Geometric Trends in the Evolution of a Small Log-Spiral Embayment on the Illinois Shore of Lake Michigan

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ABSTRACT

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The evolution of a small log-spiral embayment was monitored during nine months of 1989 on the Illinois shore of Lake Michigan adjacent to the state-owned North Point Marina at Winthrop Harbor. Maximum shoreline recession during the monitoring was 61 m. A graphical method was used to locate the log-spiral center, and regression analysis identified the best fit logarithmic-spiral curve for each of the ten mapped planforms. Geometric characteristics that progressively decreased during the evolution were the spiral angle (α) and the radius (r_o) from the log-spiral center to the origin. Geometric characteristics that progressively increased were the rotation angle (Θ) over which the planform matched a logarithmic-spiral curve, and the spiral radius (r) at the maximum rotation angle. This study demonstrates that: (1) a graphical means is an efficient way to determine the center of a log-spiral bay; and (2) a dynamic equilibrium exists during the log-spiral evolution with the planform showing excellent agreement to a logarithmic-spiral curve from the earliest bay development through the expansion landward and down-coast.

ADDITIONAL INDEX WORDS: *Headland bay, crenulate bay, zetaform bay, beach planform, beach erosion, coastal evolution, shoreline change, Great Lakes.*

INTRODUCTION

Log-spiral beaches (or bays) are hook-shaped, coastal sedimentary planforms resembling a logarithmic-spiral curve (YASSO, 1965, 1982). The spiral geometry is formed by wave erosion downdrift from a headland or other coastal promontory in response to waves from a predominant direction refracting and diffracting around the promontory (SILVESTER, 1970). The promontory can be a natural rock headland or a man-made coastal structure such as a groin, jetty or revetment. Log-spiral bays occur worldwide and have been described in such diverse locations as South Africa (SILVESTER, 1970, 1974), Australia (CHAPMAN, 1978), Singapore and Brunei (TAN and CHIEW, 1991). Examples from the United States are Bodega Bay and Half Moon Bay along the northern California coast, and Spiral Bay on the northern New Jersey coast (YASSO, 1965). FINKELSTEIN (1982) described such bays along Alaska's Kodiak Island. Other names for this type of bay and beach planform are zetaform (HALLIGAN, 1906), half-

heart (SILVESTER, 1960), crenulate (SILVESTER, 1970), hook-shaped (LEBLOND, 1972), and parabolic (HSU and EVANS, 1989). Although YASSO (1965) proposed the name headland-bay beach, the term log-spiral bay is used here in accordance with YASSO's (1965) determination that these planforms approximate logarithmic-spiral curves.

Previous field studies of the geomorphology and geometry of existing log-spiral bays have examined these bays in their post-development stage when they are typically in a state of static equilibrium with negligible net longshore sediment transport. Although the development of log-spiral bays has been studied in wave tanks (SILVESTER, 1970, 1974), their evolution in coastal settings has not been well documented. During 1989, in the late construction phase of North Point Marina on the Illinois Lake Michigan coast (Figure 1), conditions were conducive to the rapid development of a log-spiral bay. Monitoring of the bay development provided an unprecedented data base on the evolution of the log-spiral planform in an actual coastal setting. This paper presents the results of this monitoring and summarizes geometric trends in the evolution of this log-spiral bay.
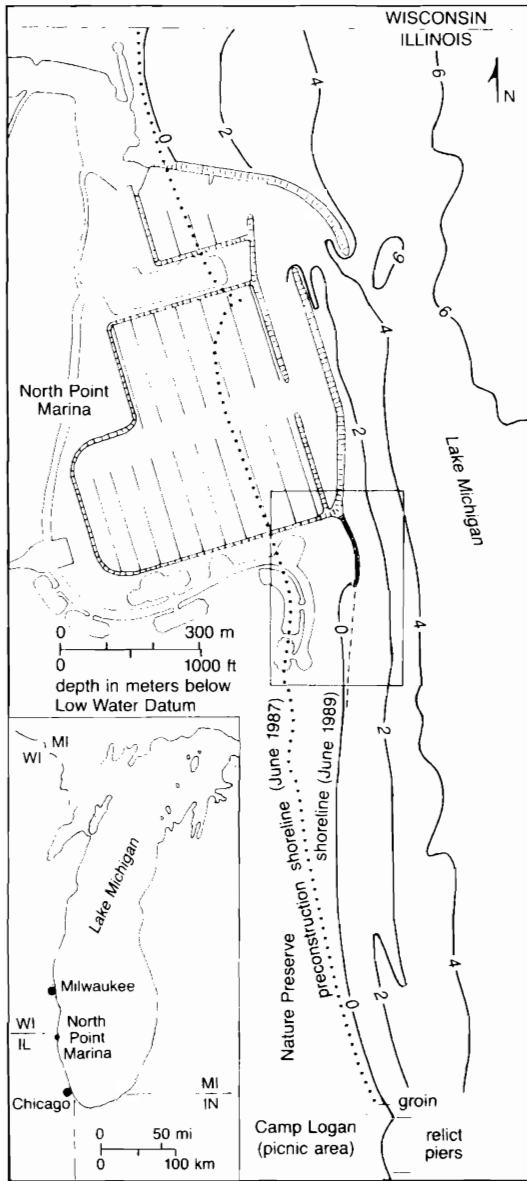
Figure 1.   Location map of study area with box at center delineating the area of log-spiral bay development.

Figure 2.   Nomenclature for a logarithmic-spiral curve.

$$r = r_0 e^{\theta \cot \alpha}$$

CHARACTERISTICS OF LOGARITHMIC-SPIRAL BEACHES AND CURVES

For a sedimentary coastline downdrift from a headland and influenced by a net oblique wave approach, the equilibrium planimetric shape is that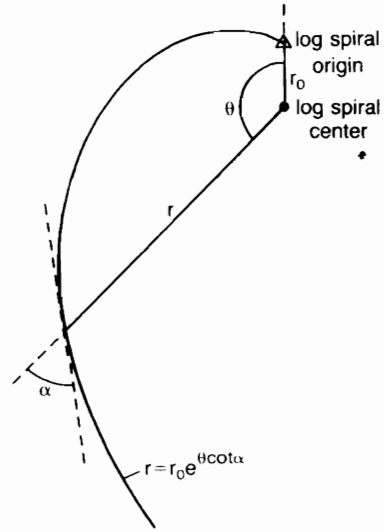 of a curving, coastal reentrant having a hook-like or half-heart shape. Three distinct curvature zones occur along this type of shoreline (SILVESTER, 1970, 1974). A nearly circular section occurs in close proximity to the headland. Accounting for the majority of the half-heart shape is a shoreline reach that has the form of a logarithmic spiral with an origin at or near the headland (Figure 2). A shoreline maintaining a true logarithmic-spiral curve would ultimately turn seaward at some point downdrift from the headland. At this point begins the third and downdrift zone of curvature which is a curvilinear to near-linear shoreline reach tangential to a downcoast headland. The distance between headlands is a factor in the maximum amount of landward reentrance of the log-spiral bay (SILVESTER, 1974; EVERTS, 1983).

This distinctive type of coastal planform is caused by wave refraction and diffraction around the headland, and by shadow effects of the headland on the distribution of wave energy (YASSO, 1965; SILVESTER, 1970, 1974). The gradual decrease in curvature of the bay with increasing distance from the headland reflects the decreasing influence of the headland on the shore. PHILLIPS (1985) suggests that the planform results from a general distance-decay function away from the headland. Such functions are typically curvilinear and can be represented by exponential or logarithmic models. YASSO (1965) demonstrated that logarithmic-spiral curves could be fit to the resulting shorelines. Although a reasonably precise

match can be obtained, the logarithmic spiral is likely a first approximation to a more complex geometric relationship (TANNER, 1982).

A logarithmic-spiral beach in static equilibrium is one that has reached harmony with the prevailing wave field. In such a case the incoming wave crests are parallel to the shoreline, the wave-approach angle (β) equals zero, and wave breaking occurs simultaneously along the shore. At such equilibrium, net longshore transport is essentially zero (SILVESTER, 1974). Research concerning the stabilization of coasts has focused on the use of log-spiral bays in static equilibrium (SILVESTER and HO, 1972). A well-studied natural example of a static equilibrium log-spiral beach is Half Moon Bay, California (YASSO, 1965; KOMAR, 1976; LEBLOND, 1979).

The logarithmic-spiral curve, also known as the Bernoulli Spiral, geometrical spiral, proportional spiral, and equiangular spiral (THOMPSON, 1961), occurs in patterns of sunflower seeds, pine cones, and various sea shells (LOCKWOOD, 1961). The fundamental property of the curve is that of continual similarity, which means that the angle between a radius vector to the curve at any point and a tangent to the curve at that point is always a constant (PROTTER and MORREY, 1964).

The equation for a logarithmic spiral is generally given in polar coordinates, with the radius r being a function of the angle Θ (LOCKWOOD, 1961). The usual form of the equation is:

$$r = r_o e^{\theta \cot \alpha} \qquad (1)$$

where, as shown in Figure 2, r is a radius from the log-spiral center to any point on the curve, r_o is the radius from the log-spiral center to the log-spiral origin, Θ is the angle between r_o and r, measured from the log-spiral center and α is the spiral angle, defined as the angle between the radius vector and the tangent to the curve at any point. For any given curve, α is a constant, so the term $\cot \alpha$ can be replaced by k in the general equation, giving:

$$r = r_o e^{k\Theta} \qquad (2)$$

so that taking the natural log of both sides gives:

$$\ln(r) = k\Theta + \ln(r_o) \qquad (3)$$

and a plot of $\ln(r)$ vs. Θ is a straight line.

STUDY AREA

The study site is located adjacent to North Point Marina on the Illinois shore of Lake Michigan

about 1 km south of the Illinois–Wisconsin state line (Figure 1). The geologic setting is a Holocene beach-ridge plain up to 1.7 km wide and 31 km long, extending from 12.5 km north to 18.5 km south of the Illinois–Wisconsin state line. Fine to medium sand is the dominant grain size, with additional clay, silt, coarse sand and gravel up to cobble size. This beach-ridge plain is a lenticular sand body up to 10 m thick along the coast and thinning landward and lakeward (HESTER and FRASER, 1973; FRASER and HESTER, 1974). The sand body overlies Wisconsinan glacial till. For at least the last two to three thousand years this beach-ridge plain has been migrating southward with erosion along its northern reach and accretion along its southern reach (LARSEN, 1985). The study site is in the historical erosional reach where 150 years of shoreline change indicate an average shoreline recession rate of 3 m/year (JENNINGS, 1990).

An estimate of the "natural state" littoral transport along the Illinois shore is about 70,000 m³/year (U.S. ARMY CORPS OF ENGINEERS, 1953; CHRZASTOWSKI, 1990). Net-southerly transport results from a maximum fetch in a north-north-east direction along the axis of Lake Michigan (Figure 1). Although wave conditions were not recorded during this study, nearshore observations at Evanston, Illinois (47 km south of North Point Marina) indicate that average wave period is 4.0 seconds and average wave height is 0.3 m. During storms, the period may increase to 9.0 seconds and height to 2.0 m. Individual waves rarely exceed 3.0 m. High-wave events are most frequent in early spring and late fall and usually last no more than two or three days. Still water level varies from year to year but generally fluctuates about 0.3 m annually, with low water in winter and high water in summer. Short period fluctuations such as storm surges and seiches can change water levels as much as 0.9 meters in 75 minutes (U.S. ARMY CORPS OF ENGINEERS, 1953).

BACKGROUND

The development of the log-spiral bay in this area is directly related to the construction of the state-owned North Point Marina. The basin of this 1500-slip marina straddles the pre-construction shoreline and was built by a combination of excavating into the beach-ridge plain and extending shore-attached, rubble-mound breakwaters into the nearshore zone. In 1987 and 1988, an estimated 1.2 million m³ of sediment were hy-

Figure 3.   Aerial photograph of the fan delta taken 14 September 1988 as the fan delta was approaching maximum areal extent.

draulically dredged from the basin and discharged via slurry pipe to the south (downdrift) side of the marina. The slurry water transported most of the silt and clay size particles into the littoral stream, leaving behind the coarser sand and gravel to aggrade in alluvial layers. The resulting fan delta extended the shoreline 200 m lakeward of the pre-construction shoreline (Figure 3). This fan delta had an artificially steep shoreface highly susceptible to erosion. The equililbrium mid-shoreface profile along this shore typically has a slope of 1:50 (MOFFATT AND NICHOL ENGINEERS, 1986). Bathymetric profiles from 1988 show the mid- to upper shoreface at the fan delta attained a slope at least as steep as 1:10. Selected bathymetric profiles in the vicinity of the fan delta are shown in Figure 4. The profiles are from annual bathymetric surveys conducted in the marina vicinity by the Illinois State Geological Survey (CHRZASTOWSKI and TERPSTRA, 1990a, 1991). Profile 42, in the zone of maximum fan-delta development and subsequent log-spiral erosion, clearly illustrates the artificially steep shoreface of the fan delta in 1988 and the subsequent smoothing of the profile in the following two years. Profiles 38 and 46, 122 meters to the north and south respectively, are included for comparison. All three profiles show net accretion between 1988 and 1989

because the dredging and discharge continued until late 1988. Shoreface erosion is evident between 1989 and 1990 on all three profiles.

The sediment dredged from the marina basin to form the fan delta was initially intended to be a feeder beach with wave erosion supplying the downdrift littoral stream (MOFFATT AND NICHOL ENGINEERS, 1986, 1987). However, subsequent plans by state officials called for a marina parking facility to be built atop the landward part of this sand reservoir. In fall 1988, the fan-delta surface was graded to an elevation ranging from 2 to 5 m above mean lake level. In January 1989, surplus breakwater stone and construction-site demolition debris were placed as riprap along the northern fan-delta shoreline. This riprap was underlain by filter fabric and piled from the water line up to about 2 m height. No stone was placed below water level. The riprap extended along the arcuate, fan-delta shoreline from the south breakwater to a point 150 m southward. The southward limit was dictated by the amount of available stone. The end point subsequently acted as a headland for wave refraction and diffraction, resulting in the development of a log-spiral bay downdrift from the riprap terminus (Figure 5). This erosion progressed with essentially no littoral sediment supply from updrift (northward) because at that time

the marina shore-attached breakwaters were a total to near-total barrier to such supply. The transport deficiency was a major factor influencing the rate at which the log-spiral embayment developed.

The Illinois State Geological Survey began intermittent monitoring of the fan-delta erosion in February 1989, and weekly to bi-weekly monitoring beginning on June 27, 1989. Other than the occurrence of nearshore ice in late January and early February, nothing interfered with the erosion and natural development of the log-spiral bay for the nine months from February to late October. During the period of 1989 monitoring, mean monthly water level of Lake Michigan had a range of 0.25 m from a low of 176.10 m above Mean Sea Level (MSL) in March to a high of 176.35 m MSL in July. By October, the level had returned to 176.15 m MSL. The evolution of the bay during these nine months is illustrated in Figure 6. The average rate of westward (landward) erosion through the monitoring period was 7 m/month, although during extreme wave events, recession rates as high as 40 cm/hour were documented (TERPSTRA and CHRZASTOWSKI, 1990). The evolution of the bay as a logarithmic-spiral landform ended on October 19 when a severe storm breached the riprap at the log-spiral origin, and rubble was placed along the southern segment of the bay as an emergency measure to defend the turn-around of the parking lot access road. In early November a concrete-cube revetment was constructed along the entire monitoring area to defend the parking facilities (CHRZASTOWSKI and TERPSTRA, 1990b).

During the nine months of log-spiral bay development, shoreface erosion and undermining caused some shifting and settling of the riprap. By October 1989, waves could overtop the riprap and erode the unconsolidated fill behind it. By fall 1990, the majority of the riprap had subsided below lake level (see Coastal Photographs by P. Terpstra, JCR, 1991, 7(2), frontispiece). The shore defense of late October and early November 1989 prevented the log-spiral bay from eroding landward to a position of static equilibrium. Continuing collapse of the riprap headland complicates modeling the potential static-equilibrium position of the shoreline even if the shore defense had not been added. A conservative model, assuming stability of the headland, places the theoretical equilibrium shoreline cutting across the access roads and becoming tangential to the preconstruction shoreline within the southern half of the
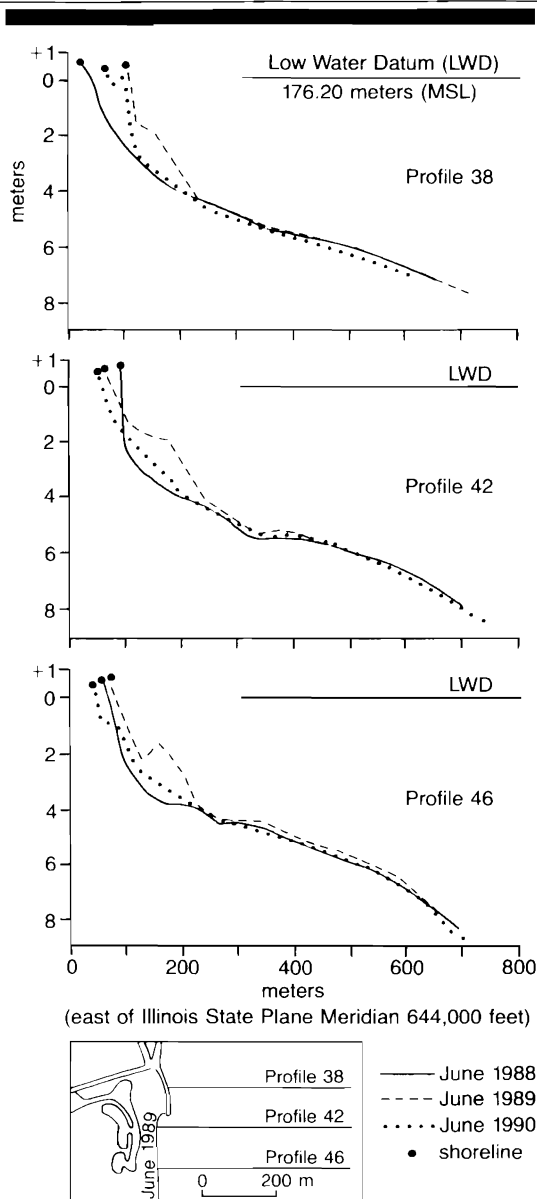
parking facilities. A steel sheetpile groin at Camp Logan, 1.1 km south of the riprap terminus, would function as the downdrift headland. Based on this model the maximum recession to a static equilibrium shoreline would have been approximately 125 m, about twice the 61 m of maximum shoreline recession measured during this study.

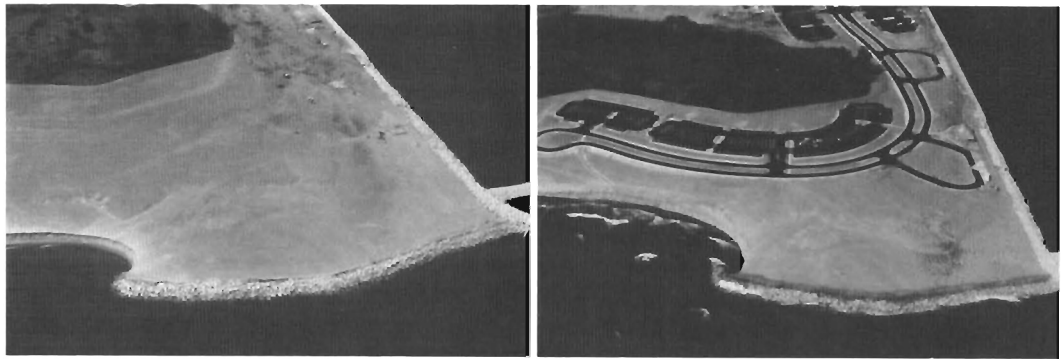Figure 4. Selected annual bathymetric profiles centered on the log-spiral embayment (Profile 42).

Figure 5.   Aerial photographs of the log-spiral bay taken (a) 1 May 1989 showing riprap along the northern fan-delta shoreline and initial bay development, and (b) 25 August 1989 showing the recessional scarp and the newly-constructed parking facility.

METHODS

Field Measurements

Through the 1989 evolution of the log-spiral bay, the planform was monitored by determining the position of the scarp crest along the top of the graded fan delta. The scarp height through the monitoring interval ranged from 2 to 4 m along the length of the log-spiral bay. The scarp-crest planform mimicked the shoreline planform, and the beach width between the scarp toe and the still-water shoreline was about 2 to 3 m (Figure 7). The monitoring was specifically focused to changes in this subaerial planform geometry. The monitoring did not include collection of bathymetric data or data of wave characteristics.

Preliminary monitoring of the log-spiral embayment began in February 1989, at which time the position of the scarp crest was measured relative to an approximate north-south baseline. No observations were made in March and April. The shoreline position for May 1 was obtained from an oblique aerial photograph rectified by computer cartography to form a plan view. Beginning on June 27, a more regular monitoring program began with the establishment of eleven stakes with a 15.2-m (50-foot) spacing along a north-south baseline. The original concern of this monitoring was to document erosion progressing toward the parking facilities, then under construction. On August 2, an additional six stakes with a 7.6-m (25-foot) spacing along an east-west baseline were added to more accurately map the northern part of the developing landform. Measurements from the monitoring stakes to the scarp crest were made

approximately every two weeks until October 19, when the log-spiral geometry was lost. During several extreme wave events the scarp recession was monitored on an hourly basis. The scarp-crest planforms were mapped by computer using the ARC/INFO Geographic Information System (GIS) to produce smoothed curves through the measurement points. A total of 23 scarp positions were mapped during the study. Ten selected scarp positions are shown in Figure 6.

Curve Fitting

A three-step method was used to find the equation of the logarithmic-spiral curve best approximating each of the ten mapped planforms.

First, the log-spiral center for each of the curves was determined by a graphical method. Tangents were drawn through several selected points along the curve on a large-scale (1:300) map. The points were considered two at a time as a sequence of pairs, each pair having two radii extending to some unknown center (Figure 8). The two radii and two tangents define a quadrilateral in which the internal angles total 360°. The center angle "d" can be determined from tangent intersection angle "c" because of the log-spiral property that α is constant for any given curve. As illustrated in Figure 8:

$$a = \alpha = 180 - b \qquad (4)$$

and because

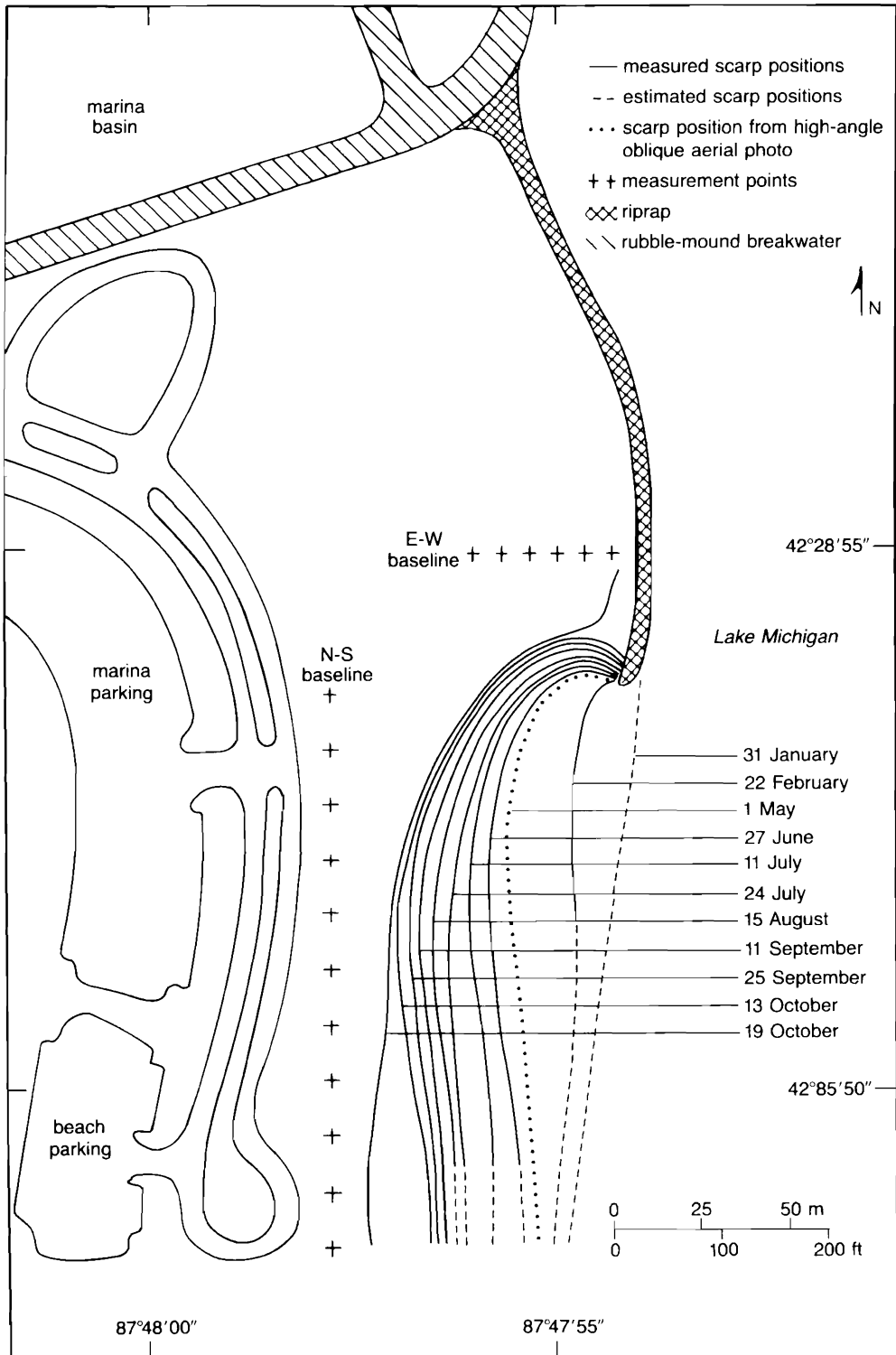$$a + b = 180° \qquad (5)$$

it follows that

Figure 6. Development history of the log-spiral bay at North Point Marina during 1989.

Figure 7. Ground photograph taken 15 August 1989 looking northward (updrift) toward the arcuate part of the log-spiral bay and the headland of riprap.

$$c + d = 180° \quad (6)$$

or

$$d = 180° - c \quad (7)$$

A circle drawn through the two points on the curve and the point where their tangents intersect will contain inscribed angle "d," and the log-spiral center will be somewhere on that circle. Repeating this procedure for additional pairs of points on the curve will yield additional circles, each containing the log-spiral center. For a perfect logarithmic-spiral curve, the circles will all intersect at one point, the true center. Some scatter to the circle intersections occurs for an imperfect log-spiral form such as a beach. In this study, a first-approximation center was assumed to be at the center of the cluster of circle intersections.

Second, after a suitable center was determined for a given curve, radii from the center to the curve were measured at five-degree increments on a 1:300 scale map starting with r_o (the distance between the center and the origin) at $\Theta = 0°$. Natural logarithms of the measured radii were calculated.

Third, the data were run through a linear-regression program to compute the equation of the best-fitting log-spiral curve for each planform. The

r_o term in the regression equation was compared to the measured r_o, and if they differed by more than 0.1 m, the first-approximation center was shifted either toward or away from the origin to match the calculated r_o value. Radii from the second-approximation center were then measured, as in Step 2 above, and a regression equation again computed. The number of times per curve that a first-approximation center required shifting in this manner ranged from 0 to 5, with an average of about 2. Significant parameters and statistics associated with the final run for each curve are listed in Table 1.

To ensure the validity of this three-step method of curve fitting, the procedure was also applied to Half Moon Bay and Drakes Bay on the California coast, for which YASSO (1965) previously evaluated the fit of logarithmic-spiral curves by computer-assisted trial and error. For both bay shorelines a spiral center was determined graphically, and radii measurements from the spiral center to the shoreline were made in five-degree increments. Natural logarithms of the radii were calculated, and linear regression was used to determine the best-fit log-spiral equation for each planform. The results are given in Table 2. Compared to the correlation coefficients obtained from

the log-spiral bay at North Point Marina, the results for Drake's Bay show a relatively low correlation coefficient. This less perfect match is probably due to the nearly circular planform of the bay, which results in the radius changing little as the rotation angle increases. The spiral angle was virtually the same as that determined by YASSO (1965) (85°). Half Moon Bay yielded a high correlation coefficient, similar to those found for this study's log-spiral bay, although the spiral angle for the equation of the Half Moon Bay shoreline is slightly larger than that determined by YASSO (46° instead of 41°). In general, the differences between this graphical approach and the computer-assisted, trial-and-error, curve-fitting approach are not significant, and this three-step method of fitting logarithmic-spiral curves appears to be valid when applied to these previously studied log-spiral bays.

RESULTS

All ten scarp planforms showed excellent agreement with computer-generated, logarithmic-spiral curves. Goodness of fit did not significantly improve through time as the bay enlarged, but was already excellent at the beginning of the monitoring and remained so throughout the nine-month monitoring period. Correlation coefficients ranged from a low of 0.99913 to a high of 0.99994 with no noticeable temporal trend. The extremely high correlation coefficients reflect in part the smoothing of the curves between measurement points, especially near the spiral origin, where small changes in spiral radius represent relatively large changes in the natural logarithm of the radius. The shifting of the log-spiral center described in the previous section also contributed slightly to maximizing the correlation coefficients.
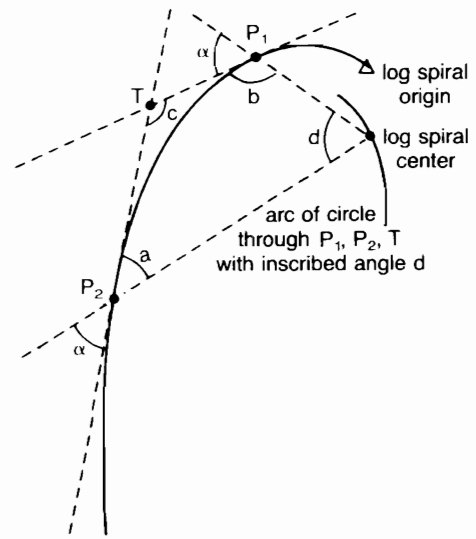

Figure 8. Graphical method for determining the geometric center of a logarithmic spiral.

However, the major factor explaining the excellent fits appears to be the excellent log-spiral geometry.

Temporal trends were observed in specific geometric characteristics. The distance r_o between the spiral center and the spiral origin in general decreased through time as the spiral developed (Figure 9a). It was anticipated that this distance might increase with time, as shown theoretically by BREMNER and LEBLOND (1974), but the graphically-determined center migrated from a location 3.4 m away from the origin on May 1 to a location 0.7 m away from the origin on October 19. The February 22 position of the center is regarded as

Table 1. *Summary of geometric trends for the log-spiral bay evolution at North Point Marina.*

Date (1989)	N (obs. pts.)	r_o (m)	α (deg)	Θ max (deg)	Radius at Θ max (m)	Azimuth of r_o (degrees)	Standard Deviation of ln (r)	Correlation Coefficient [ln(r) and Θ] at Θ max
22 Feb	26	1.448	46.99	126	11.6	4°	0.611	0.999389
01 May	30	3.377	44.91	146	46.0	7°	0.758	0.999142
27 Jun	32	2.925	42.34	155	58.4	12°	0.885	0.999608
11 Jul	34	2.070	40.41	166	63.5	14°	1.006	0.999872
24 Jul	36	1.873	37.38	175	110.0	18°	1.187	0.999241
15 Aug	36	1.331	36.49	178	90.9	31°	1.226	0.999944
11 Sep	38	1.951	36.88	185	155.0	30°	1.276	0.999184
25 Sep	39	1.304	33.87	190	199.0	27°	1.465	0.999126
13 Oct	40	0.989	32.81	196	206.2	34°	1.563	0.999769
19 Oct	40	0.733	31.08	194	211.0	41°	1.672	0.999784
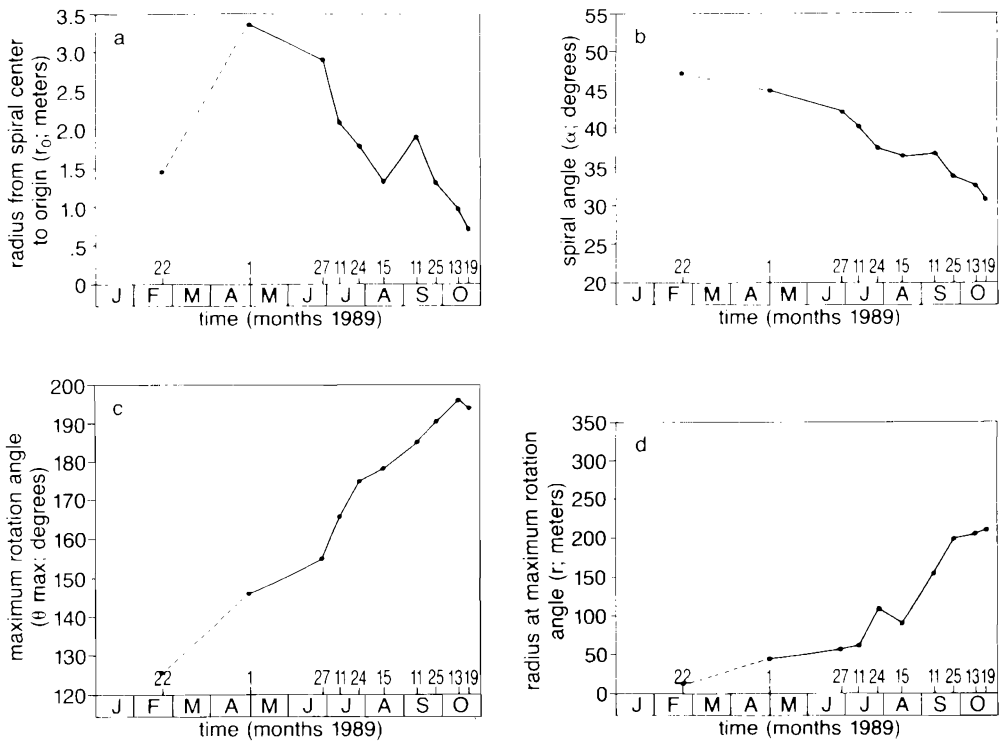
Figure 9.   Temporal trends in geometric characteristics of the log-spiral bay: (a) r_o; (b) α; (c) Θ max; (d) spiral radius at Θ max.

an anomaly possibly caused by the scarp planform being mapped from a small number of points referenced to an approximate baseline. Neither the spiral center nor the spiral origin remained fixed in location during the monitoring interval. Both shifted northward (updrift) through time, with the center migrating more rapidly than the origin, resulting in the temporal trend of decreasing distance r_o (Figure 10). Measured along the migration track, the total northward shift of the spiral center and spiral origin were respectively 14.9 m and 11.0 m. These are small changes relative to the overall changes of the bay planform. The consistent trend of the shifts suggests these are not an artifact of some measurement error in the

method of graphic determination of the log-spiral center.

The spiral angle α decreased with time as the spiral grew (Figure 9b). From February 22 to October 19, the spiral angle decreased from 47° to 31°. The amount of decrease in α gradually increased with time as the development of the log-spiral bay proceeded. The range of α values in this study (47° to 31°) is slightly lower than the range reported by SILVESTER (1974) from wave-tank experiments (54° to 39°).

Another temporal trend was the areal extent of the match between the measured planform and the ideal logarithmic-spiral curves. The angle Θ corresponding to the maximum correlation coef-

Table 2.   *Summary of geometric characteristics for two static equilibrium log-spiral bays of the California coast.*

Bay	N (obs. pts.)	r_o (km)	α (deg)	Θ max (deg)	Radius at Θ max (km)	Standard Deviation of ln(r)	Correlation Coefficient [ln(r) & Θ] at Θ max
Drake's Bay	18	3.564	85.23	85.0	4.09	0.039	0.973872
Half Moon Bay	30	0.475	46.30	145.0	5.49	0.722	0.999498

ficient (Θ max) increased with time as shown in Figure 9c. As the logarithmic-spiral erosion advanced landward and farther down the coast, the angular extent of the log-spiral geometry increased from 126° in February to 196° on October 13. A severe storm of October 17–19 impaired the log-spiral geometry both by breaching the riprap and by preferentially eroding the southern half of the planform. This loss of the ideal log-spiral configuration is shown in Figure 6 with the October 19 planform altered at both its northern and southern extent. The slight decrease in maximum angle to 194° on October 19 reflects this impaired geometry. Figure 9d shows that the spiral radius at which the maximum correlation coefficient occurred, r at Θ max, increased as the spiral grew, but the rate of increase was not uniform throughout the bay development. This fluctuation in rate of change subsequent to August 15 possibly relates to a change in the azimuth between the origin and the center as discussed below.

Figure 11 summarizes the relationship between the natural logarithm of the radius and Θ for the downdrift portion of all ten beach planforms. This summary diagram reiterates that the goodness of fit to a log-spiral curve did not significantly improve with time, but the angular extent of the log-spiral curve consistently increased as the bay evolved.

DISCUSSION

Two physical characteristics peculiar to this study site are worth noting. First, the updrift headland consisted of riprap placed above water adjacent to an eroding shoreface. Due to shoreface erosion the bathymetric profile was changing to an unknown extent during the log-spiral bay evolution, and this temporal change in profile would have caused temporal changes in the degree of refraction of the waves as they approached and rounded this headland. Second, the unconsolidated fan-delta sediments were highly erodible and unstable. As a result, the development of the log-spiral bay occurred extremely rapidly.

Two trends observed in the log-spiral bay of this study are the decrease through time in both the spiral angle α and the radius r_o. Yasso (1965) found little correspondence in the value of the spiral angle for the various shorelines he studied. He speculated, however, that if a developing log-spiral beach "is well fitted by a log-spiral curve, then a sequential change in spiral angle or posi-
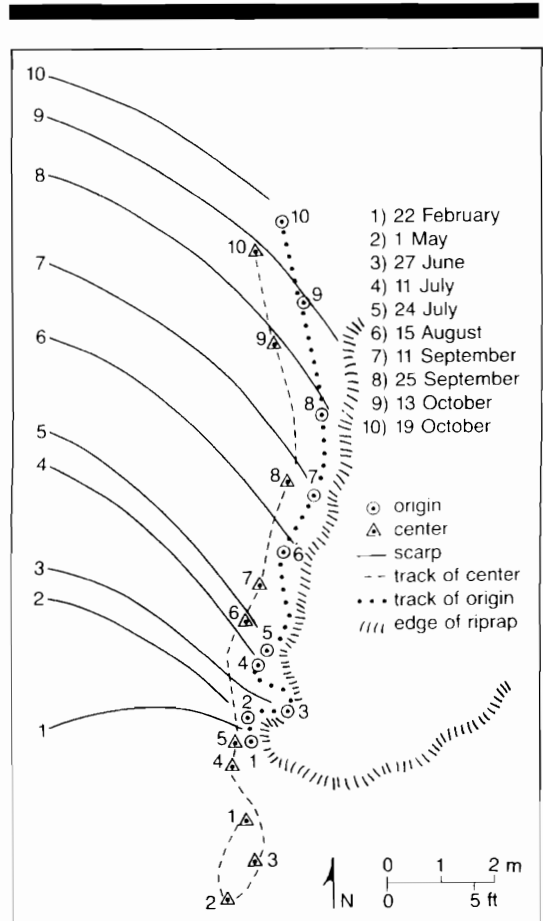

Figure 10. Temporal trends in the position of the log-spiral center and log-spiral origin. Scarp positions are approximate.

tion of log-spiral center might truly represent change in shoreline dimensions occurring within an equilibrium shape configuration" (Yasso, 1965 p. 713). This study provides data supporting Yasso's speculation. It is uncertain whether the observed decrease in r_o and α should be considered a general characteristic for an evolving log-spiral bay, or if these trends result specifically from the temporal changes to the shore and nearshore geomorphology near the headland of this study site. Another uncertainty arises from the extremely rapid rate of development of this log-spiral bay. It is possible that this case history represents such a brief interval in the evolution toward static equilibrium, that some of the trends observed may not reflect long-term trends or net effects.

The rate of increase of r (spiral radius) at Θ max is generally greater after August 15 than during
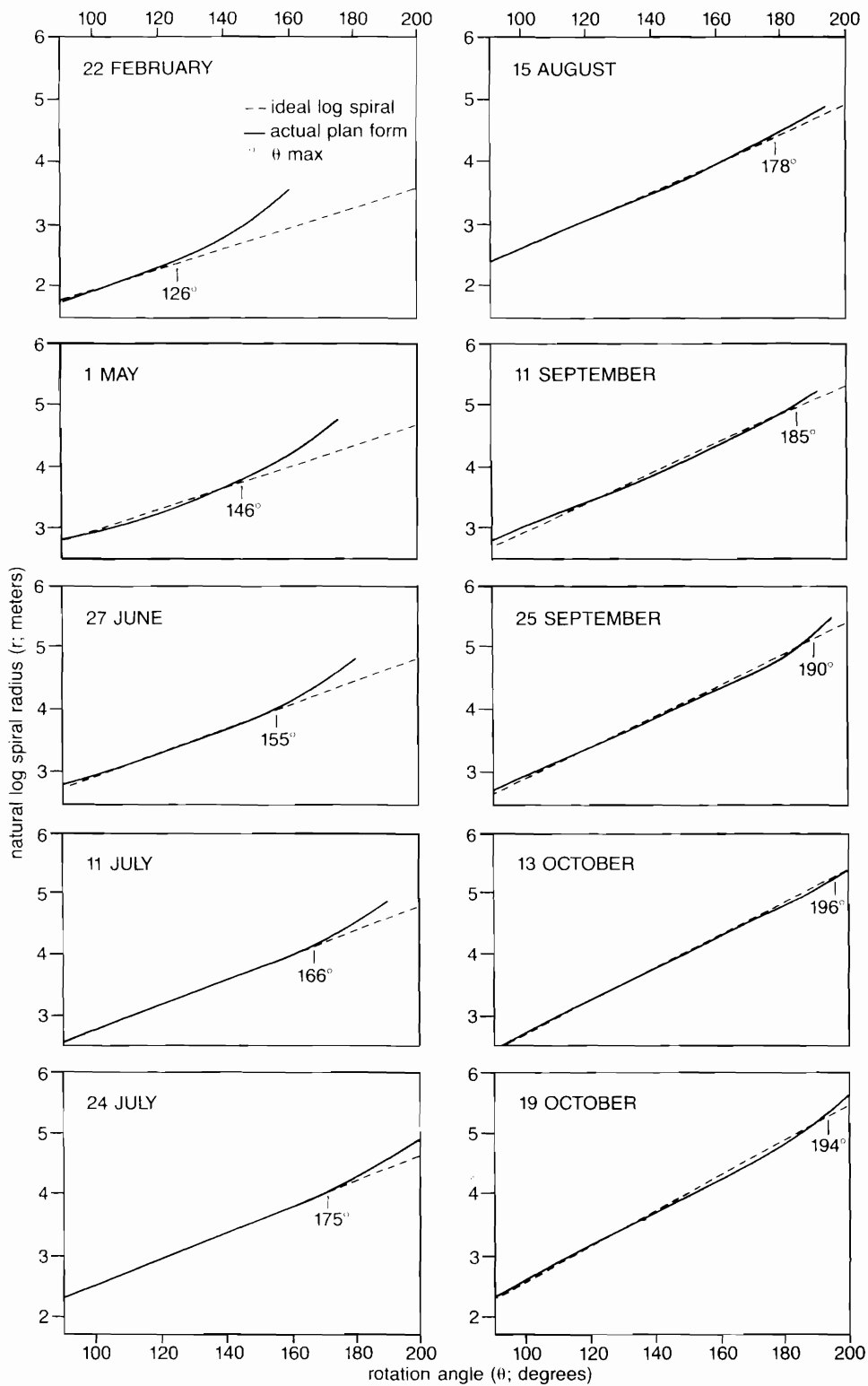
Figure 11. Plot of ln(r) vs. Θ for Θ greater than 90° for each of the 10 mapped scarp positions.

the earlier part of the monitoring period (Figure 9d). This difference in rate of change may be explained by a temporal change in the azimuth from the center to the origin. This azimuth from center to origin originally had a north-northeast orientation, but rotated clockwise to a more northeast orientation as both center and origin migrated northward (Figure 10). Table 1 shows this clockwise rotation of azimuth r_o. This westward shift of the center relative to the origin changed the azimuth of r_o to be more favorable to larger values of r at Θ max as the bay continued to develop. The shift of the center location likely relates to changes in wave dynamics brought about by shoreface erosion below the riprap, erosion south of the riprap, shifting of the riprap at its southern end, or some combination of these factors.

The most significant finding in this study is the consistency in high values for goodness of fit between ideal logarithmic-spiral curves and the measured planforms. This suggests that a developing log-spiral planform is in a state of dynamic equilibrium throughout its evolution as the form expands landward and downcoast, with successive planforms fluctuating within a family of log-spiral curves which are gradually approaching a log-spiral configuration of static equilibrium. The excellent fits even in the early stages of development suggest that poor-fitting log-spiral beaches found elsewhere are not poor-fitting because they are in a pre-equilibrium stage, but because there is a factor affecting their geometry, such as islands in the bay (BREMNER and LEBLOND, 1974), minor headlands along the beach (YASSO, 1965), or possibly some nearshore bathymetric irregularity, any of which will influence wave dynamics and littoral-sediment transport. At North Point Marina, the beach and shoreface are sandy and uniform, free of shoals, islands, and minor headlands. The log-spiral embayment evolved with the naturally occurring variance in wave dynamics. Because of the natural wave influence and the excellent fits observed, it is believed that the observed temporal changes in geometric trends are significant for general cases of rapid and early evolution of log-spiral bays, and can be useful in predicting the geometry of log-spiral shoreline recession in similar geologic settings elsewhere on Great Lakes and ocean coasts.

CONCLUSIONS

Based on this nine-month study of geometric trends in the evolution of a small log-spiral em-

bayment along the Illinois shore of Lake Michigan, the following conclusions are made:

(1) At all stages of development, this log-spiral bay had an excellent match to an ideal logarithmic-spiral curve.
(2) The log-spiral center remained in close proximity to the log-spiral origin throughout the bay development, and the center moved closer to the origin as erosion progressed.
(3) The spiral angle progressively decreased from 47° to 31° through the monitoring period.
(4) The angular extent of the log-spiral portion of the shoreline progressively increased from 126° to 196°.
(5) The approximate center of a log-spiral bay can be determined rapidly by a graphical method.
(6) A developing log-spiral planform is in a state of dynamic equilibrium from the earliest stage of development and as the form expands landward and downcoast.
(7) The observed geometric trends have potential implications in predicting rates and temporal geometric changes in shoreline recession behind and downdrift from the terminus of shore-parallel defense structures.

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□ RESUMEN □

La evolución de una pequeña bahía con forma de espiral-logarítmica fue medida en 1989 durante nueve meses sobre la costa de Illinois de Lago Michigan, adyacente a la Marina North Point en el Puerto Winthrop. La máxima recesión de la costa durante la medición fue de 61 m. Un método gráfico fue utilizado para localizar el centro de la espiral-logarítmica, y por medio de un análisis de regresión se identificó el mejor ajuste a la curva espiral-logarítmica para cada una de las diez formas planas graficadas. Las características geométricas que progresivamente decrecieron durante la evolución eran: el ángulo de la espiral (α) y el radio (r_0) desde el centro de la espiral-logarítmica al origen. Las características geométricas que aumentaron progresivamente eran el ángulo de rotación (Θ) sobre la cual la forma plana se asemejaba a una curva espiral-logarítmica, y el radio de la espiral (r) en el máximo ángulo de rotación. Estos estudios demuestran que: (1) un medio gráfico es una forma eficiente para determinar el centro de una bahía de forma espiral-logarítmica; y (2) durante la evolución de la espiral-logarítmica existe un equilibrio dinámico con la forma plana mostrando una excelente concordancia con una curva espiral-logarítmica a partir de los primeros desarrollos de la bahía a través de la expansión hacia el continente y costa abajo.—*Néstor W. Lanfredi, CIC-UNLP, La Plata, Argentina.*

□ RÉSUMÉ □

L'évolution d'une petite baie en spirale logarithmique située sur les plages illinoises du lac Michigan a été controlée durant neuf mois de l'année 1989. Elle touche North Point Marina à Wintrop Harbor. Le maximum de retrait du rivage durant le contrôle a été de 61 m. Une méthode graphique a été utilisée pour localiser le centre de la spirale logarithmique, et une analyse de régression a identifié le meilleur ajustement à la courbe d'une spirale logarithmique pour chacune des dix formes cartographiées. Les caractéristiques de la géométrie qui ont progressivement décru durant l'évolution sont: l'angle de la spirale ( ) et le rayon (r_0) mesuré entre le centre de la spirale et l'origine. Les caractéristiques géométriques qui se sont accrues sont l'angle de rotation (θ) sur lequel la forme cartographiée s'ajuste à la courbe de la spirale logarithmique et le rayon (r) de la spirale au niveau de l'angle de rotation maximum. Cette étude démontre 1° qu'un moyen graphique est adéquat pour déterminer le centre d'une baie en spirale logarithmique; 2°) qu'il existe un équilibre dynamique pendant l'évolution de la spirale: du début du développement de la baie jusqu'à l'expansion vers la terre et le long de la côte, dont la forme s'ajuste très bien avec la courbe d'une spirale logarithmique.—*Catherine Bousquet-Bressolier, Géomorphologie E.P.H.E., Montrouge, France.*