The Analysis of Exponential Beach Profiles

Paul D. Komar† and William G. McDougal‡

†College of Oceanography
Oregon State University
Corvallis, OR 97331, U.S.A.

‡Ocean Engineering
Oregon State University
Corvallis, OR 97331, U.S.A.

ABSTRACT ■

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The overall form of many beach profiles has an exponential shape where the depth h is given by h = (S./k)(1 e ⁱ·) with S. the beach-face slope at x = 0 and k is an adjustable coefficient that determines the degree of concavity. The cross-shore variation in beach slope is then S = S.e ⁱ· or S = S.. – kh. Beach profiles can be analyzed in terms of both the cross-shore variations in h and S. Based on previous studies, the beach-face slope S., is predictable as a function of the sediment grain size and wave parameters. The evaluation of k can be based on best-fit comparisons with the measured profile depths or from bottom-slope variations across the profile. Equations are derived for the evaluation of k from the offshore closure depth of the envelope of profile changes, or from some arbitrarily selected coordinate of the profile. An example of the analysis approach is provided by a beach profile from the Nile Delta coast of Egypt. This measured profile shows good agreement with the exponential form for cross-shore variations in both the absolute depth h and local bottom slope S. There is poor agreement with the h = Ax profile relationship, in part because this Nile Delta profile is more reflective and has a greater concavity than allowed by the x dependence. The failure of the x profile form is still more evident in analyzing the beach-slope variations since it predicts an infinite slope at the shoreline. The exponential beach profile is a convenient mathematical relationship that should be useful in many applications.

ADDITIONAL INDEX WORDS: *Beaches, beach profiles, equilibrium profiles, beach profile models.*

INTRODUCTION

Although beach profiles can be complex due to series of bars and troughs, in overall form they are steepest at the shoreline and have a progressively decreasing slope as the water depth increases in the offshore direction. This regularity has inspired attempts to develop mathematical expressions to describe the profile shape. These formulations can then be used in analyses of wave dynamics during shoaling, in examinations of the generation of nearshore currents and sediment transport, and in computer models of beach morphology and shoreline change.

Best known and most commonly used in applications is the Bruun/Dean profile model where the offshore depth increases in proportion to x ' where x is the offshore distance from the shoreline (BRUUN, 1954; DEAN, 1977, 1991). More recently, BODGE (1992) has employed an exponential beach profile which he concluded shows better agreement with measured beach profiles than does the Bruun/Dean profile. We have also been exploring the use of exponential profiles in comparisons with a large number of beach profiles that have been measured along the coast of the Nile Delta (NAFAA

et al., in preparation). The form we are employing is somewhat different than that used by BODGE (1992); our beach-profile expression in essence is an exponential decrease of the slope in the offshore direction from its value at the beach face which depends on the sediment grain size and wave conditions. Having evaluated the beach slope at the shoreline, the resulting profile equation contains only one adjustable coefficient which may be evaluated from the offshore closure depth of the envelope of profile changes. The objective of the present paper is to develop this form for exponential beach profiles, and to illustrate its use in analyzing one example profile from the Nile Delta.

BEACH PROFILE MODELS

The beach profile expression derived by BRUUN (1954) and DEAN (1977, 1987, 1991) has the general form

$$h = Ax^m \qquad (1)$$

where h is the still-water depth, x is the horizontal distance from the shoreline, and A and m are empirical coefficients based on a best fit to the measured profile. The original application by BRUUN (1954) was limited to the profile seaward

of the breaker zone, having been derived on an assumption of an equal bottom stress exerted by the waves. DEAN (1977) extended the application through the surf zone to the shoreline, demonstrating that the form of equation (1) reasonably represents the 504 beach profiles compiled by HAYDEN et al. (1975) along the U.S. east and Gulf coasts. DEAN (1977) adopted a mean value m = ⅔ to be used as a functional constant, but values for individual profiles within the data base ranged from about 0.2 to 1.2. Justification for selecting ⅔ was based in part on the apparent Gaussian distribution of values and observed central tendency of this ⅔ exponent. In a study of beaches on islands in the Caribbean Sea, BOON and GREEN (1988) found an average exponent of approximately ½, and concluded that this lower value for the Caribbean beaches resulted from the more reflective nature of those beaches with a greater concavity than typical of the continental quartz-sand beaches in the U.S. analyzed by DEAN (1977). Boon and Green correctly concluded that since the concavity of beaches varies as they range from reflective to intermediate to dissipative, the exponent will necessarily change rather than always being ⅔. The proportionality coefficient A in Equation 1 has been empirically related to the grain size (mean or median) of the beach sediment (MOORE, 1982) and to the corresponding grain settling velocity (DEAN, 1987).

The simplicity of the Bruun/Dean profile expression of Equation 1 has resulted in its use in a variety of applications (McDOUGAL and HUDSPETH, 1983a,b; DEAN, 1991). However, there are several shortcomings in this model which adversely affect the analysis results in applications. One problem is its dimensionality, where the units for A depend on the value of the exponent m [with m = ⅔, the units for A are (length)$^{1/3}$]. The physical interpretation of A is unclear; BOON and GREEN (1988) have noted that A is a scale parameter which is numerically equal to the depth at a unit distance from the shore, making it something of a surrogate measure of the beach slope, but one having dimensions. This would account for the empirical increase in A with increasing beach sediment grain size or settling velocity found by MOORE (1982) and DEAN (1987). It has been well established that the beach slope depends on wave parameters as well as on the sediment grain size (BASCOM, 1951, 1954; KING, 1972; KOMAR, 1976; SUNAMURA, 1984, 1989), so one would expect a comparable dependence of A on wave conditions.

This is suggested by the results of BOON and GREEN (1988).

A shortcoming of the Bruun/Dean beach profile relationship of Equation 1 is its prediction of an infinite slope at the shoreline. The derivative of the relationship with m = ⅔ yields the beach slope variation:

$$S = \tan \beta = \frac{dh}{dx} = \frac{2A}{3x^{1/3}} \qquad (2)$$

which becomes infinite when x = 0 at the shore. This is true for any model having the form of Equation 1 so long as the exponent is less than 1 (which it is for concave profiles). This problem is shared by the revised expressions developed by WORK and DEAN (1991) who explored models incorporating a cross-shore variation in A due to changing grain sizes along the profile. Because of its prediction of an infinite slope at the shoreline, the Bruun/Dean models can be expected to agree best with measured profiles in the offshore as originally analyzed by BRUUN (1954), but will progressively fail as the shoreline is approached.

Following its introduction by BALL (1967) in analyses of edge waves, BODGE (1992) has developed an exponential beach-profile model having the form

$$h = B(1 - e^{-kx}) \qquad (3)$$

where B and k are empirical coefficients. BODGE (1992) demonstrated that this exponential profile more closely approximates the measured profiles in the HAYDEN et al. (1975) data set than does the form of the Bruun/Dean profile of Equation 1, even when one allows the exponent m to empirically depart from the mean ⅔ value. The coefficient k in Equation 3 determines the concavity of the profile, and according to BODGE (1992) has a range 3×10^{-5} to $1.16 \times 10^{-3}\,m^{-1}$ for the HAYDEN et al. (1975) data set. The proportionality coefficient B is the offshore depth to which the sloping profile of equation (3) approaches asymptotically. The slope is given by:

$$S = \tan \beta = \frac{dh}{dx} = kBe^{-kx} \qquad (4)$$

At the shoreline (x = 0) the beach face slope is S_o = kB; a finite value but one that combines the two empirical coefficients. This suggests the alternate form:

$$h = \frac{S_o}{k}(1 - e^{-kx}) \qquad (5)$$

the one we have been using in analyses of profiles on the Nile Delta. The variation in bottom slope along the profile is now:

$$S = \tan \beta = \frac{dh}{dx} = S_o e^{-kx} \qquad (6)$$

This can be viewed as the most fundamental relationship of the model—an exponential decrease in the profile slope with distance offshore from the S_o value at the shoreline.

Equations 5 and 6 contain only one empirical coefficient, k, since there has been considerable study of how the beach face slope S_o at the shoreline varies with sediment grain size and wave conditions so that it is a predictable parameter (BASCOM, 1951, 1954; KING, 1972; KOMAR, 1976; SUNAMURA, 1984, 1989). Alternately, S_o can be taken as the measured slope of the beach face, and k determined by a best fit comparison with the total measured profile. We can also simply approximate k from a single offshore depth, in particular the closure depth of offshore profile changes that has been analyzed by HALLERMEIER (1981) and BIRKEMEIER (1985). For closure depth coordinates (x_c, h_c), the rearrangement of Equation 5 yields

$$\frac{(h_c/x_c)}{S_o} = \frac{1 - e^{-kx_c}}{kx_c} \qquad (7)$$

a dimensionless relationship from which the curve of Figure 1 has been calculated, making possible an evaluation of k from the closure-depth coordinates. The profile is therefore completely defined in terms of the beach face slope, S_o, governed by the beach sediment grain size and wave conditions, and by the closure depth coordinates (x_c, h_c) which according to the analyses of HALLERMEIER (1981) and BIRKEMEIER (1985) depend on the wave height and steepness.

The exponent e^{-kx} can be expanded using the Taylor series, such that Equation 5 becomes

$$h = \frac{S_o}{k}\left[(kx) - \frac{(kx)^2}{2!} + \frac{(kx)^3}{3!} - \cdots\right] \qquad (8)$$

The first-order profile is a linear slope $h = S_o x$, the second-order is a parabola, and so on. Using this Taylor series expression for h, its substitution in Equation 7 and solution for k to a third-order approximation yields:

$$k = \frac{1}{x_c}\left[\frac{3}{2} - \left(\frac{6h_c}{S_o x_c} - \frac{15}{4}\right)^{1/2}\right] \qquad (9)$$
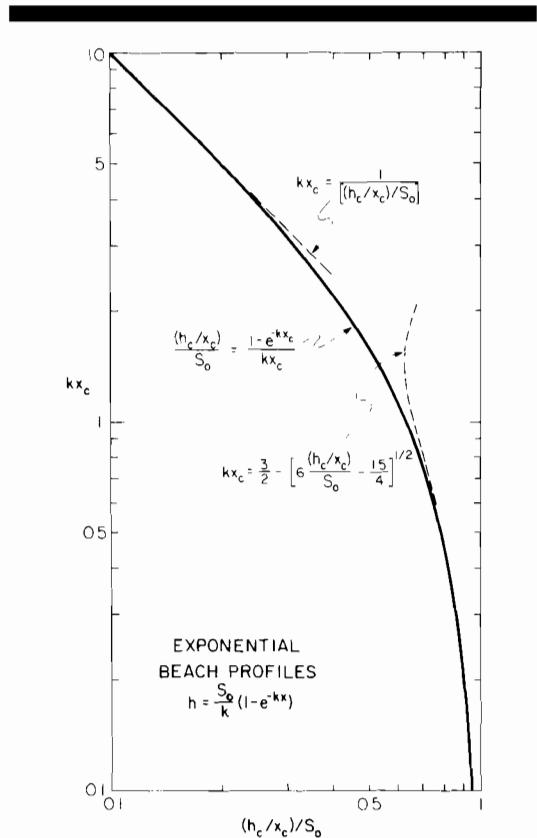

Figure 1. The curve of Equation 7 based on the exponential profile relationship, Equation 5, that can be used to evaluate the exponent coefficient k from the beach-face slope S_o and coordinates of the closure depth (x_c, h_c) or some other selected offshore depth. Equations that can serve as satisfactory approximations are shown.

which gives real solutions when $(h_c/x_c)/S_o > 0.625$. The dependence again is one of k or kx_c on the ratio of h_c/x_c to S_o. This approximate solution is graphed in Figure 1 where it is seen to converge with the curve from Equation 7 when $(h_c/x_c)/S_o > 0.75$ or $kx_c < 0.6$. Also shown in Figure 1 is the approximation $kx_c = 1/[(h_c/x_c)/S_o]$, which is equivalent to $k = S_o/h_c$, to which the general solution of Equation 7 approaches when $e^{-kx_c} \ll 1$, that is when kx_c is large.

The form of Equation 7 is interesting in that it relates the ratio of the average beach slope out to the closure depth, equal to h_c/x_c, to the higher slope of the beach face, S_o; this ratio depends on the value of k which governs the profile's degree of concavity. As expected, the higher the value of k the greater the decrease in the overall slope

compared with S_o. It is apparent from this that there are trends in both S_o and k that parallel the classification of beaches in a series from reflective to dissipative as developed by WRIGHT and SHORT (1983, 1984). Reflective beaches will tend to have a combination of high beach-face slope S_o and high values for k due to their stronger degree of concavity. For reflective beaches of high curvature such that $e^{-kx_c} \ll 1$, Equation 7 reduces to the $kx_c = 1/[(h_c/x_c)/S_o]$ approximation graphed in Figure 1, or simply to $k = S_o/x_c$. Equation 5 for the general beach profile then becomes

$$h/h_c = (1 - e^{-S_o x/h_c}) \qquad \text{[reflective beaches with}$$
$$e^{-kx_c} \ll 1] \qquad (10)$$

one which is dimensionless and completely in terms of dependencies on the beach face slope S_o and an offshore closure depth h_c. Dissipative beaches will tend to have a combination of low S_o and low values of k due to their smaller degree of concavity. As noted in the Taylor Series expansion, Equation 8, as k decreases the solution approaches $h = S_o x$, so the exponential beach profile for dissipative beaches will approach a planar beach as a limit. Expressed in terms of the closure depth, this becomes $h_c = S_o x_c$ or $(h_c/x_c)/S_o = 1$; it is seen in Figure 1 that the general curve calculated from Equation 7 asymptotically approaches $(h_c/x_c)/S_o = 1$ for small values of kx_c as would be expected for dissipative beaches. From this it can be seen that the exponential beach profile and its dependence on S_o and k can be conveniently viewed within the scope of reflective to dissipative beach types, and could be incorporated into that classification scheme.

Profiles could also be analyzed in terms of their slopes using Equation 6. That relationship can be expressed as:

$$\log_e(S) = \log_e(S_o) - kx \qquad (11a)$$

or

$$\log_{10}(S) = \log_{10}(S_o) - kx/2.3 \qquad (11b)$$

which shows that a semilog graph should yield a straight line having a slope that reflects the value of k and a x = 0 intercept of $\log(S_o)$. But from Equation 6 we have $e^{-kx} = S/S_o$, and if this is substituted in Equation 5, we obtain the simple linear form

$$h = \frac{S_o - S}{k} \qquad (12a)$$

or

$$S = S_o - kh \qquad (12b)$$

which shows a linear decrease in the beach slope S from its value at the shoreline as the offshore depth increases. Thus, the exponential beach profile has the interesting property that it predicts linear relationships between both log(S) versus x and S versus h, with k being the slope in each case. It is apparent from these relationships that analyses of measured profiles in terms of their changing slopes in the offshore direction could potentially be more straight forward than analyzing the depth variations as is usually done. This will be seen in the example developed below.

PROFILE ANALYSIS: AN EXAMPLE FROM THE NILE DELTA

Through an analysis of the extensive set of beach profiles compiled by HAYDEN et al. (1975), BODGE (1992) demonstrated that the exponential forms of Equations 3, 5 and 6 are inherently better than the Bruun/Dean model of Equation 1. BODGE (1992) reached this conclusion through best-fit comparisons of the entire profiles. From this, it is unnecessary in the present paper to make further comparisons with the Bruun/Dean model using large numbers of beach profiles. Instead, the objective is to illustrate the use of the exponential-profile relationships formulated above that are in terms of the beach face slope S_o and the closure depth.

The analysis presented here is for a beach profile from the Nile Delta, a typical example of the immense number of beach profiles that have been measured on that coastline by investigators at the Coastal Research Institute in Alexandria. A study is underway that will include analyses of more beach profiles from the Delta, including comparisons between measured profiles and the various model relationships (NAFAA et al., in preparation). The measured profile is shown in Figure 2. The shoreward half of the profile is clearly concave, but the outer half shows a broad bar of low height; the slope is everywhere seaward in spite of the bar form. Wave heights on the Nile Delta coast are less than three meters except during unusual storms (NAFAA et al., 1991), so that the breaker zone would normally be within 200 meters of the shoreline, that is, within the concave portion of this profile which extends to water depths and an offshore distance that is well beyond any expected closure depth.
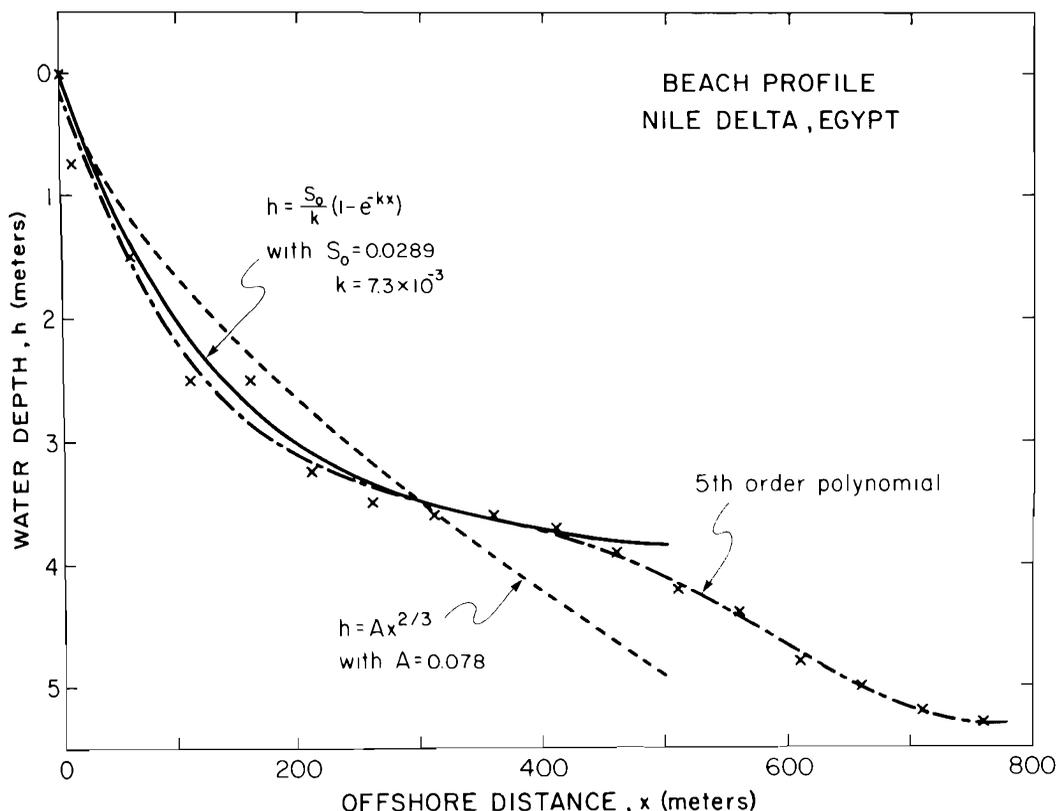
Figure 2. An example profile from the Nile Delta, Egypt, with the measured depths (symbol X) approximated by the 5th-order polynomial of Equation 13. There is good agreement to an offshore distance of 400 meters with an exponential profile having a beach-face slope $S_o = 0.0289$ and $k = 0.0073$ m⁻¹.

A 5th-order polynomial has been fitted to the measured profile, yielding:

$$h = 0.14859 + (2.8876 \times 10^{-2})x$$
$$- (9.5416 \times 10^{-5})x^2 + (1.3873 \times 10^{-7})x^3$$
$$- (5.4695 \times 10^{-11})x^4$$
$$- (1.711 \times 10^{-14})x^5$$
$$[R^2 = 0.993] \qquad (13)$$

This polynomial is seen in Figure 2 to provide a good representation of the water-depth variations of the measured profile, including the curvature of the offshore bar. In this example, a 5th-order polynomial was used as still higher orders provided negligible improvement; in other examples, this choice would depend on the complexity of the measured profile. Of interest is the similarity in form of this polynomial of Equation 13 to the Taylor series expansion of Equation 8, with there being the same alternation of signs of the coefficients up through the 4th order term. Accordingly, from the correspondence of the x coefficient, we have $S_o = 0.028876$ for the beach-face slope; the x^2 coefficient gives $S_o k/2! = 9.5416 \times 10^{-5}$ or $k = 6.609 \times 10^{-3}$ m⁻¹, which is close to the values determined below using a closure depth and beach-slope variations in the cross shore. The main use of the polynomial of Equation 13 comes later in analyzing the profile slope variations.

Also shown in Figure 2 is the exponential profile of Equation 5 with beach-face slope $S_o = 0.0289$ and $k = 0.0073$ m⁻¹. This value for S_o again is obtained from the polynomial fit [the x^1-term coefficient], while the value for k is based on the offshore profile coordinates $x_c = 300$ meters and $h_c = 3.50$ meters that approximately incorporate

the inner half of the measured profile which is more clearly concave, and probably also approximates an expected closure depth of profile variations. These coordinates give $(h_c/x_c)/S_o = 0.403$, and from the graph of Figure 1 we have $kx_c = 2.2$, yielding $k = 0.0073$ m^{-1}. In effect, the fitted profile starts off at the shoreline with a slope $S_o = 0.0289$, and is forced to have an overall curvature (k value) that will cause the profile to have a depth of 3.50 meters at 300 meters offshore. The resulting exponential profile fits the concave form of the measured profile out to approximately 400 meters offshore, and is nearly the same as the 5th-order polynomial over that profile range (Figure 2).

Also shown in Figure 2 is the Bruun/Dean profile of Equation 1 with exponent m = $\frac{2}{3}$ and A = 0.078 m$^{\frac{1}{3}}$. This value for A again was based on forcing the profile through a depth of 3.50 meters at 300 meters offshore. It is apparent that there is poor agreement with the measured profile, the $\frac{2}{3}$ exponent not providing sufficient concavity for this more reflective beach. The profile of Equation 1 with m = 0.5 does provide a better fit, just as found by BOON and GREEN (1988) for Caribbean beaches. One objective of our more comprehensive analyses of the Nile Delta profiles is to provide such comparisons where the exponent m in the Bruun/Dean profile is allowed to vary (NAFAA *et al., in preparation*).

The analysis of this Nile Delta beach profile, but in terms of the bottom slope variations, is presented in Figures 3 and 4. In Figure 3, the comparison is between log(S) and the offshore distance x, the dependence suggested by Equation 11. If one directly converts the measured depths of the profile into slopes, the small irregularities or errors in the depth determinations result in large variations in the directly calculated slopes. This particularly affects the log(S) versus x comparison as the logarithm of S accentuates the scatter. Another problem is dealing with horizontal bottoms such as occur at the top of a bar or within a trough, since S = 0 cannot be plotted on the log(S) scale. We found in this example that the scatter of the measurements in the log(S) versus x graph was much too large to allow direct comparisons with Equations 6 and 11. This is where the 5th-order polynomial of Equation 13 becomes particularly useful, since its derivative yields a smoothed description of the cross-shore variation in local beach slope. Accordingly, we have:

$$S = dh/dx$$

$$= 2.8876 \times 10^{-2} - (1.9083 \times 10^{-4})x$$
$$+ (4.1619 \times 10^{-7})x^2$$
$$- (2.1878 \times 10^{-10})x^3$$
$$- (8.555 \times 10^{-13})x^4 \qquad (14)$$

This relationship is graphed in Figure 3 where it is seen that the broad, low-amplitude offshore bar in the original profile (Figure 2) produces a significant curvature in the corresponding variation in the bottom slope. It is apparent from this that the presence of a significant bar-trough system would make analyses of the bottom-slope variations particularly difficult. Of interest in the profile under consideration is the existence of a nearly straight-line dependence of the inner half of the profile variations in the bottom slope (Figure 3). The intercept of the straight line again yields $S_o = 0.0289$, while the slope of the line gives $k = 0.0081$ m^{-1}. The exponential form for this slope variation is then $S = 0.0289 \exp[-0.0081x]$, corresponding to Equation 6. This analysis in particular demonstrates that the inner half of this Nile Delta beach profile corresponds closely to the exponential form of Equations 5 and 6. The failure of the Bruun/Dean model of Equation 1 with m = $\frac{2}{3}$ is again evident, particularly close to shore where it predicts that $S_o = \infty$. However, it is seen that the upward turn of the Bruun/Dean profile slope does not occur until very close to the shoreline, and this would in part explain the success of that model in comparisons with measured profiles in spite of this inherent failing.

The comparison in Figure 4A is between the linear S and offshore depth h predicted by Equation 12. With linear scales there is no problem in plotting S = 0, and the overall scatter of the data is sufficiently small that trends are readily discernable. Regression of the data out to the bar at 400 meters offshore distance yielded the straight line shown in Figure 4A with $S_o = 0.037$ and $k = 0.0099$ m^{-1}; for regression over the entire profile, $S_o = 0.033$ and $k = 0.0069$ m^{-1} (Figure 4A). This difference in S_o values determined by the two regression analyses, an 11% difference, has little effect on the overall forms of the calculated profiles. More significant is the 30% difference in k values which results in contrasting degrees of profile curvature as shown in Figure 4B. The k values and resulting curvature significantly affect the offshore depths calculated from the exponential profile. The regression using data out to 400 meters yields the higher curvature and gives a profile
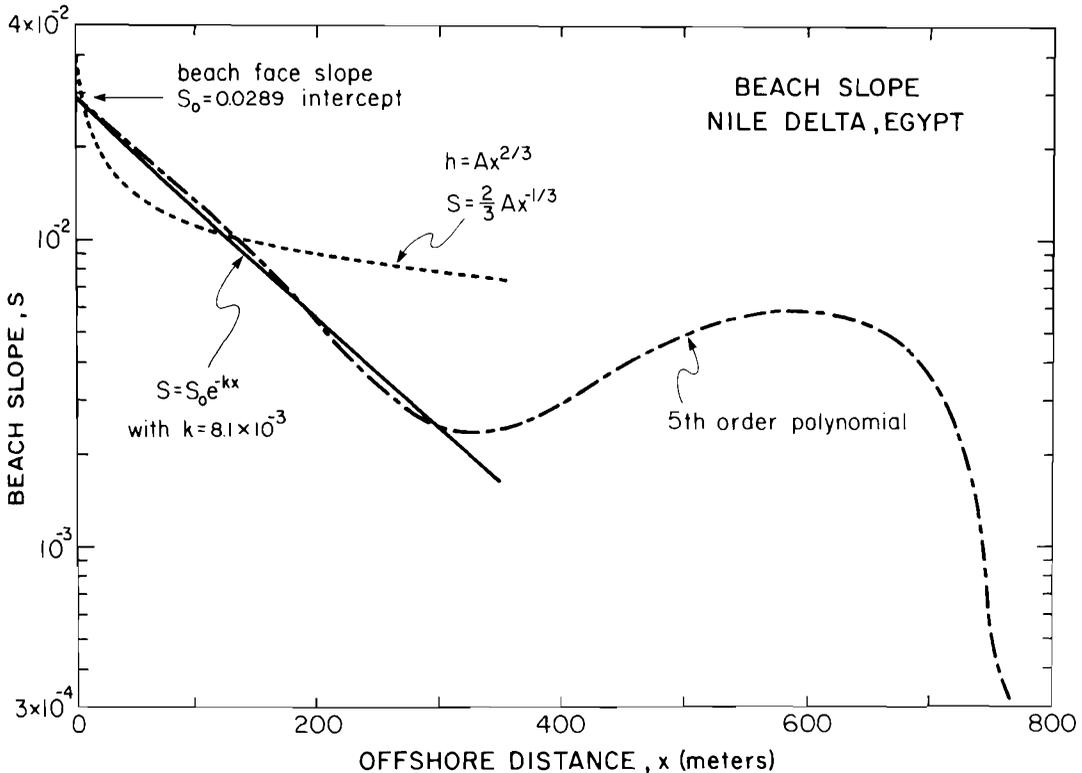
Figure 3. An analysis of the cross-shore variations in the bottom slope of the Nile Delta beach profile, with semi-log paper being used so that the exponential relationship of Equation 11 plots as a straight line. The derivative of the 5th-order polynomial of Figure 2 is used to describe the beach slope variations. The straight-line trend out to 300 meters offshore distance yields $S_o = 0.0289$ and $k = 0.0081$ m^{-1} in $S = S_o e^{-kx}$.

that levels off at a depth of approximately 3.7 meters in the offshore (Figure 4B); the regression of the complete profile gives a lower curvature and a profile that levels off at 4.7 meters. This difference raises the question of what portion of the profile should be used in the analysis to determine the coefficients. In the present low-energy case from the Nile Delta, the profile inshore of the bar at 400 meters is directly influenced by the waves while the offshore profile is beyond the plunge point for typical storms (NAFAA et al., 1991). Therefore, it seems reasonable to regress only the inner portion of the profile. This supposition is supported by a lower RMS error for the inshore regression.

GENERAL PREDICTIONS OF EXPONENTIAL BEACH PROFILES FOR APPLICATIONS

Many applications require predictions of beach profiles for given conditions of beach sediment grain sizes and wave parameters. The exponential beach profile is particularly suitable for such analyses in that there has been considerable study of factors governing the beach-face slope, S_o, and the determination of k can be based on one set of offshore coodinates such as the closure depth, or on a bottom slope in the offshore. For example, based on a compilation of measurements from many field studies, SUNAMURA (1984) obtained

$$S_o = 0.12 \left[\frac{H_b^2}{gDT^2} \right]^{\cdot} \tag{15}$$

where H_b is the wave breaker height, T is the wave period, and D is the sediment diameter. Using $L_x = gT^2/2\pi$ and the KOMAR and GAUGHAN (1972) relationship for the breaker height as a function of the deep-water wave parameters, the above relationship can be converted into dependencies on D/H_x and $H_x L_x$:
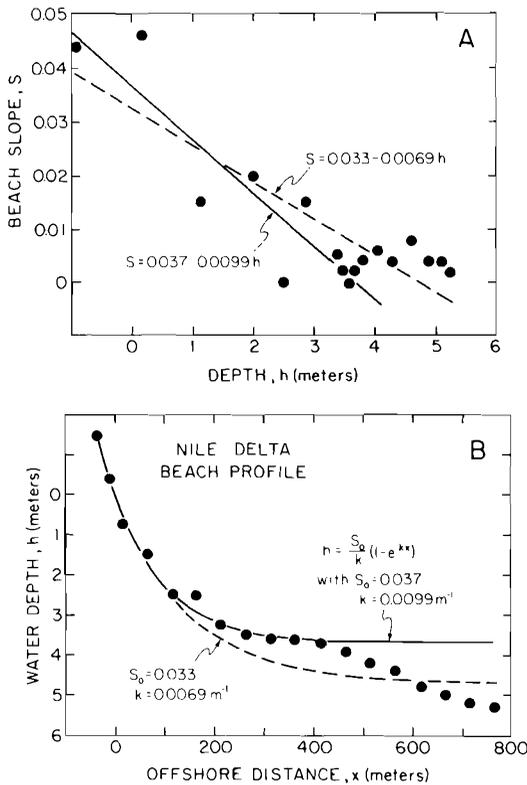
Figure 4. A. An analysis of the cross-shore variations in measured bottom slopes versus the water depths to provide comparisons with the linear relationship of Equation 12b derived from the exponential profile. The solid line is based on a regression of the data out to a depth of 3.7 meters, corresponding to a 400 meter offshore distance, while the dashed line is for the entire measured profile. B. The corresponding calculated beach profiles based on the evaluations of S_o and k found in A.

$$S_o = 0.25\left(\frac{D}{L_\infty}\right)\left(\frac{H_\infty}{L_\infty}\right)^{0.15} \qquad (16)$$

Of significance is that S_o is reasonably predictable or can be easily measured, leaving only an assessment of k in the exponential profile relationships for a particular application.

As discussed earlier, a logical basis for the assessment of k comes from the offshore closure depth of profile changes. It is observed that with repeated profiles obtained at a fixed location, vertical variations are greatest in the nearshore, particularly where bar-trough systems develop, with the envelope of change rapidly thinning and pinching out toward the offshore. HALLERMEIER

(1981) has examined this zonation and attempted to relate it to the annual wave climate. The "close-out depth" h_c, the limit of the zone of extreme bottom changes for quartz sand beaches, was found to be given by

$$h_c = 2.28H_e - 68.5\left(\frac{H_e^2}{gT_e^2}\right) \qquad (17)$$

where H_e is the nearshore storm wave height that is exceeded only 12 hours per year and T_e is the associated wave period. This is in effect a dependence on the wave height, with an adjustment for the wave steepness. BIRKEMEIER (1985) has compared this relationship with profile variations at the Field Research Facility in Duck, North Carolina, and found that a replacement of the coefficients in Equation 17 with the values 1.75 and 57.9 yields a better fit to the data. BIRKEMEIER (1985) also noted that the simple proportionality $h_c = 1.57H_e$ provides a satisfactory prediction of the closure depth. For an evaluation of k in the exponential profile, it is necessary to know the approximate offshore distance x_c from the shoreline to the closure depth. Alternately, as outlined earlier, the assessment of k can be based on the bottom slope at the closure depth.

A family of curves generated by this approach is presented in Figure 5, providing an example that focuses mainly on the effects of varying the sediment size and hence the beach-face slope. In the evaluation of S_o from Equation 15, we have arbitrarily set $H_b = 2$ meters and $T = 10$ seconds, so that the remaining variations depend only on the sediment diameter D; we could as easily have set the value of D and varied H_b and T to examine profile variations at one beach location resulting from changing wave conditions. In the example graphed in Figure 5, the closure depth is set at $h_c = 8$ meters, which corresponds to $H_e = 5.1$ meters according to the simplified $h_c = 1.57H_e$ proportionality determined by BIRKEMEIER (1985). The offshore distance to this closure depth is set at $x_c = 500$ meters, such that $h_c/x_c = 5.1/500 = 0.0102$. In that the value of k in the exponential profile depends on $(h_c/x_c)/S_o$ as given in Figure 1, k will vary with S_o and thus with the sediment grain size D. The series of exponential profiles graphed in Figure 5 range from steep, reflective beaches for large D, to dissipative beaches for small sediment diameters. The corresponding Iribarren Number values have been calculated, using the form $\xi = S_o/(H_\infty/L_\infty)^{1/2}$ based on the beach-face slope and deep-water wave parameters.
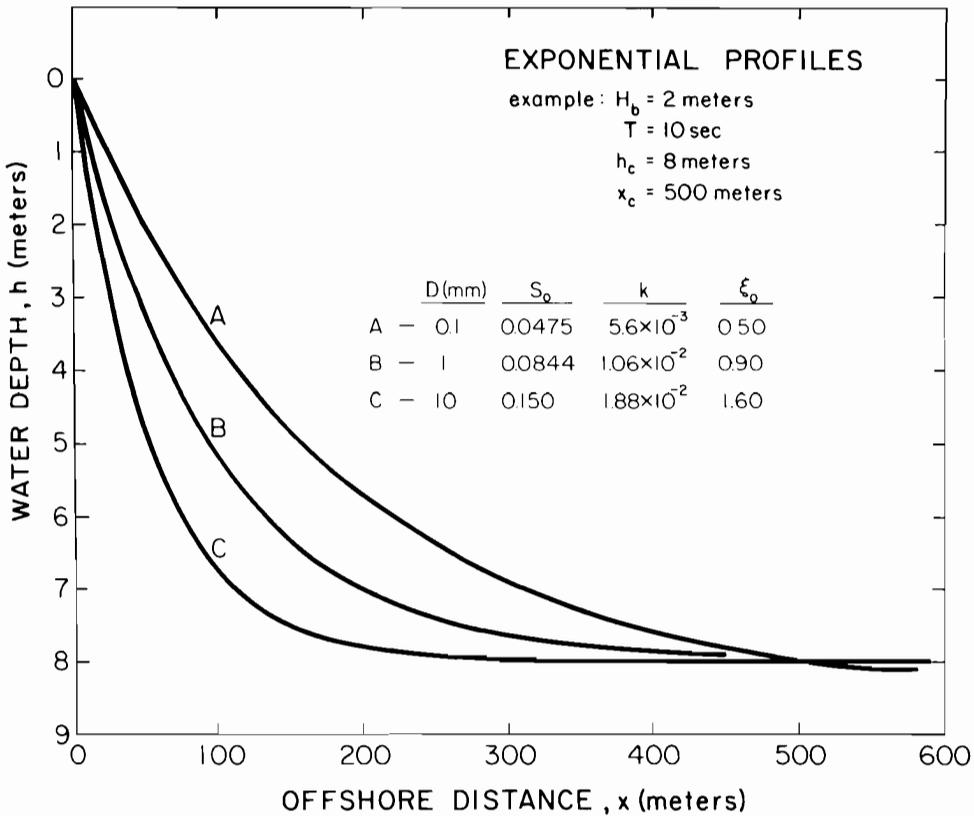
Figure 5. Three exponential profiles calculated for the wave conditions shown. The closure depth is based on the relationship of BIRKEMEIER (1985), while Equation 15 from SUNAMURA (1984) was used to calculate the S_o beach-face slopes for the three sediment grain sizes, D. The k coefficients were determined from the graph in Figure 1. The three examples range from dissipative (A) to reflective (C) as shown by their respective Iribarren numbers, ξ.

The three profiles in Figure 5 are seen to level off to fixed offshore depths, which in the exponential profile is equal to S_o/k [Equation 5]. The more reflective the beach, the closer inshore that depth is effectively reached. However, in some applications the profile will approach an offshore slope which the bottom maintains over the inner continental shelf. It is simple to match the exponential profile with that offshore slope. This is done in Figure 6 for the same conditions used in the example of Figure 5, except that the offshore boundary condition is a bottom slope $S_t = 0.010$ at the closure depth $h_c = 8$ meters. In this case it is not necessary to specify the offshore distance x_c of the closure depth, since that can be calculated from the exponential profile relationship of Equation 6, giving

$$x_c = \frac{1}{k} \log_e (S_o/S_t) \tag{18a}$$

which differs for each profile in Figure 6, being approximately twice as great for the dissipative profile A compared with the reflective profile C. Equation 12b permits the calculations of k for the combination of beach face and offshore slopes:

$$k = \frac{1}{h_c} (S_o - S_c) \tag{18b}$$

The value for k also differs for each profile in Figure 6 due to the dependence on S_o which in turn depends on the sediment diameter, D.

It is apparent that the calculation of exponential beach profiles, based on known dependencies of the beach-face slope and closure depth on wave
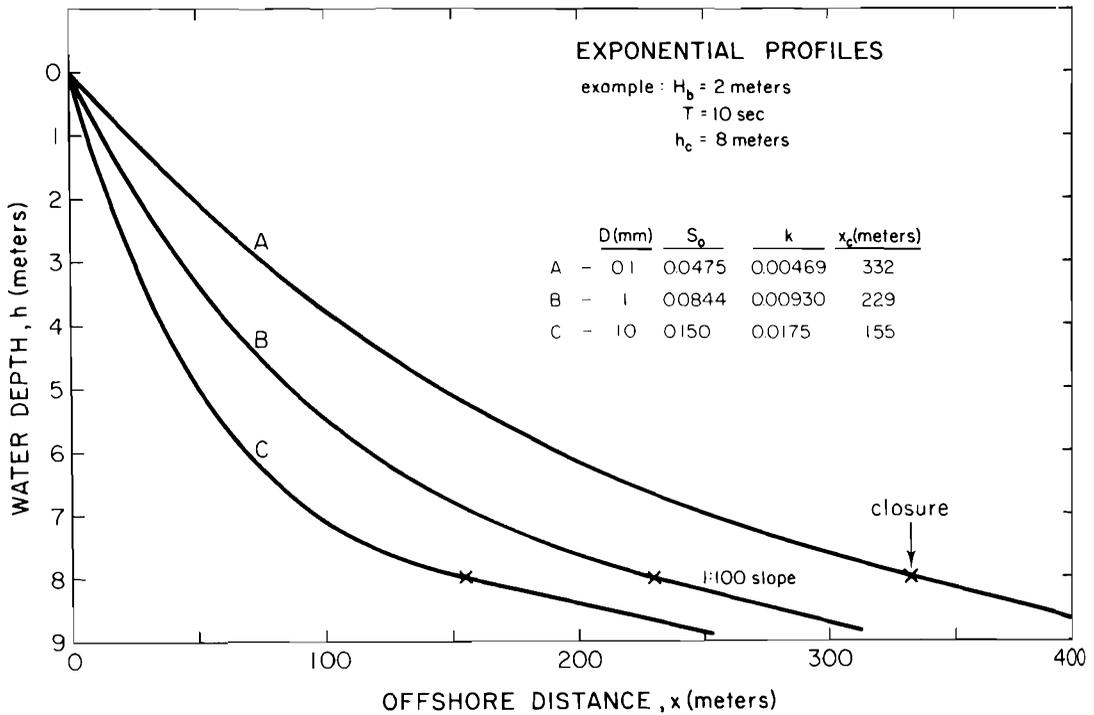
Figure 6. Example exponential beach profiles corresponding to the conditions used in Figure 5, but where the profile is matched with an offshore bottom slope S, = 0.010 at the closure depth h, = 8 meters.

and sediment conditions, or matching the profile to an offshore slope, is a straight-forward procedure. The resulting profiles can then be used in analyses of cross-shore wave transformations, of longshore current and sediment transport distributions, and in applications examining profile elevation changes in response to beach nourishment projects or due to elevated water levels.

SUMMARY AND DISCUSSION

The overall exponential form of beach profiles has been explored with application of the results to a typical beach profile from the Nile Delta. The general exponential form was employed by BODGE (1992), who utilizing a large data set, demonstrated that it shows better agreement with measured beach profiles than does the Bruun/Dean x ' profile. The form we are using, Equation 5, is somewhat different than that used by BODGE (1992) in that it depends directly on the slope of the beach face, S_0, and contains only one adjustable coefficient, k, which governs the degree of concavity of

the profile. Calculations of exponential beach profiles, therefore, can be based on the many past studies that relate S to beach sediment grain sizes (or settling velocities) and wave conditions. Furthermore, as demonstrated here, the evaluation of k can be based on the coordinates of the closure depth of profile change or equally on some arbitrarily chosen offshore depth coordinate (x_c, h_c). The beach-profile expression presented here in essence is an exponential decrease in the local bottom slope in the offshore direction from its S_0 value on the beach face, the dependence given by Equation 6, or the linear decrease in slope with offshore depth h as given by Equation 1. This opens the possibility of analyzing beach profiles in terms of their offshore variations in slopes as well as water depths.

The example analysis of a concave, reflective beach profile from the Nile Delta has illustrated the general procedures involving both cross-shore variations in water depths and bottom slopes. In all analyses, good agreement was found with re-

lationships based on the exponential profile, particularly within the inner portion of the profile out to a depth of 3.5 meters and offshore distance of 400 meters, the zone expected to be under the more active influence of waves on the low-energy Nile Delta coastline. The respective analyses showed that the results are relatively insensitive to the evaluated beach-face slope, S_o, but are sensitive to the exponent coefficient k which governs the overall curvature of the profile and calculated depths in the offshore. It is, therefore, important in applications of the exponential profile to rationally determine what portion of the profile should be used in the regression analyses. In some cases it may be desirable to have different k values for the prediction of water depths and cross-shore variations in the bottom slope.

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