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Tidal Energy Dissipation in Well-mixed Estuaries

Panagiotis D. Scarlatos

Civil Engineering Program
Department of Ocean Engineering
Florida Atlantic University
Boca Raton, FL 33431, U.S.A.

ABSTRACT

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Long water waves are strongly distorted and lose substantial energy as they propagate through shallow estuarine waters. The energy dissipation is mainly due to increased bottom frictional effects. Using the linearized Telegrapher's equations, an analytical expression is obtained for tidal energy dissipation in terms of estuarine geometric features and tidal harmonic parameters. For estimating the harmonic parameters, the full St. Venant system of equations is numerically solved and a large amount of data is generated for hypothetical one-dimensional, well-mixed estuaries, subject to semi-diurnal tidal action. Relations are then developed expressing the tidal energy dissipation as a function of prevailing macroscopic estuarine features. The effectiveness and applicability limitations of linearization are discussed. Simulation results are verified with actual data. The average tidal energy dissipation was found to be proportional to the third power of the tidal Reynolds number. This result is in agreement with experimental data and theories pertaining to rate of energy dissipation, diffusion in homogeneous turbulent flow and dispersion in shear flow.

ADDITIONAL INDEX WORDS: *Friction, dispersion, harmonic analysis, hydrodynamics, waterways.*

INTRODUCTION

Estuaries are very complicated natural systems subject to a variety of physical effects such as tides, winds, riverine flow, overland runoff, groundwater seepage, direct precipitation, and evaporation (SCARLATOS, 1988). In most cases, however, astronomical tides are the predominant driving force. Therefore, it is very important to quantify the tidal oscillations before initiating any engineering or environmental estuarine study. In spite of the fact that tides can be accurately predicted in the deep ocean (SILVESTER, 1974), their prediction in shallow estuarine waters is very complicated due to the nonlinear effects induced by advection and bottom friction (SCARLATOS and SINGH, 1987a). Theoretically, the propagation of tides can be described by the St. Venant system of partial differential equations. Due to the complexity of these equations (nonlinear PDE of the hyperbolic type), direct solution of the full system is feasible only by means of numerical techniques such as the methods of characteristics (ABBOTT, 1966), finite differences (DRONKERS, 1964), or finite elements (SCARLATOS, 1982). Closed-form solutions can be obtained only by using harmonic analysis; *i.e.*, linearizing the governing equations and decomposing the predominant tidal constituents into a series of periodic functions (DRONKERS, 1964; PROUDMAN, 1957). One of the most advantageous methods of harmonic analysis is the one developed by Ippen and Harleman for channels of infinite and/or finite length (IPPEN, 1966). Analytical solutions for the tidal wave motion can be used to derive closed-form expressions for the energy dissipation. For channels of finite length, this can be accomplished by defining energy dissipation as the difference between the energy of the incident and reflected waves (IPPEN, 1966). Knowledge of the tidal energy dissipation is useful for estimation of other important physical parameters such as the relative diffusion D_{ii} and dispersion coefficient D_L (IPPEN, 1966; SCARLATOS and SINGH, 1985). Although the Ippen and Harleman method has the advantages of simplicity, efficiency, and adequate accuracy, its main drawback is the need for estimation of certain harmonic parameters such as the dimensionless parameter ϕ and friction damping modulus μ. These quantities require actual field data for their calibration and verification (IPPEN, 1966). SCARLATOS and SINGH (1987b) using computer simulation data presented a methodology for calibration of the dimensionless parameter ϕ.

The purpose of this paper is to investigate the behavior of tidal energy dissipation using the Ippen

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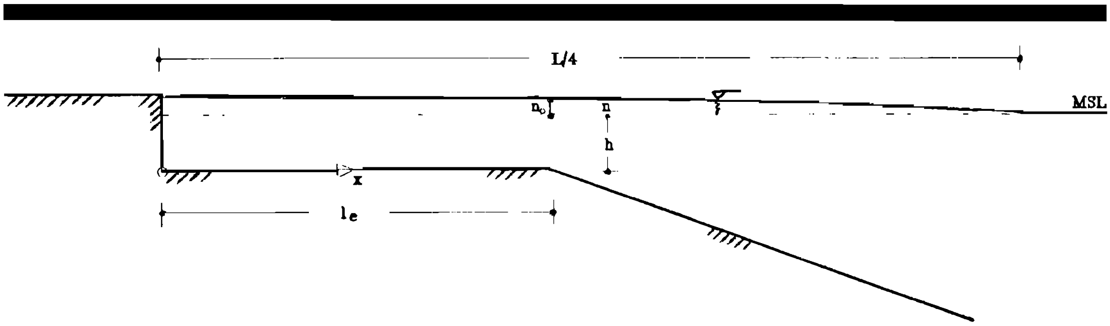

Figure 1. Schematic features of a tidal estuary.

and Harleman harmonic analysis and computer simulation results of the full St. Venant system of equations. The energy dissipation is related to certain macroscopic estuarine and tidal features (*e.g.*, water depth, wave amplitude, bottom friction, current velocities, *etc.*). Energy dissipation in well-mixed estuaries as related to diffusion and dispersion processes is also discussed. The results are compared with available experimental data and existing theories.

ANALYSIS OF TIDAL EQUATIONS

Tidal wave propagation in one-dimensional estuaries can be accurately quantified by the St. Venant system of equations written as

$$A_{,t} + (uA)_{,x} = q \tag{1}$$

$$(uA)_{,t} + (u^2A)_{,x} + gAH_{,x} = gA(S_0 - S_f) + qu \tag{2}$$

where A = wet-cross section area, u = mean velocity of the cross-section, q = lateral inflow, H = water surface elevation measured from a reference datum, S_0 = bottom slope, S_f = energy gradient, and g = acceleration due to gravity. The subscripts x and t represent the independent variables; *i.e.*, longitudinal distance and time respectively. A comma in front of the subscripts denotes partial differentiation with respect to the subscript, *e.g.*, $A_{,t} = \partial A/\partial t$. Equation (1) describes the conservation of mass while Eq. (2) describes the momentum balance. A schematic representation of the estuarine system is given in Figure 1.

For prismatic estuaries without lateral inflow and negligible side effects (*i.e.*, unit width approximation), Eqs. (1) and (2) can be reduced to

$$\eta_{,t} + [u(h + \eta)]_{,x} = 0 \tag{3}$$

$$u_{,t} + uu_{,x} + g\eta_{,x} + gS_f = 0 \tag{4}$$

where h = water depth below mean sea level (MSL), and η = water surface elevation above (or below) MSL. The energy gradient term can be expressed as a quadratic velocity formula:

$$S_f = u|u|/(C^2H^{1+\beta}) \tag{5}$$

where C = coefficient of friction, and β = numerical constant. For $\beta = 0$, the coefficient C equals to the Chezy's coefficient of friction C_z. The absolute value of the velocity in Eq. (5) is used to maintain the directionality of the flow. The system of Eqs. (3) and (4) is nonlinear with respect to the depended variables u(x, t) and η(x, t). For negligible bottom slope; *i.e.*, $S_0 \ll 1$ (h ≈ constant) and small wave amplitude; *i.e.*, $h \gg \eta$ and $\eta/L \ll 1$, where L = wave length, Eq. (3) is linearized as

$$\eta_{,t} + hu_{,x} = 0 \tag{6}$$

Similarly, Eq. (4) can be linearized by omitting the nonlinear advective term and modifying the energy gradient term as

$$S_f = Mu \tag{7}$$

where M = linearized friction factor defined by Parsons (IPPEN, 1966) as

$$M = [8/(3\pi)]u_m/(C_z^2h) \tag{8}$$

where u_m = idealized maximum velocity that can only be a spatial function; *i.e.*, $u_m = u_m(x)$. In terms of the Darcy-Weisbach friction coefficient f, Eq. (8) reads

$$M = [f/(3\pi)]u_m/(gh) = [f/(3\pi)]u_m/c^2 \tag{9}$$

where $c = [g(h + \eta)]^{1/2} \approx (gh)^{1/2}$ = phase velocity. Combination of Eqs. (4), (6) and (7) yields the Telegrapher's equation

$$c^2\eta_{,xx} = \eta_{,tt} + gM\eta_{,t} \quad (10)$$

or

$$c^2u_{,xx} = u_{,tt} + gMu_{,t} \quad (11)$$

Double subscript denotes a second partial derivative with respect to that subscript, *e.g.*, $h_{,tt} = \partial^2 h/\partial t^2$. Assuming a sinusoidal disturbance at the ocean end of the estuary and total reflection conditions at the upstream end of the estuary, the solution of Eq. (10) is given as

$$\eta(x, t) = a[e^{-\mu x}\cos(\sigma t - \kappa x) + e^{\mu x}\cos(\sigma t + \kappa x)] \quad (12)$$

where a = tidal amplitude at the closed end, μ = damping modulus, $\kappa = 2\pi/L$ = tidal wave number, $\sigma = 2\pi/T$ = tidal frequency. L and T are, respectively, the tidal wave length and tidal period. Substituting Eq. (12) into Eq. (6) and integrating with respect to x, the tidal velocity u is defined as

$$u(x, t) = (ac/h)[\kappa_0/(\kappa^2 + \mu^2)^{1/2}] \times [e^{-\mu x}\cos(\sigma t - \kappa x + \alpha) - e^{\mu x}\cos(\sigma t + \kappa x + \alpha)] \quad (13)$$

where $\kappa_0 = (\kappa^2 - \mu^2)^{1/2}$ = wave length corresponding to frictionless bottom conditions, and α = phase shift between tidal wave height and tidal current. Based on the solutions given by Eqs. (12) and (13) the linearized friction factor can be re-defined as

$$M = (\sigma/g)\tan[2\tan^{-1}(\mu/\kappa)] = 2(\sigma/g)\mu\kappa/(\kappa^2 - \mu^2) \quad (14)$$

The total wave energy within the channel is comprised of both kinetic and potential energy. Assuming a channel of length l_e, the average energy dissipation per unit mass E_{Dx} between any station x and the tidal boundary can be estimated as the difference of the energy flux between the incident and reflecting wave (IPPEN, 1966). Thus,

$$E_{Dx} = gca^2[\sinh(2\mu l_e) - \sinh(2\mu x)]/[h(l_e - x)] \quad (15)$$

Based on Eq. (15), the energy dissipation per unit mass for the entire channel E_D is obtained by setting x = 0, *i.e.*,

$$E_D = gca^2\sinh(2\mu l_e)/(hl_e) \quad (16)$$

Eq. (15) can be rewritten as

$$E_D = gc\eta_0^2/\sinh(2\mu l_e) \times \{[(\cos(2\kappa l_e) + \cosh(2\mu l_e)](2hl_e)\} \quad (17)$$

where η_0 = wave amplitude at the ocean end of the channel. From Eqs. (15) to (17), it is evident that whenever $\mu \to 0$, $\sinh(2\mu l_e) \to 0$ and, therefore, $E_{Dx} = E_D \to 0$. For small estuaries where $l_e/L \ll 1$, $\cos(2\kappa l_e) \to 0$. In that case, Eq. (17) is reduced to

$$E_D = gc\eta_0^2\tanh(2\mu l_e)/(2hl_e) \quad (18)$$

Knowledge of the energy dissipation rate is very useful for understanding saline, thermal and solid particle mixing processes in estuarine waters.

EDDY DIFFUSION AND LONGITUDINAL DISPERSION

The flow field in estuarine waters is always turbulent. Under homogeneous turbulent conditions, energy is transferred from large eddies to small eddies through vortex stretching and pairing (energy cascade) (LANDAHL and MOLLO-CHRISTENSEN, 1986). These small eddies are controlled by viscosity and as suggested by Kolmogorov, they are in a state of statistical equilibrium (BATCHELOR, 1953). Assuming u' and λ as the characteristic velocity and length of these small-scale eddies, the energy dissipation rate E is approximated as

$$E \approx \nu(u'/\lambda)^2 \quad (19)$$

where ν = kinematic viscosity. Also, for these small surviving eddies, the Reynolds number is of the order of one, so that inertial forces are in balance with viscous forces, *i.e.*,

$$u'\lambda \approx \nu \quad (20)$$

Combination of Eqs. (19) and (20) yields the well-known Kolmogorov's length scale

$$\lambda \approx (\nu^3/E)^{1/4} \quad (21)$$

Based on Eq. (21) which is valid only for fully developed turbulence, BATCHELOR (1952) derived the coefficient of relative diffusion D_{ii} for a quasi-asymptotic state as

$$D_{ii} = \epsilon E^{1/3}\lambda^{4/3} \quad (22)$$

where ϵ = proportionality constant of order one. In case of solid particle suspensions, the characteristic length λ is indicative of the particle displacement distance within the flow field. Experimental evidence of the validity of Eq. (22) was provided by ORLOB (1961) for a two-dimensional particle diffusion under homogeneous turbulence conditions. MONIN and OZMIDOV (1985) confirmed the 4/3-power law by performing oceanic experiments. Whenever Kolmogorov's length does not change substantially, the relative diffusion coef-

Table 1. *Data of study cases.*

Length of estuary, l_e, in km	45, 60, 75, 90
Depth of estuary, h, in meters	10, 20, 30
Chezy's coefficient, C_z, in $m^{1/2}$/sec	30, 45, 60, 75, 90, 105
Ratio η_0/h	0.025, 0.050, 0.075, 0.100
Tidal period, T, in hours	12

ficient is directly proportional to the one-third power of the energy dissipation rate; *i.e.*,

$$D_{ii} = \epsilon' E^{1/3} \qquad (23)$$

where ϵ' = proportionality constant. In shallow, well-mixed estuaries with regular configuration, the assumption $\lambda \approx$ constant seems to be a reasonable approximation since variability of the size of the vertical eddies is restricted by the water depth. For estuarine flows with a fully developed turbulent boundary layer, the turbulent energy dissipation E can be assumed to represent the energy losses E_D generated by bottom friction; *i.e.*, $E \approx E_D$. In that case,

$$D_{ii} \approx \epsilon' E_D^{1/3} \qquad (24)$$

For one-dimensional parallel flows, the diffusion coefficient is replaced by the dispersion coefficient D_L. The dispersion coefficient incorporates the mixing effects due to the vertical velocity profile whenever an average along the depth velocity is used. Based on TAYLOR's (1954) experiments in flow through pipes, and on the generalized analysis given by FISCHER *et al.* (1979), the dispersion coefficient for tidal flow is given as

$$D_L = 14.3(2g)^{1/2}\nu R_e/C_z = 7.15f^{1/2}\nu R_e \qquad (25)$$

where R_e = tidal Reynold's number defined as

$$R_e = 2(h + \eta)|u_m|/(\pi\nu) \qquad (26)$$

Based on Eq. (5), the rate of energy dissipation per unit mass of fluid within a tidal estuary can be generally estimated as

$$E_D = guS_f = gu^3/(C^2H^{1+\beta}) \qquad (27)$$

Combining Eqs. (25) to (27) and taking the time-averaged velocity over half a tidal cycle; *i.e.*, $u_{av} = (2/\pi)u_m$, the dispersion coefficient reads

$$D_L = 20.22g^{1/6}[(h + \eta)^{4/3}/C_z^{1/3}]E_D^{1/3} \qquad (28)$$

As described in Eqs. (24) and (28), both the relative diffusion and the dispersion coefficients are proportional to the one-third power of the rate of energy dissipation. These conclusions have been verified experimentally (IPPEN, 1966).

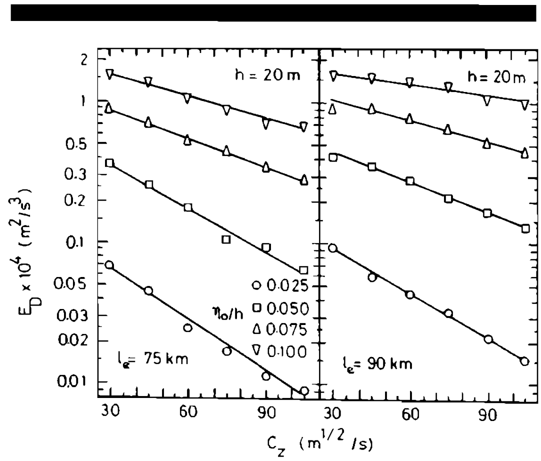

Figure 2. Energy dissipation E_D versus Chezy's coefficient of friction C_z.

METHODOLOGY AND STUDY CASES

The average energy dissipation per unit mass in a tidal estuary can be estimated by means of Eq. (18). Application of Eq. (18), however, requires tidal records at the estuarine mouth, geometric data along the estuary and the frictional damping modulus μ. The damping modulus μ can be obtained only through analysis of tidal data from different stations within the estuary (IPPEN, 1966). For this study, a large amount of data was generated by means of computer simulation using a finite element model. The model used for simulation is described in detail elsewhere (SCARLATOS, 1982; SCARLATOS and SINGH, 1987b). The simulation results provided tidal wave heights η(x, t) and current velocities u(x, t) at a number of stations. From this data, the damping modulus μ and the frictional wave number κ were calibrated according to the Ippen-Harleman harmonic analysis. The estuarine and tidal characteristics of the cases studied were combinations of the quantities given in Table 1. It should be noted, however, that the estuarine length l_e never exceeded 20% of the tidal wave length L.

A tidal period of 12 hours was used for all of the simulations. The 12 hour period is close to the period of the semi-diurnal tidal constituent M1 (12.46 hours). Once the harmonic parameters μ and κ were estimated, the average tidal energy dissipation per unit mass E_D and the linearized friction parameter M were obtained respectively from Eqs. (18) and (14). Knowing the parameter M, the idealized maximum velocity u_m was de-

Table 2. *Values of constants m_1, m_2 ($E_D = m_1 10^{(m_2 C_z)}$).*

	l_e = 75 km, h = 20 m		l_e = 90 km, h = 20 m	
η_0/h (1)	$m_1 \times 10^5$, in m^2/sec^3 (2)	$m_2 \times 10^3$, in $sec/m^{1/2}$ (3)	$m_1 \times 10^5$, in m^2/sec^3 (4)	$m_2 \times 10^3$, in $sec/m^{1/2}$ (5)
0.025	1.594	−12.126	1.988	−10.688
0.050	7.372	−10.375	7.179	−6.761
0.075	13.669	−6.708	15.728	−5.176
0.100	21.679	−4.857	20.234	−2.521

fined, and based on the velocity u_m the tidal Reynolds number R_e was estimated.

Eq. (25) indicates dependency of the dispersion coefficient D_L to the tidal Reynolds number R_e. Thus, the behavior of R_e was investigated with respect to energy dissipation.

RESULTS AND DISCUSSION

Due to space limitations, only part of the simulated data is presented in this paper. Indeed, results are primarily given for two estuarine lengths; *i.e.*, 75 and 90 km for a mean depth of 20 m. However, the results presented in this paper are similar to the results of the unreported simulation cases. Any particular observation pertaining to the unreported cases is included. Tidal energy dissipation as expressed by Eq. (18) depends on linearized bottom frictional effects (μ). However, since the damping modulus μ is used only in harmonic analysis, it is interesting to see the relation of E_D versus the Chezy's coefficient C_z. In Figure 2, data plotted on a semi-log paper yields the relation

$$E_D = m_1 10^{(m_2 C_z)} \qquad (29)$$

where m_1, m_2 = dimensional coefficients. Coefficient m_1 depends on the dimensionless ratio η_0/H; *i.e.*, $m_1 = m_1(\eta_0/H)$. On the other hand, changes

Table 4. *Ratio of u_m/u_{max} for different wave heights and bottom friction.*

	l_e = 75 km, h = 20 m, T = 12 hr			
	u_m/u_{max}			
η_0, in meters:	0.50	1.00	1.50	2.00
C_z, in $m^{1/2}$/sec:	(1)	(2)	(3)	(4)
30	0.834	0.699	0.580	0.554
45	1.172	0.853	0.782	0.716
60	1.071	0.976	0.892	0.817
75	1.100	0.839	1.093	0.859
90	1.049	0.997	1.094	0.978
105	1.072	0.883	1.139	1.145

on coefficient m_2 ($m_2 < 0$) affect only slightly the behavior of Eq. (29). Therefore, an average, constant value for m_2 can be used for a particular estuary of constant length and depth. The values of the coefficients m_1 and m_2 for the study cases presented in Figure 2 are given in Table 2. The negative slope of the E_D versus C_z curves is in qualitative agreement with the energy dissipation per unit mass under steady-state uniform flow conditions defined as

$$E_{DU} = gU^3/(C_z^2 H) \qquad (30)$$

where U = uniform velocity. Equation (30) is similar to Eq. (27) with the only difference based on the interpretation of the velocity; *i.e.*, U versus u or u_m. The values of the uniform velocity U obtained from Eq. (30) and the linearized maximum velocity u_m as obtained from Eqs. (8) and (14) are presented in a number of cases in Table 3. In the same table, the maximum velocity u_{max} as computed by the numerical simulations is also provided. By looking at the results, it is evident that the corresponding uniform velocity U is always smaller than either the linearized velocity u_m or the actual maximum velocity u_{max}. This is in

Table 3. *Uniform, linearized and simulated maximum velocities.*

l_e [km] (1)	h [m] (2)	η_0 [m] (3)	C_z [$m^{1/2}$/sec] (4)	$E_D \times 10^4$ [m^2/sec^3] (5)	U [m/sec] (6)	u_m [m/sec] (7)	u_{max} [m/sec] (8)
45	10	0.50	60	0.088	0.318	0.386	0.386
45	20	2.00	45	0.480	0.271	0.358	0.679
60	10	0.25	30	0.049	0.165	0.180	0.250
60	20	1.50	90	0.225	0.719	1.012	0.741
75	20	2.00	45	1.400	0.834	0.859	1.199
75	30	2.25	60	0.550	0.357	1.171	0.952
90	20	1.00	30	0.420	0.426	0.382	0.528
90	30	0.75	75	0.017	0.308	0.358	0.393

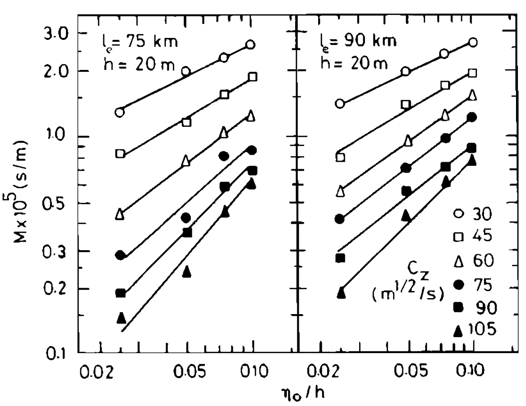

Figure 3. Linearized friction factor M versus dimensionless wave amplitude η_0/h.

agreement with experimental data reported by Ippen and Harleman (IPPEN, 1966). Indeed, in their test No. 28, $U = 0.851$ m/sec, while $u_m = 1.834$ m/sec. In Table 4, the values of u_m/u_{max} are given for an estuary of length $l = 75$ km and depth $h = 20$ m. From this table, some trends can be identified for the ratio u_m/u_{max}. For example, the ratio generally increases with increasing Chezy's number (decreasing roughness) and decreases with increasing tidal amplitude. Generally, the range of u_m/u_{max} was found to be $0.3 < u_m/u_{max} < 3.0$. Since u_m is being used to estimate the linearized friction parameter M, it is clear that measured maximum velocities u_{max} cannot be deliberately used in place of u_m.

The computer simulations revealed that $0 < \mu/\kappa < 0.637$, and $0 < 2 \tan^{-1}(\mu/\kappa) < 1.134 < \pi/2$. Therefore, from Eq. (14) it can be seen that the linearized friction factor M follows monotonically the behavior of μ/κ. Plots of M versus η_0/h and C_z on a log-log scale for different study cases are given in Figure 3. From Figure 3, the following relation is obtained

$$M = n_1(\eta_0/h)^{n_2} \quad (31)$$

where n_1 = proportionality constant with the same units as the M (s/m), and n_2 = dimensionless exponent. The values of the constants n_1 and n_2 for the cases presented in Figure 3, are listed in Table 5. It is generally evident from the data that n_2 depends on bottom friction; *i.e.*, $n_2 = n_2(C_z)$. Constant n_1 does not seem to follow any recognizable pattern.

Table 5. *Values of constants n_1, n_2 ($M = n_1(\eta_0/h)n_2$).*

	l_e = 75 km, h = 20 m		l_e = 90 km, h = 10 m	
Cz, in m$^{1/2}$/sec:	$n_1 \times 10^{5}$, in sec/m (1)	n_2 (2)	$n_1 \times 10^{5}$, in sec/m (3)	n_2 (4)
30	7.819	0.466	6.319	0.364
45	6.921	0.577	5.166	0.384
60	7.580	0.781	5.096	0.445
75	7.149	0.900	4.352	0.466
90	7.500	1.000	4.206	0.510
105	9.958	1.192	3.800	0.532

Once the velocity u_m is determined, the tidal Reynolds number R_e can be estimated by using Eq. (26). In Figure 4, the energy dissipation per unit mass E_D is plotted on a log-log paper against the tidal Reynolds number. From this figure, it is evident that

$$E_D = r_1 R_e^{r_2} \quad (32)$$

where r_1 = dimensional constant and r_2 = dimensionless exponent. The values of the exponent r_2 for all of the simulated cases were $2.8 < r_2 < 3.2$, with an average value of 3.0. Similar results were found in the experiment data reported by Ippen and Harleman (IPPEN, 1966). A straight line is obtained by plotting constant r_1 against Chezy's coefficient C_z on a log-log paper (Figure 5). Therefore,

$$r_1 = s_1 10^{(s_2 C_z)} \quad (33)$$

where s_1, s_2 = constants. Equation (33) is in agreement with Eq. (29). Combination of Eqs. (25) and (32) yields

$$D_L = a_1 E_D^{(1/r_2)} \approx a_1 E_D^{1/3} \quad (34)$$

where a_1 = proportionality constant. Eq. (34) verifies Eq. (28) which was derived by analytical means. Experimental verification of Eq. (34) was documented by ORLOB (1961). Physical interpretation of Eqs. (34) and (24) implies that for a homogeneous turbulent field, the scale of turbulent eddies are proportional to the estuarine depth. Inaccuracies resulting from harmonic analysis applications were observed in cases where the estuarine length exceeded one-fourth of the tidal wave length. The reason for the inaccuracies was due to excessive frictional damping. In these cases, the wave amplitude at the closed end was not maximum (SCARLATOS and SINGH, 1985, 1987b) so the harmonic analysis was not applicable.

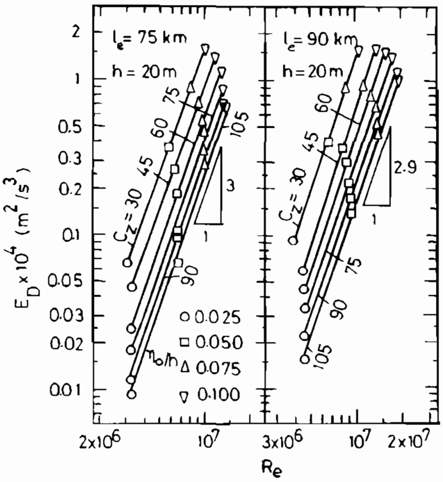

Figure 4. Energy dissipation E_D versus tidal Reynolds number R_e.

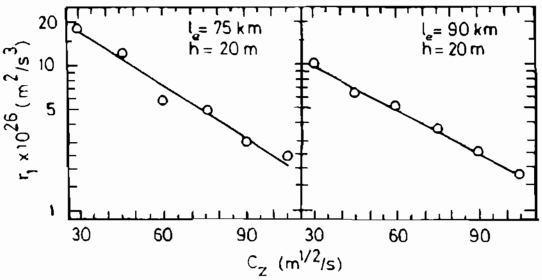

Figure 5. Constant r_1 versus Chezy's coefficient of friction C_z.

CONCLUSIONS

A large number of estuaries under co-oscillating tidal conditions were studied by means of the Ippen and Harleman's harmonic analysis. The data required for calibration of the harmonic parameters are generated by computer simulation using the full St. Venant system of equations. Emphasis was placed on the energy dissipation as related to various physical parameters.

The conclusions of the study are as follows:

(a) Energy dissipation is related exponentially to Chezy's coefficient of friction (Eq. 29). Since the exponent is a negative number, the result is in qualitative agreement with energy dissipation under steady-state, uniform flow conditions. The proportionality constant depends on the ratio η_0/h (Table 2).

(b) The same amount of energy dissipation is produced with much less velocity in a uniform steady state than in tidal flow; *i.e.*, $U < u_{max}$ (Table 3).

(c) The idealized maximum velocity u_m introduced by Parsons is different than the actual maximum velocity u_{max}. The range of variation is $0.3 < u_m/u_{max} < 3.0$.

(d) The linearized frictional factor M is exponentially related to the ratio η_0/h (Eq. 31). The exponent is proportional to bottom friction (Table 5).

(e) Energy dissipation is exponentially proportional to tidal Reynolds number (Eq. 32). The exponent of this relation varies between 2.8 and 3.2 with an average value of 3.0. This number is in agreement with available experimental and theoretical data. Also, the proportionality constant is exponentially related to Chezy's coefficient of friction (Eq. 33).

(f) In agreement with theoretical analysis, the simulated dispersion coefficient was proportional to the one-third power of energy dissipation. The physical interpretation of this result is that the energy carrying turbulent eddies in an estuarine environment are of the order of estuarine depth.

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Notation

The following symbols are used in this paper:

A = wet-cross section area, $[L^2]$
a = tidal amplitude at the closed end, $[L]$
a_1 = proportionality constant, $[L^{1/3}]$
C = coefficient of friction, $[L^{0.5}T^{-1}]$
c = phase velocity, $[LT^{-1}]$
C_z = Chezy's coefficients of friction, $[L^{1/2}T^{-1}]$
E = rate of energy dissipation, $[L^5T^{-3}]$
D_0 = relative diffusion, $[L^2T^{-1}]$
D_l = dispersion coefficient, $[L^2T^{-1}]$
E_D = energy dissipation per unit mass, $[L^2T^{-3}]$
E_{Dt} = energy dissipation for uniform, steady-state flow, $[L^2T^{-3}]$
E_{Ds} = energy flux between incident and reflecting wave, $[L^2T^{-3}]$
f = Darcy-Weisbach friction $[-]$
g = acceleration due to gravity, $[LT^{-2}]$
H = water surface elevation measured from a reference datum, $[L]$
h = water depth below mean sea level, $[L]$
L = tidal wave length, $[L]$
l_e = estuarine length, $[L]$
M = linearized friction factor, $[L^{-1}T]$
m_1 = dimensional proportionality constant, $[L^2T^{-2}]$
m_2 = dimensional exponent, $[L^{-1}T]$
n_1 = dimensional proportionality constant, $[L^{-1}T]$
n_2 = dimensionless exponent, $[-]$
q = lateral inflow, $[L^2T^{-1}]$
R_e = tidal Reynold's number, $[-]$
r_1 = dimensional proportionality constant, $[L^2T^{-1}]$
r_2 = dimensionless exponent, $[-]$
S_f = energy gradient, $[-]$
S_0 = bottom slope, $[-]$
s_1 = dimensional proportionality constant, $[L^2T^{-1}]$
s_2 = dimensionless exponent, $[L^{-1}T]$
t = time, $[T]$
U = uniform velocity, $[LT^{-1}]$
u = mean velocity of the cross-section, $[LT^{-1}]$
u_{av} = time-averaged velocity over half tidal cycle, $[LT^{-1}]$
u_m = idealized maximum velocity, $[LT^{-1}]$
u_{max} = actual maximum velocity, $[LT^{-1}]$
u' = characteristic velocity of energy dissipating eddies, $[LT^{-1}]$
x = longitudinal distance, $[L]$

Greek letters:

α = phase shift between tidal wave height and tidal current, $[-]$
β = numerical constant, $[-]$
ϵ = proportionality constant, $[-]$
ϵ' = proportionality constant, $[L^{1/3}]$
η = water surface elevation above (or below) mean sea level, $[L]$
η_0 = wave amplitude at the ocean end of the channel, $[L]$
κ = wave number, $[L^{-1}]$
κ_0 = frictionless wave number, $[L^{-1}]$
λ = Kolmogorov's length scale, $[L]$
μ = friction damping modulus, $[L^{-1}]$
ν = kinematic viscosity, $[L^2T^{-1}]$
σ = tidal frequency, $[T^{-1}]$
ϕ = dimensionless parameter, $[-]$