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Numerical Simulation of Shoreline Evolution Using A One Line Model

Jayakumar and R. Mahadevan

Ocean Engineering Centre
Indian Institute of Technology
Madras 600 036, India

ABSTRACT

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Two numerical schemes for the solution of a one-line model equation for shoreline evolution are reviewed. The influence of the order of occurrence of the combination of ocean wave parameters, *viz* wave height, direction and period which do not follow a predictable pattern, on the shoreline evolution are studied. Case studies of shoreline evolution adjacent to Madras and Beypore harbours on the east and west coasts of India respectively are presented. Limitations of the one-line simulation model in predicting the shoreline evolution are indicated.

ADDITIONAL INDEX WORDS: *Shoreline evolution, numerical simulation, one line model, case study, Madras and Beypore, India*

INTRODUCTION

Coastal Engineers frequently encounter the problem of changing shorelines in regions where variations in the wave climate and longshore littoral transport rates result from construction of coastal structures (such as breakwaters, groynes, jetties, *etc.*) which act as partial barriers to wave propagation and littoral transport. A review of littoral transport, shoreline configurations and their evolution may be found in BRUUN (1990).

Most studies of shoreline simulation are based on the one-dimensional sediment balance equation which is referred to as the one-line model equation. LEMEHAUTE and SOLDATE (1980) obtained the analytical solution to the linearized form of the one-line model equation, for a shoreline which is initially straight with a breakwater projecting perpendicular to it when unidirectional waves of constant amplitude approach the shoreline. But, because the general one-line model equation is nonlinear and the environmental conditions are complex (due to the variations in the wave field and consequent long shore littoral transport rates), the simulation of shoreline evolution requires numerical methods for its solution.

In this paper, the numerical schemes of LEMEHAUTE and SOLDATE (1980) and KRAUS and HARIKAI (1983) for shoreline simulation are reviewed. When numerical schemes are used for long-term prediction of shoreline evolution, the forecast of wave statistics is required in the form of a time series. Unfortunately, it is seldom possible to know in advance the exact sequence in which the wave parameters will occur in an ocean environment. Hence, numerical experiments are conducted in an effort to study the model response to changes in a wave parameter sequence.

Two case studies of shoreline evolution, one adjacent to Madras harbour and the other near Beypore harbour on the east and west coasts of India (Figure 1) are presented. The instabilities observed in the evolution of the shoreline for certain input wave parameters and the limitations of the one line model are explained.

Model Formulation

The mathematical modelling of shoreline evolution correlates the change of beach volume due to the change in shoreline to the rate of material transported from the beach along the shore. When changes in the mean sea level and the changes in the beach slope are not significant, the one-dimensional sediment balance equation (after LEMEHAUTE and SOLDATE, 1980) is given by

$$(b + h_c)(\partial y/\partial t) = -(\partial Q/\partial x) + q(x) \qquad (1)$$

In this formulation, the orientation of the coordinate system is chosen so that the x-axis lies roughly parallel to the beach. The shoreline is represented by y(x, t), where t is the time, b is

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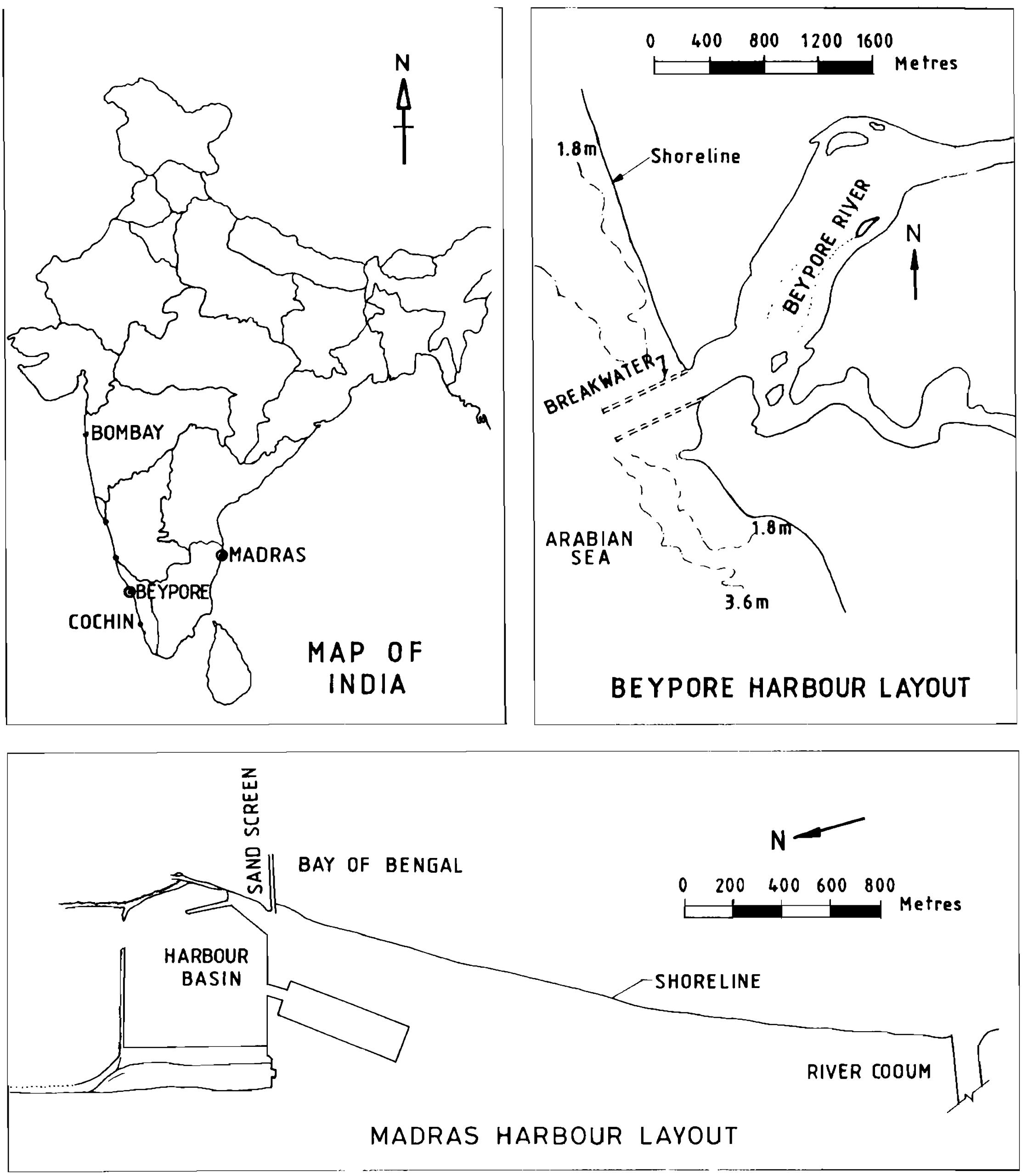

Figure 1. Location map.

the height of the beach berm, h_c is the water depth at the limit of active sand transport beyond which profile changes can be assumed to be negligible and q(x) is the quantity of sand dredged or deposited along the beach.

The long shore littoral transport rate Q in Equation 1 which varies along the shoreline is responsible for the evolution of the shoreline. There are several formulae for the estimation of Q which relate Q with the angle of approach of

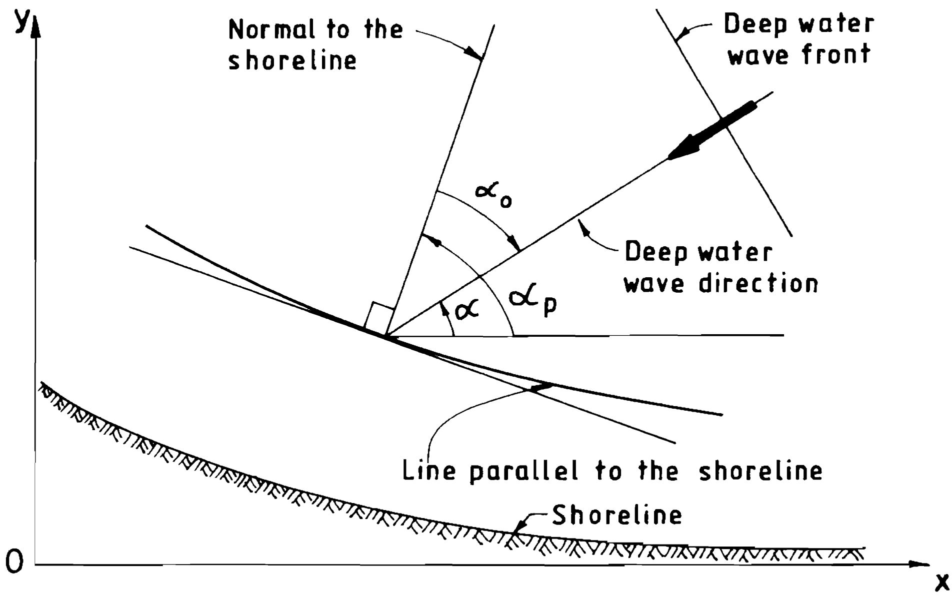

Figure 2. Sketch defining angles α, α_o and α_p.

incident waves and their energy. But BRUUN (1990) points out that it is more realistic to expect the littoral drift rate to be related to beach profile and beach material characteristics in addition to the parameters of the approaching waves. The most recent formula is of KHAMPHUIS (1991) given by

$$Q = (\rho H^3/T)(H/L_o)^p m^q (H/D_{50})^r \sin^s(2\alpha_b) \quad (2)$$

where H and T are the wave height and period, ρ = fluid density, L_o = deep water wave length, D_{50} = characteristic grain size, m = beach slope, α_b = breaking wave angle and p, q, r and s are exponents. The breaker angle α_b and the bed slope change with time and along the shoreline as it evolves and therefore their time histories are needed to accurately estimate the changing longshore transport.

In the present study, the average wave parameters obtained from the wave roses reported in the quarterly publications of the ports are used in the computation of longshore transport rate. As the wave roses give only the direction of approach of waves in deeper waters, the breaker angles are estimated using the empirical relation (LEMEHAUTE and KOH, 1967)

$$\alpha_b = [0.24 + 5.5(H_o/L_o)]\alpha_o \quad (3)$$

where L_o and α_o are respectively the deep water wave length and wave angle with respect to the seaward normal to the shoreline. The deep water wave angle α measured counter clockwise from x-axis is related to α_o (Figure 2) by

$$\alpha_o = \alpha - \tan^{-1}(dy/dx) \pm \pi/2 \quad (4)$$

As the wave parameters used in the present computations are only statistical averages and as there are no records of the beach profiles and beach material characteristics, the longshore transport Q in Equation 1 is estimated using the relation

$$Q = AK_r^2 K_d^2 \cos\alpha_b \sin\alpha_b \quad (5)$$

which essentially has the same form as that of Equation 2. In Equation 5, $A = A_1 H_o^2 T$ and A_1 is a dimensional constant which depends on the units used for the deep water wave height H_o and wave period T. $K_r^2 = (\cos\alpha_o/\cos\alpha_b)$ and K_d are respectively the refraction and diffraction coefficients at the breaking point. $K_d = 1$ for $\alpha \leq \pi/2$. For $\alpha > \pi/2$ and in the diffraction region of the breakwater, K_d is estimated by an approximate procedure (see chapter 4, DEAN and DALRYMPLE, 1984).

Expressing the variables in the nondimensional form Equations 1 and 2 become

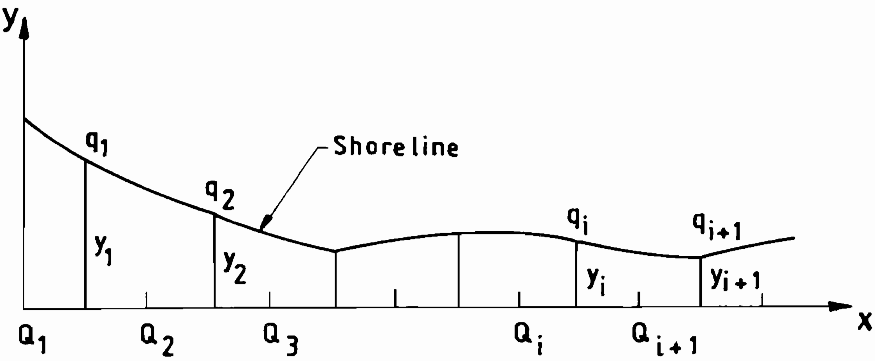

Figure 3. Grid specification for finite difference schemes.

$$(\partial\bar{y}/\partial\bar{t}) = -(\partial\bar{Q}/\partial\bar{x}) + \bar{q}(\bar{x}) \quad (6)$$

and

$$\bar{Q} = Q/A = K_d^2\cos\alpha_o\cos\alpha_b \quad (7)$$

where $\bar{x} = x/(b + h_c)$, $\bar{y} = y/(b + h_c)$, $\bar{t} = At/(b + h_c)^3$. An alternative form to Equation 7 given by

$$(\partial\bar{Q}/\partial\bar{t}) = a(\partial^2\bar{Q}/\partial\bar{x}^2) - a(\partial\bar{q}/\partial\bar{x}) \quad (8)$$

where

$$a = K_d^2(\beta\cos\alpha_o\cos\alpha_b - \sin\alpha_o\sin\alpha_b) \div [1 + (d\bar{y}/d\bar{x})^2]$$

is derived by differentiating Equation 6 with respect to x and using Equation 4. In subsequent discussions, the overbars on all nondimensional variables will be dropped.

Numerical Schemes

Scheme-A: In this scheme, the non-dimensional equations of sediment balance and longshore transport (Equations 6 and 7) are expressed in finite difference form (KRAUS and HARIKAI, 1983). Using a staggered grid system, Equation 6 in implicit finite difference form is given by

$$Y_{n,t+1} = B(Q_{n,t+1} - Q_{n+1,t+1}) + C \quad (9)$$

where $B = \delta t/(2\delta x)$ and $C_n = Y_{n,t} + B(Q_{n,t} - Q_{n+1,t} + 2\delta x q_{n,t})$.

Note that in the staggered grid system, the set $\{Q_n\}$ is specified at the grid points, whereas the sets $\{q_n\}$ and $\{Y_n\}$ are defined at the centre of the grid spacing (Figure 3). δx is the distance between two consecutive grid points, δt is the time interval chosen for the integration and subscript t refers to quantities at time step t. Then the longshore transport Q at the time step (t + 1) is expressed in terms of Y, by first isolating the terms involving α_p (α_p is defined in Figure 2). Expressing one of the terms involving α_p as a first order quantity in Y at the time step (t + 1), we get

$$Q = K_d^2\cos(\alpha - \alpha_p)\sin\alpha_b$$

$$= K_d^2(\cos\alpha\sin\alpha_p\cos\alpha_p + \sin\alpha\sin\alpha_p)\sin\alpha_b$$

and

$$Q_{n,t+1} = E_n(Y_{n-1,t+1} - Y_{n,t+1}) + F_n \quad (10)$$

where

$$E_n = (K_d^2\cos\alpha\sin\alpha_{p,t}\sin\alpha_{b,t})/\delta x$$

and

$$F_n = K_d^2\sin\alpha\sin\alpha_{p,t}\sin\alpha_{b,t}$$

As the boundary conditions are expressed in terms of Q, by eliminating Y between Equations 9 and 10, we get the difference equation as

$$BE_nQ_{n-1,t+1} - (1 + 2BE_n)Q_{n,t+1} + BE_nQ_{n+1,t+1} = E_n(C_n - C_{n-1}) - F_n \quad (11)$$

For n = 2 to N, Equation 11 represents a set of (N − 1) linear equations in (N − 1) unknowns. The end values Q_1 and Q_{N+1} are specified by boundary conditions. For a breakwater which prevents longshore transport, $Q_1 = 0$. At the other boundary, it is assumed that $Q_{N+1} = Q_N$; *i.e.*, $(\partial Q/\partial x) = 0$ for large x. This linear system of equations is in tridiagonal form which can be solved using standard procedures. Then the set $\{Y_{n,t+1}\}$ is determined using Equation 9, and this procedure is repeated to simulate the evolution of shoreline.

Scheme-B: In this scheme, the finite difference equation is formed by discretizing Equation 8 (LEMEHAUTE and SOLDATE, 1980). In the staggered grid system, the difference equation in implicit form is given by

$$Q_{n,t+1} = (Q_{n-1,t+1} - 2Q_{n,t+1} + Q_{n+1,t+1}) \cdot (a\delta t/2\delta x^2) + F_n \qquad (12)$$

where

$$F_n = Q_{n,t} + (Q_{n-1,t} - 2Q_{n,t} + Q_{n+1,t})(a\delta t/2\delta x^2) - (q_{n,t} - q_{n-1,t})(a\delta t/\sigma x)$$

and a is a function of shoreline coordinates $\{Y_n\}$ which is calculated at the previous time step. For n = 2 to N, Equation 12 forms a linear system of equations similar to that in Scheme-A.

Numerical Experiments

Before using numerical schemes for case studies, we have first used them to simulate the evolution of shoreline adjacent to a breakwater with waves from deep water approaching the coast continuously from one direction. A typical simulated shoreline obtained using both the schemes is presented in Table 1. Computations carried out for various deep water wave directions show that, for numerical stability, Scheme-A generally requires a smaller integration time step δt than that required by Scheme-B for the same spatial descretization interval δx. Hence, in subsequent numerical experiments, Scheme-B is used.

For case studies on shore line evolution, the input required is the wave parameters (α, T, H_0) in the form of time series. This is seldom available and the required information has to be derived from the monthly statistical summaries or wave roses of the near shore region. These monthly summaries are used to estimate the average wave direction, average wave height and period. But in estimating these averages, we lose information on the sequence in which the combination (α, T, H_0) occurs. In order to study the effect of change in sequence of occurrence of wave parameters on the shore line evolution, numerical experiments are carried out. For all the experiments, initial shoreline is taken to be straight with a shore-connected breakwater perpendicular to the coast and q(x) = 0.

In the first experiment, the waves are assumed initially to approach the breakwater at a constant

Table 1. *Shoreline coordinates after (non-dimensional) $\bar{t}$ = 400.*

		Shoreline Coordinates	
		y-Coordinate	
No.	x-Coordinate	Scheme-A (K & H)	Scheme-B (LMS)
1	2.5	6.8023	6.8014
2	7.5	3.5048	3.5049
3	12.5	1.4497	1.4500
4	17.5	0.3851	0.3848
5	22.5	0.0522	0.0521
6	27.5	0.0041	0.0061
7	32.5	0.0006	0.0019
8	37.5	-0.0005	0.0011
9	42.5	0.0005	0.007
10	47.5	0.004	0.005
11	52.5	0.0002	0.0005
Non-dimensional integration time step		0.20	0.50

Note: Shoreline coordinates are non-dimensionalised with respect to (b + D_c). Real time t = $\bar{t}$(b + D_c)^3/A. Non-dimensional grid spacing δx = 5. Wave parameters: α = 45°. T = 8 sec and H_0 = 0.5 m

angle of 45° to the shoreline for a period of 6 months. Subsequently for the next six months, the wave angle is changed to 60°. The other two wave parameters, namely, average wave height and the wave period are taken as 0.5 m and 8 sec respectively. The computations are repeated reversing the order of occurrence of incident wave angles, but keeping the other wave parameters the same. From the simulated shoreline profiles presented in Table 2, it can be seen that the order in the sequence of occurrence of incident wave direction has significant influence on shoreline evolution closer to the breakwater. Hence, these models require the exact sequence of occurrence of wave parameters over long periods of time (several months and years) for the shoreline simulation. But this data requirement for the shoreline forecasting cannot be met from wave forecasting models as they predict the wave field only for a short period (a few days). Also, the exact sequences of observed wave data are not published by Port authorities; they publish only the summaries of wave data in the form of wave roses which are again not in the required form for shoreline hindcasting. Therefore, one has to be cautious in interpreting the predicted shoreline close to the breakwater, obtained from numerical models.

Similar numerical experiments are performed to study the effect of changes in the sequence of

Table 2. *Shoreline coordinates after time t = 12 months using Scheme-B ($\alpha \neq$ constant). Run I: α = 45° for the first six months and 60° for the next six months. T = 8 sec; H_o = 0.5 m. Run II: α = 60° for the first six months and 45° for the next six months. T = 8 sec; H_o = 0.5 m.*

	Shoreline Coordinates		
		y-Coordinate	
No.	x-Coordinate	Run-I	Run-II
1	2.5	8.7987	10.6230
2	7.5	6.3865	6.9358
3	12.5	4.4224	4.2531
4	17.5	2.8864	2.3759
5	22.5	1.7504	1.1609
6	27.5	0.9713	0.4757
7	32.5	0.4871	0.1635
8	37.5	0.2198	0.0508
9	42.5	0.0896	0.0154
10	47.5	0.0333	0.0042
11	52.5	0.0114	0.0014

Note: Shoreline coordinates are non-dimensionalised with respect to ($b + D_c$)

occurrence of other wave parameters, *viz.* wave heights and wave periods, on shoreline evolution. No significant differences in the shoreline evolution over the same period (12 months) is observed when the shoreline was simulated considering the occurrence of these wave parameters (wave height or period) in one particular order or in the reverse order, keeping the other parameters constant.

Case Studies

Madras Harbour

The Madras harbour which lies on the east coast of India (Figure 1) was developed along a straight open coastline by the construction of breakwaters perpendicular to the shore.

A case study of the shoreline evolution adjacent to the Madras harbour is carried out using the numerical Scheme-B. The input data used in this simulation are listed in Table 3. The average wave parameters are extracted from the annual hydrographic survey reports released by the Madras Port Trust Authorities. The shoreline coordinates south of Madras harbour corresponding to the March, 1977, survey are taken to form the initial shoreline. From the details of annual accretion and erosion of material outside the harbour and dredging particulars available from the Port Authorities, the percentage of sediment by-passing the breakwater is computed.

The simulation of shoreline is carried out for a

Table 3. *Parameters used in the simulation of shoreline adjacent to Madras Harbour.*

Depth of water at the limit of active sand transport (h_c)	12.20 m	
Height of berm (b)	3.05 m	
Length of breakwater	788 m	
Depth of water at the seaward end of breakwater	3.05 m	
No. of grid points (N)	50	
	1977 Pre-Monsoon	1977 Post-Monsoon
Average wave direction ($\bar{\alpha}$) (with respect to the line perpendicular to the break water)	59.6°	105.8°
Average wave period ($\bar{T}$)	8 sec	8 sec
Average wave height ($\bar{H}$)	0.95 m	1.28 m
Percentage of sediment by passing	44.7	27.09

period of three years. During that period, no major change in the shoreline was reported in the annual survey reports. The predicted shoreline for the year 1978 is found to agree only qualitatively with the measured shoreline of the annual 1978 survey (Figure 4).

A possible reason for getting only a qualitative agreement between the measured and the predicted shoreline is that when the average wave parameters are used as input to the simulation model, the average measured shoreline should have been used as the initial shoreline to estimate the evolution of the average profile. Instead, in the present investigation, we used the measured shoreline as the initial profile and the average wave parameters as input to the model.

Further studies are carried out to investigate the effect of waves which make a large incident angle in deep water on simulation (*i.e.*, the effect of large α_o). Taking the 1977 annual shoreline south of Madras harbour as the initial profile, the shoreline is simulated for waves approaching at an angle of 55° (in deep water) with the normal to the shoreline at the extreme southern end of the study area (Figure 5). The simulated shoreline after six months from the 1977 annual profile showed instabilities at certain locations along the beach (locations A and B in Figure 5). In this context, it is worthwhile to note the following:

(a) For a given wave height and wave period, the sediment transport rate (Q in Equation 5) increases continuously with α_o, and attains the maximum value around α_o = 50° (Figure 6).

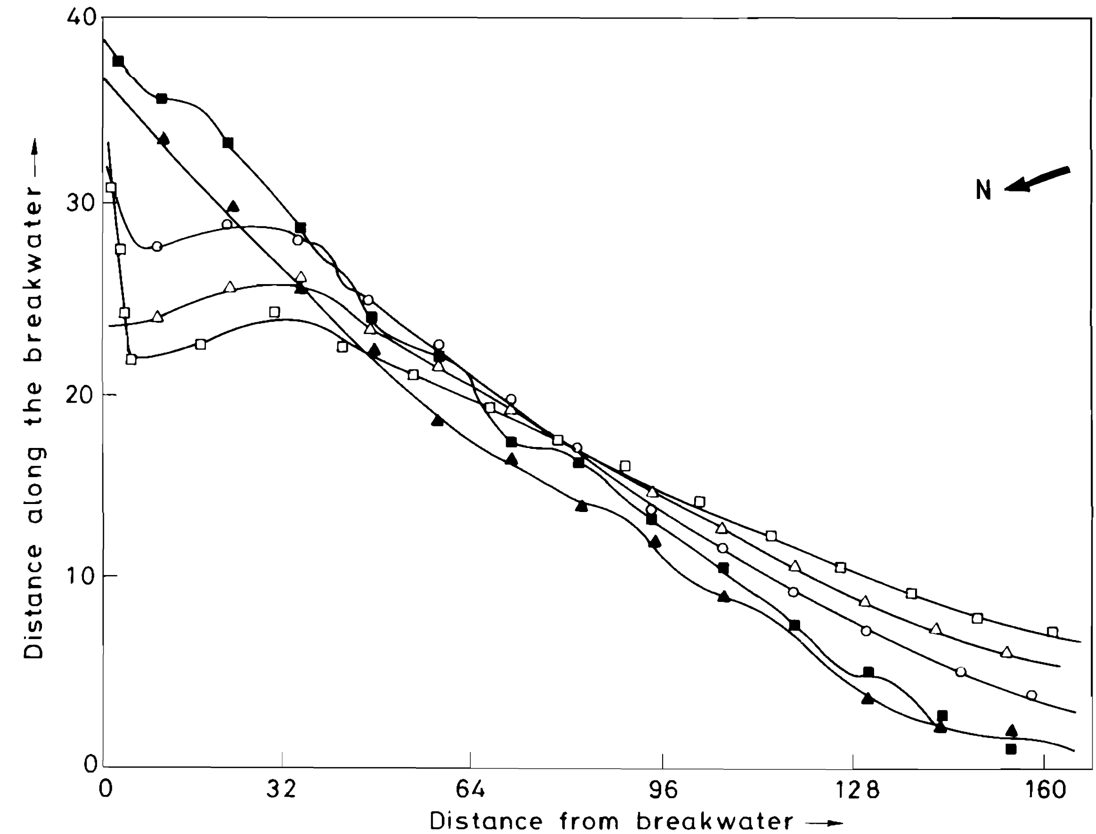

Figure 4. Predicted and observed shorelines south of Madras Harbour. Distances are non-dimensionalised with respect to (b + h_c). □—Initial shoreline (1977) measured, ○—Predicted shoreline (6 months), ○—Predicted shoreline (12 months), △—Measured shoreline (12 months), △—Predicted shoreline (24 months), ■—Predicted shoreline (36 months).

For the shoreline corresponding to the year 1977, α_o can be seen (Figure 5) to reduce continuously towards the breakwater from the value assumed (*i.e.*, 55°) at the southern end of the study area. This implies that inside the study area where $\alpha_o = 50°$, the sediment transport would be maximum and that transported into this area through the southern end would be less than the maximum value, since at this end $\alpha_o = 55°$. This obviously leads to the instability in the shoreline evolution at certain locations (locations A and B in Figure 5) where the sediment transport rate becomes maximum.

(b) In one-line simulation models, approximate methods are used to estimate the refraction co-efficient and the wave breaker angle in the nearshore region. In the present investigation, the breaker angle α_b is estimated using LEMEHAUTE and KOH's (1967) approximate formula which is only valid for small deep water angle α_o, since it predicts α_b to increase monotonically with α_o. But in reality, α_b will not be increasing uniformly without any limit since the wave fronts approaching the shoreline always tend to align themselves parallel to the shoreline even for large deep water wave angles. Hence, an accurate method of estimation of α_b is needed especially for large α_o, to get improved estimates of littoral transport rate and shoreline evolution.

The present study suggests that since the phenomena of refraction, wave breaking and reflec-

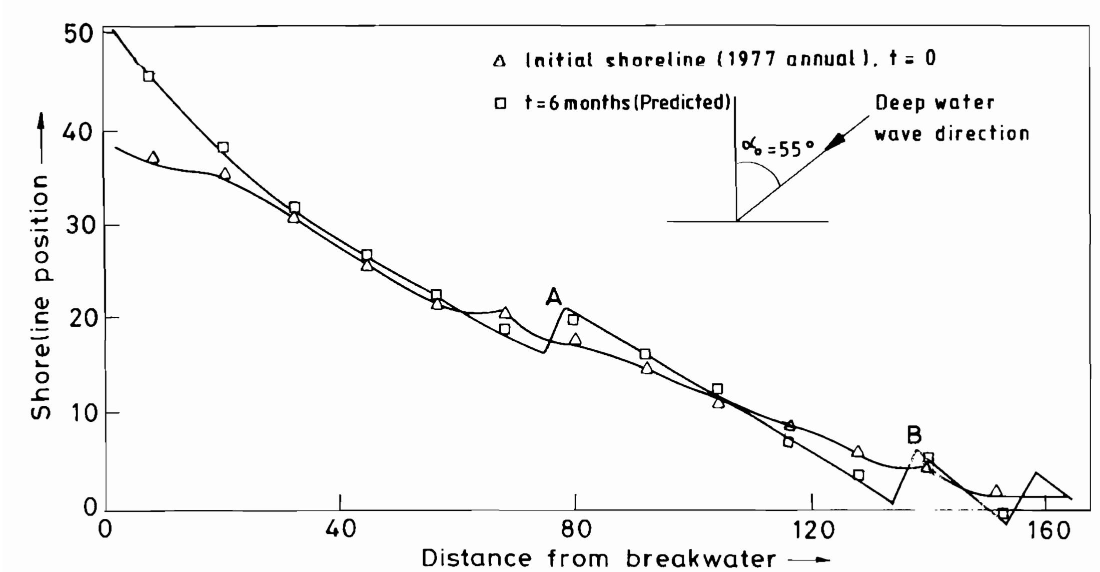

Figure 5. Predicted shoreline south of Madras Harbour for α = 55°. Distances are non-dimensionalised with respect to (b + h_c).

tion from breakwaters and entrance channel are not represented properly in the one line models, these models have serious limitations in their applicability to realistic environmental conditions.

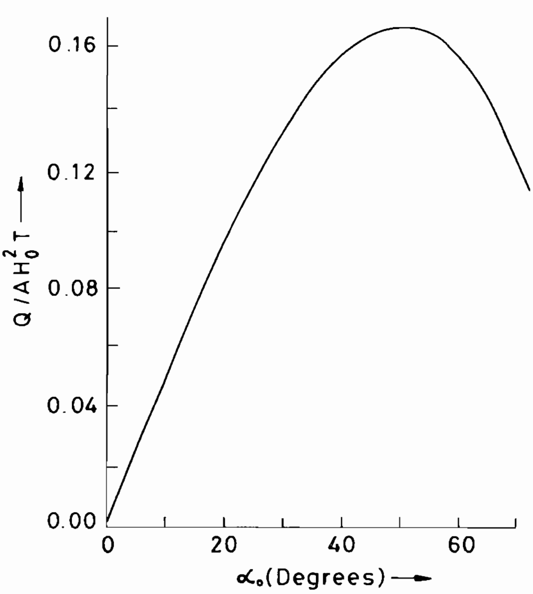

Figure 6. Variation of Q/AH_o^2T with α_o.

Beypore Harbor

Beypore Harbour on the west coast of India is located near the inlet of the Beypore estuary (Figure 1). The inlet is flanked by two parallel breakwaters (constructed in the year 1984) to maintain the navigability of the approach channel to the harbour. The coastline on the southern side of the inlet channel consists of rocky outcrops whereas on the northern side it consists of sandy beaches. The evolution of this sandy shoreline as a con-

Table 4. *Parameters used in the simulation of shoreline adjacent to Beypore Harbour.*

Depth of water at the limit of active sand transport (h_c)	3.47 m	
Height of berm (b)	1.50 m	
Length of breakwater	483 m	
Depth of water at the seaward end of the breakwater	2.01 m	
No. of grid points (N)	50	
	January to June	July to December
Average wave direction ($\bar{\alpha}$) (with respect to the line perpendicular to the break water)	86.2°	83.4°
Average wave period ($\bar{T}$)	10.5 sec	10.9 sec
Average wave height ($\bar{H}$)	0.50 m	0.64 m

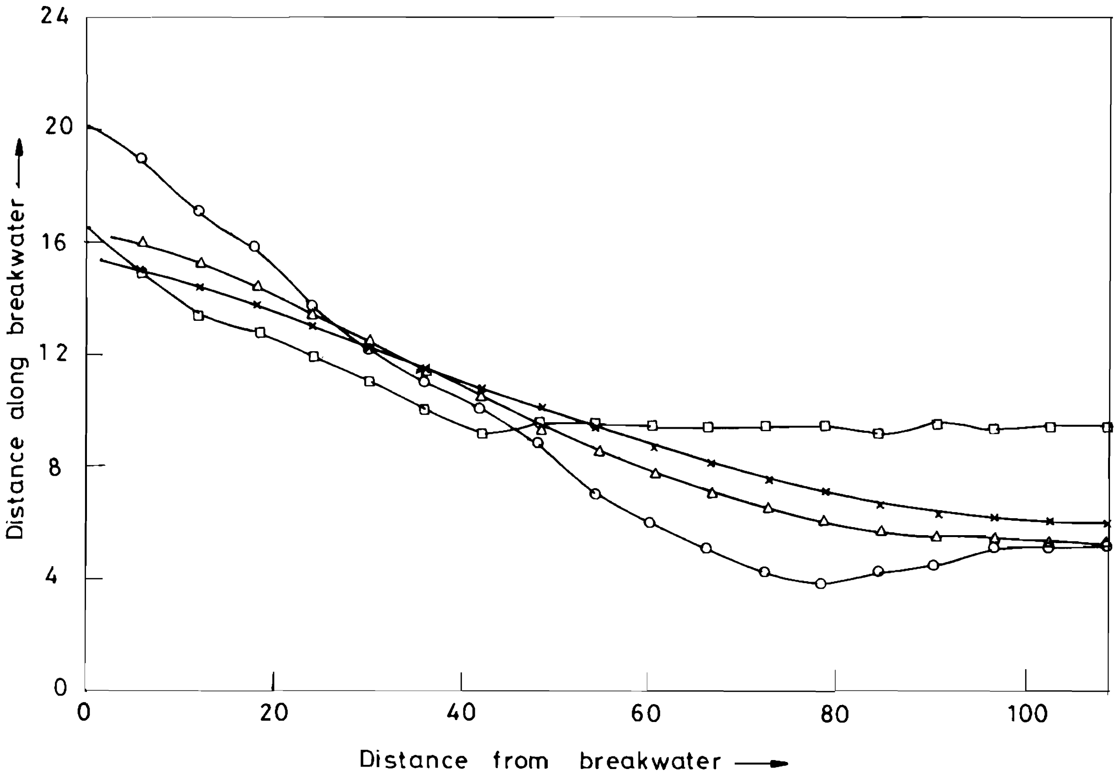

Figure 7. Predicted and observed shorelines north of Beypore Harbour. Distances are non-dimensionalised with respect to (b + h,). O—Initial shoreline (1984) measured, △—Predicted shoreline (1991), □—Measured shoreline (1991), X—Predicted shoreline (1999).

sequence of the breakwater construction is studied.

The input data, used in the simulation (Table 4) are extracted from the hydrographic survey reports published by Port authorities. The initial shoreline considered for this simulation study is the shoreline profile measured during December, 1984.

Figure 7 shows the initial shoreline (1984) and the measured and simulated shorelines of 1991. The simulated shoreline of 1991 can be seen to retreat from the 1984 shoreline near the breakwater and to advance into the sea away from the breakwater and agrees qualitatively with the measured shoreline of 1991. The simulated shoreline of 1999 (Figure 7) shows no significant deviations from that of 1991 which is to be expected as the annual average wave direction is nearly normal to the coast.

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