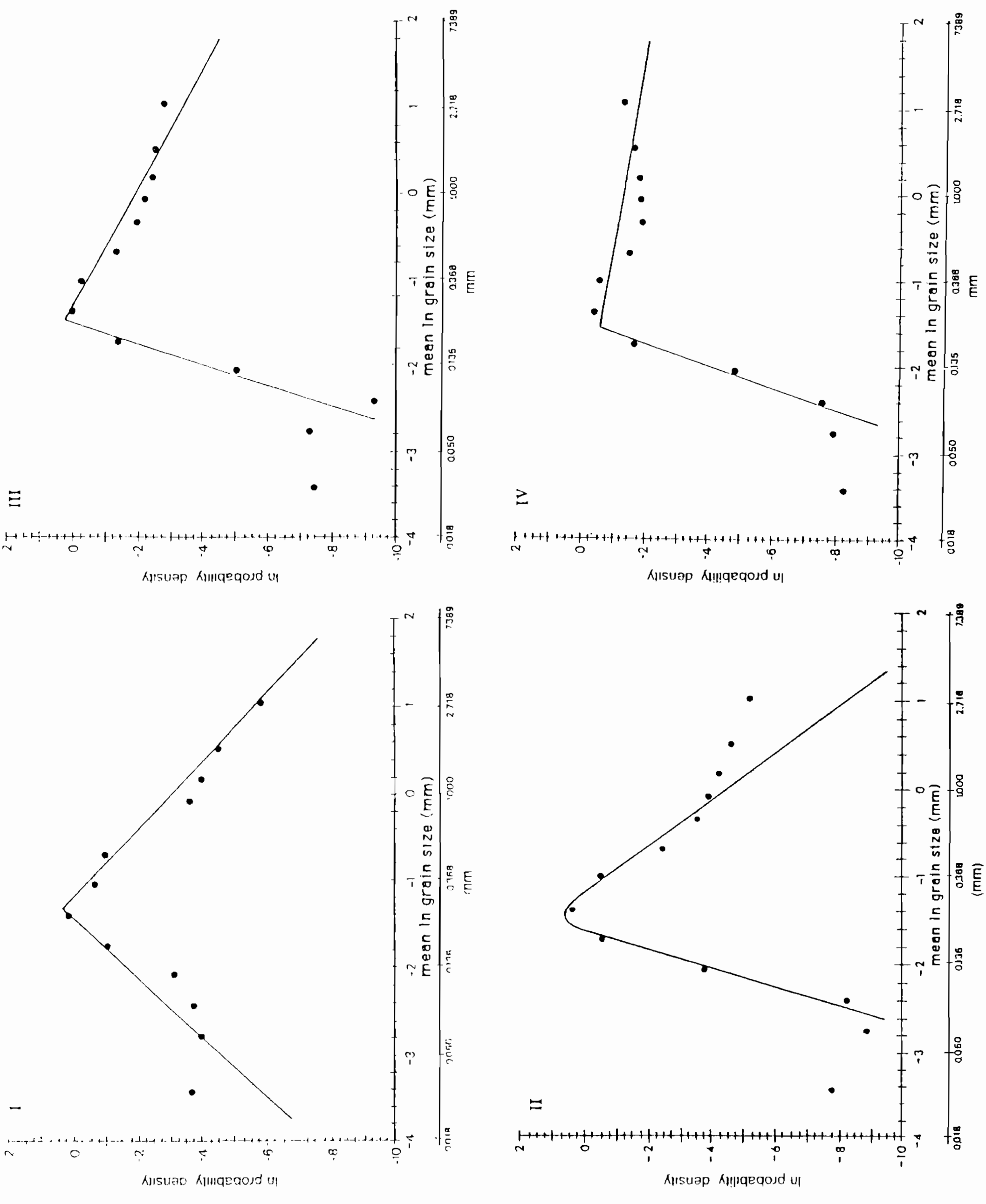


Figure 4. Observed and estimated log-hyperbolic distributions of The Zwin environmental grain size supersamples. Figure labels I–IV refer to facies sub-environments.

mated log-hyperbolic curves, one should recall that the ordinate is on a logarithmic scale; *i.e.*, Ln probability density.

Comparison of the berm sediments at the inlet throat with the furthest inland site studied suggests two very different sedimentary environments: Group IV (Figure 4), at the inlet throat, is a medium to coarse sandy environment. Group I, the uppermost inland site of the inlet studied, is much finer. It shows a wide range of distributional shapes (Figure 6a) of predominantly depositional facies. It includes moderate-flat, fine-tailed and symmetrical distributions, as well as moderate-flat, coarse-tailed ones. The sediments at the far end of the inlet are reflecting both erosional and depositional processes. They consist of grains which have shifted inland as suspension and as bed load. Dominance of the flood currents

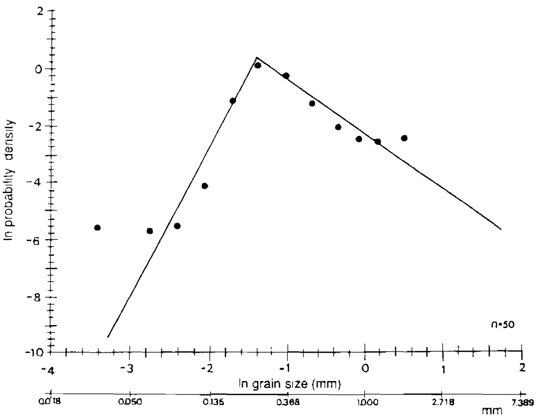


Figure 5. The overall Zwin supersample representing the fifty combined grain size samples.

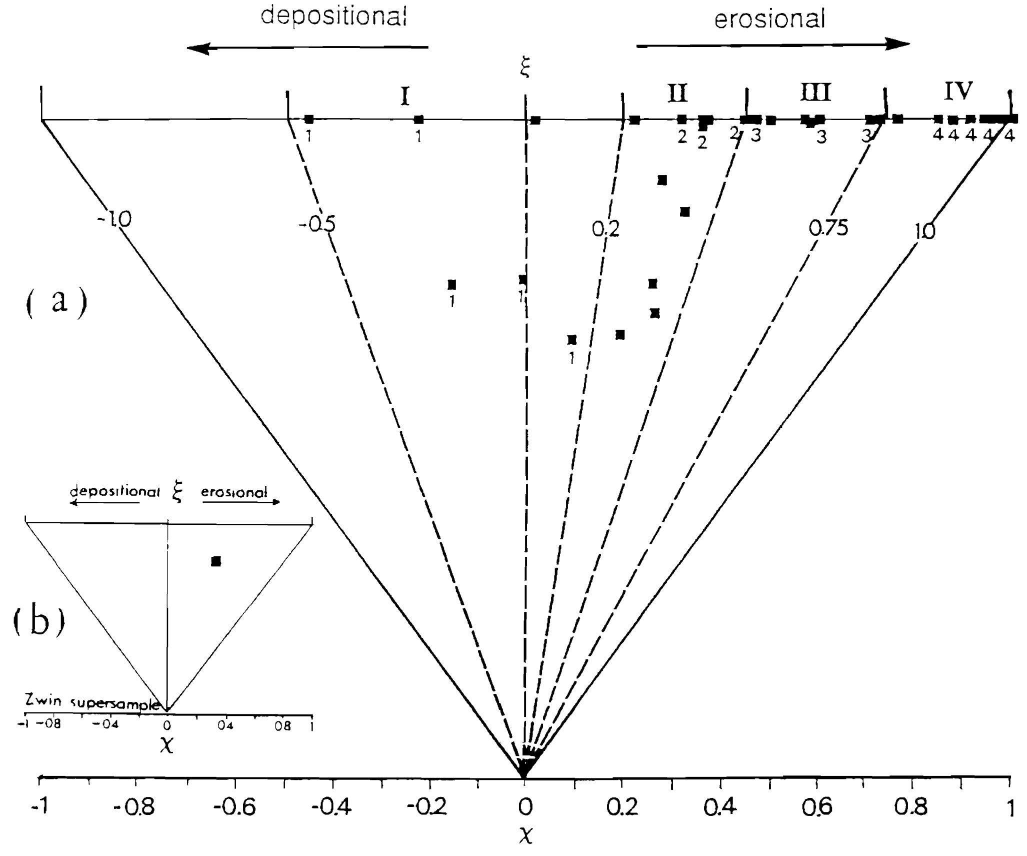
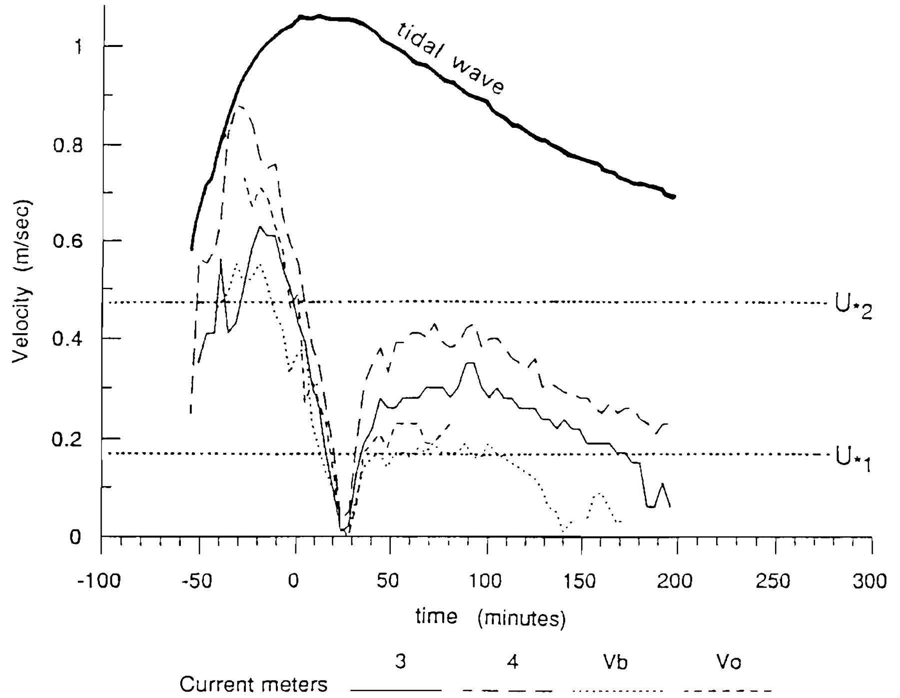


Figure 6. Hyperbolic shape triangle plots of The Zwin sediments: (a) Single samples, (b) The overall Zwin supersample.



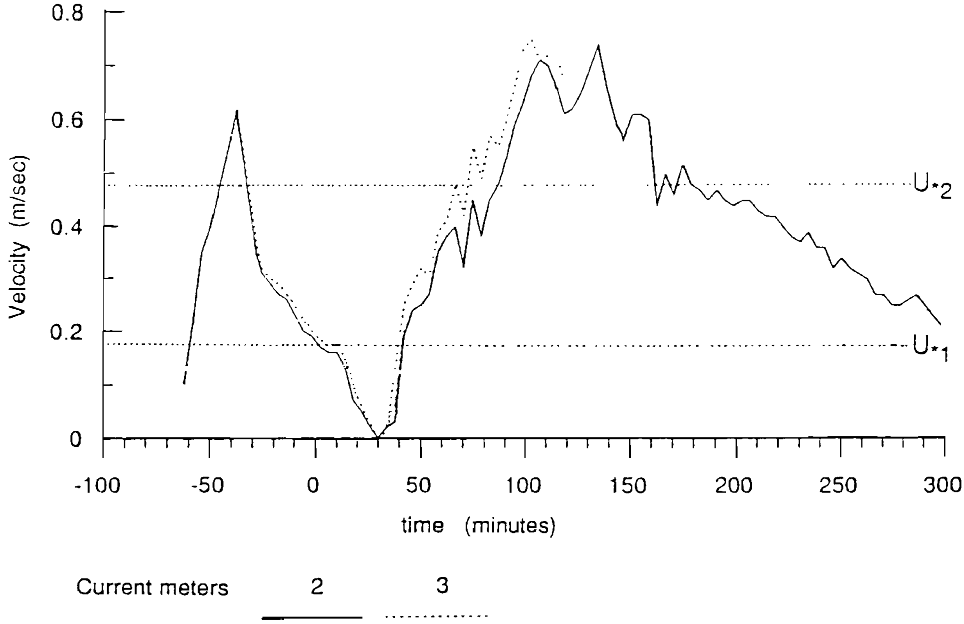


Figure 7. Typical velocity patterns for Group I (top) and for Group IV (bottom). U_{*1}, U_{*2} indicate sand entrainment threshold velocity between small-scale and large-scale ripples, respectively.

(Figure 7a) is demonstrated here by the high strength index, *i.e.*, U_{max} flood $\gg$ $U_{critical}$ (TERWINDT and BROUWER, 1986). The ebb-directed currents do not attain velocities beyond those forming small ripples within the lower flow regime.

Group IV (Figure 4) represents ebb-dominated berm samples at the lower Zwin channel. These sediments always have a very steep, fine tail, as well as a relatively flat, coarse one. The group is delimited within the far right half of the hyperbolic shape triangle (Figure 6a), almost approaching an exponential distribution, which is one of the limiting forms of the log-hyperbolic model (Figures 1 and 6). This position is spectacularly indicative of the dominance of erosional or winnowing processes. The sediments at the throat are dominated by strong flood and ebb flows which cause lag facies to form. Grains in suspension are out of equilibrium in such an energetic environment and the sediment consists mainly of bed load material. Both the flood and the ebb currents reach velocities typical to megaripple formation in the lower flow regime, with ebb velocities even exceeding those of the flood (Figure 7b). Such a velocity cycle with large bed shear values characterizes only the lower part of the inlet channel because of the converging flows and the channeling effect.

Groups II and III are located in the erosion-dominated part of the hyperbolic shape triangle (Figures 1 and 6) indicating a regime of intermediate erosion. Group II represents the upper Zwin channel with a higher portion of erosional processes over depositional ones. This group is more erosive than group I of the berm at the upper inlet where ebb currents are less effective. Group III reflects the wave-induced bedforms at the lower part of the inlet. From its position in the shape triangle, we conclude that its erosive component is dominant over the depositional one.

DISCUSSION

The hyperbolic shape triangle allows discrimination between four main sediment groups in Het Zwin tidal-inlet. Changes between the environments are gradual and the general trend is clear. Each of the environments is statistically unique and significantly different from the others. The tilt parameter, the ratio $\rho = \chi/\xi$, seems to be the best parameter to express the shape of the studied grain size distributions. The four environments are ordered according to ascending ρ values (Figure 6a), starting with the most symmetrical log-hyperbolic distribution, located near the center of the shape triangle and ending up with the most asymmetrical one at the right corner. As can be expected from a natural probabilistic model, the sorting processes do not create sharply defined populations but rather show environments which partly overlap. Following the erosion-deposition model presented by BARNDORFF-NIELSEN and CHRISTIANSEN (1988), we suggest a dynamic link of sedimentary subenvironments within this sequence.

When the ρ index of the hyperbolic shape is plotted on a bivariate diagram against the mean grain size (Figure 8), good separation of the grain size populations of the closely-related sub-environments is discerned. A clear trend is seen along the inlet, starting at the throat with the dominantly erosional, coarse, lag facies (Group IV) and ending in the upstream, inner inlet depositional facies (Group I). The most symmetrical, relatively fine-grained distributions are found in the upstream, whereas the asymmetrical coarse ones, with a very steep fine tail, are those near the beach, indicating a seaward coarsening trend as reported by KLEIN (1970).

Many of the samples lie on the upper borderline of the hyperbolic shape triangle (Figure 6a), indicating closeness to the Laplace distribution, another limiting form of the hyperbolic model (FIELLER *et al.*, 1984). The concentration at the upper borderline may, however, according to our experience, also result from the measurement technique; *i.e.*, sieving at 0.5 phi interval may cause measurement noise without any sedimentological meaning.

Study of the Zwin bedform fields by moments statistics (BOWMAN, 1993) similarly suggests an overall fining-inland trend, accompanied by improvement of sorting and increase of the negative skewness. These findings indicate inland winnowing of the fine fraction, while leaving at the throat the coarsest sediment fraction as a lag. These trends, however, reflect mainly the summer low-energy wave regime of The Zwin. The fingerprint of storms is found at the upper inlet as coarse shell deposits alternating with sand and mud. Similar trends have been observed along the main channel. The most dramatic change along the channel is the improvement of sorting inland. At the throat (1,300–1,500 m downstream), the sediment is the coarsest, the poorest sorted and the least skewed.

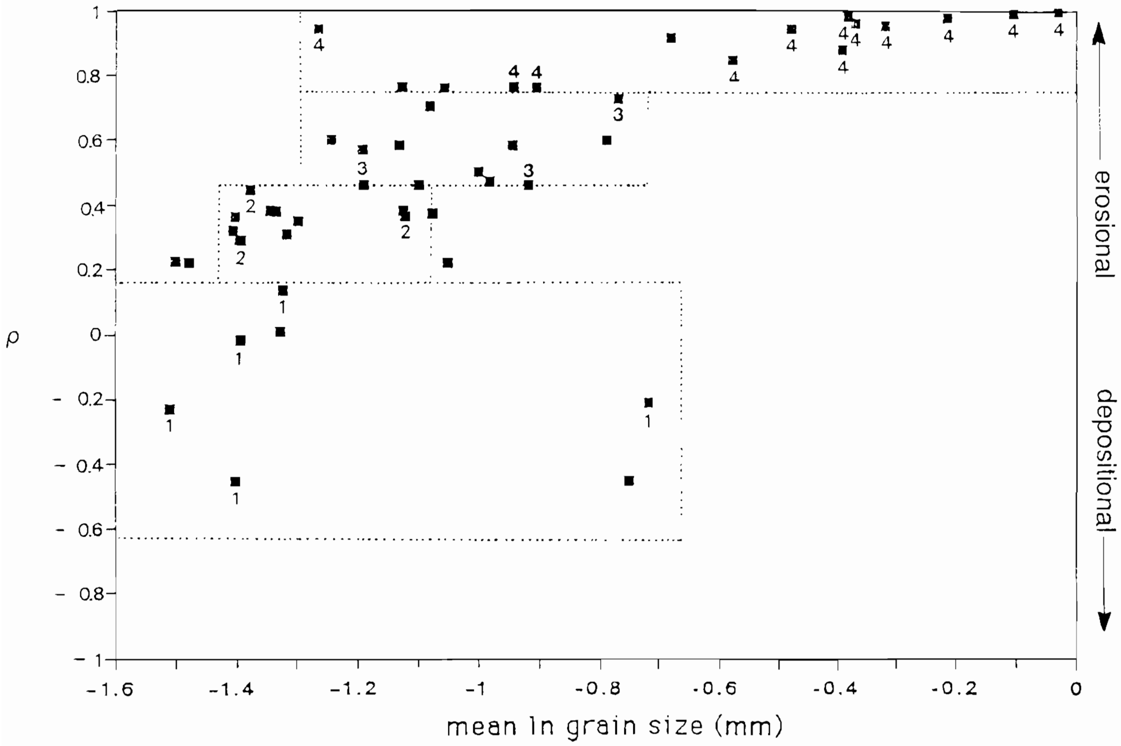


Figure 8. Bivariate plot of the hyperbolic tilt parameter ρ of the main four sedimentary populations of The Zwin Inlet in relation to Ln mean grain size.

The four grain size supersamples from the subenvironments discriminated so far along Het Zwin Inlet make four different log-hyperbolic curves (Figure 4) and occupy four distinct fields in the hyperbolic shape triangle (Figures 6a). Although there is some overlap, the distributional shapes of the defined supersamples are significantly different. This population approach demonstrates the probabilistic nature of the sampled sediments and the efficiency of the hyperbolic model in environmental discrimination and processes reconstruction.

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