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Natural Stability of Beaches Around a Large Bay

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ABSTRACT

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A numerical wave refraction and longshore sediment transport model was employed to examine the alignment of two long sweeping beaches and a series of headland beaches in an essentially enclosed embayment, Port Phillip Bay, Australia. The model, which is suitable for general application to irregular bathymetry and variable wave conditions, indicated that net sediment transport along these beaches was essentially zero under the prevailing wind-driven wave-energy climate. As the bay is almost fully enclosed, the results suggest that the beaches have obtained an alignment which is in equilibrium with the overall shape of the bay and the local weather conditions. The analysis highlights the importance of beach alignment in a sediment-limited environment and the need to conform with the equilibrium alignment when establishing artificial beaches in this bay and bays of a similar type.

ADDITIONAL INDEX WORDS: Numerical model, longshore transport, wave shoaling, enclosed bay, beach equilibrium.

INTRODUCTION

A number of beach renourishment projects have been undertaken around Port Phillip Bay, in southern Australia (BIRD, 1990). Some of these were successful but others were short-lived. While reviewing these projects in some detail, BIRD (1990) did not attempt to inter-relate their success or failure, being primarily concerned with each beach at a local scale. In this paper, we place the locations of these beach renourishment projects in a bay-scale context. This was achieved by relating the orientation of the beaches to the overall shape of the bay and prevailing weather patterns, using numerical modelling and semi-analytical techniques. The model is a coupled simulation incorporating wave generation, wave refraction, and longshore sediment transport.

WAVE HEIGHT DETERMINATION

The unsteady 2-dimensional refraction model of BLACK and HEALY (1988) was extended to include longshore sediment transport and validated with new data for this application. The model presented here is a wave height simulation in steady conditions. A more comprehensive non-steady model is described by BLACK and ROSENBERG (in press). Numerical techniques and model verification over natural bathymetry, including a method to treat "zig-zagging," are described in the Appendix.

The basis of the refraction model is the steady-state equation of wave power conservation in 2 dimensions (NODA, 1972), *i.e.*

$$\frac{\partial}{\partial x}(F \cos \theta) + \frac{\partial}{\partial y}(F \sin \theta) = -F_D \qquad (1)$$

where x and y are orthogonal coordinates (Figure 1), θ is the wave angle and F_D ($= F_f + F_b$) is a combined bed friction (F_f) and wave breaking (F_b) power dissipation term. F is the wave power which, for linear Airy waves, is

$$F = Ec_g = 0.125\rho g H^2 c_g \qquad (2)$$

The wave height is H while ρ is water density, g is gravitational acceleration and c_g is the wave group speed, and E is the wave energy. The wave height shoaling is treated using the method described by BLACK and ROSENBERG (in press).

The dispersion relation for linear waves is

$$\omega^2 = gk \tanh kd \qquad (3)$$

where k is the wave number ($k = 2\pi/L$, L the wavelength), ω the radian frequency ($\omega = 2\pi/T$, T the wave period) and d the total water depth. An iterative Newton-Raphson technique is employed to find k, knowing the radian frequency.

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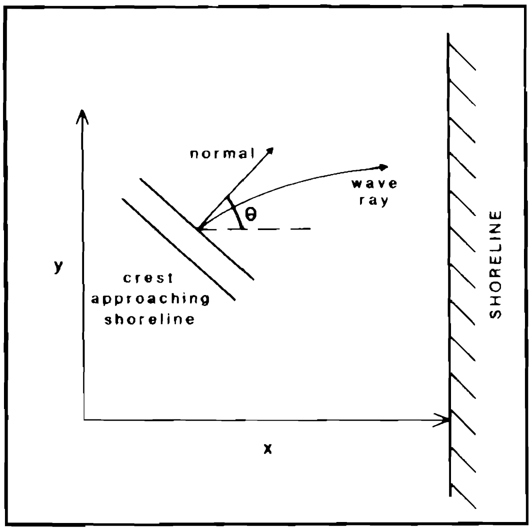

Figure 1. Definitions for wave modelling.

The bed frictional dissipation term (BRETSCHNEIDER and REID, 1954) is,

$$F_f = \overline{\tau \cdot U_b} = \frac{\rho C_f}{6\pi}[H\omega/\sinh(kd)]^3 \qquad (4)$$

In this formula, F_f depends on the maximum wave orbital velocity only, rather than the combination of wave orbital motion and low frequency currents. The coefficient C_f has the same definition as that used by THORNTON and GUZA (1983) but is half the magnitude of the coefficient f utilised by DALLY, DEAN and DALRYMPLE (1985).

A similarity law is employed to govern the broken wave height. The height can never exceed the breaking criterion, *i.e.*

$$H < \gamma d \qquad (5)$$

where γ is a constant, usually about 0.8. This height limitation substitutes for F_b.

WAVE ANGLE

The wave angle was obtained from the equation for conservation of wave number,

$$\frac{\partial}{\partial x}(|k| \sin \theta) - \frac{\partial}{\partial y}(|k| \cos \theta) = 0 \qquad (6)$$

In the model, height and angle are directly obtained on a regular finite difference grid (see the Appendix), which eliminates the need for interpolation, as required when a ray tracking procedure is used.

LONGSHORE SEDIMENT TRANSPORT

The longshore sediment transport is calculated using the formula recommended by KOMAR (1976),

$$I_l = K_l[cS_{xy}]_b \qquad (7)$$

where c is the wave celerity, the subscript b indicates values at the break point, I_l is the immersed-weight transport rate, K_l is a constant and S_{xy} is the longshore component of the radiation stress (LONGUET-HIGGINS and STEWART, 1964) and is,

$$S_{xy} = En \cos \theta \sin \theta \qquad (8)$$

where $n = c_g/c$ the ratio of group and phase speeds and $n = 1$ in shallow water. The volume of transported sediment is,

$$S_l = I_l/[(\rho_s - \rho)gP] \qquad (9)$$

where ρ_s and ρ are the sediment and water densities respectively, and P is a correction factor for the pore spaces ($P = 1.0$ in this paper as absolute quantities are not being considered, although more common values of P lie in the range 0.6 to 0.7). K_l was taken as 0.77 (KOMAR, 1976) although other values have been obtained experimentally (*e.g.* WHITE and INMAN, 1989; DEAN, 1989), suggesting that K_l is a function of wave parameters and/or beach slope.

SUB-GRID SCALE SHOALING AND BREAKING

The selected model grid size is usually determined by computer limitations, the required speed of solution and available bathymetry resolution. With these limitations, the wave shoaling and breaking near the beach most often occurs at a sub-grid scale. Accordingly, an iterative sub-grid scale scheme was developed to find the breaking height and angle for the longshore sediment transport calculations. The scheme operates as follows.

Along each on/offshore-directed row, the cell immediately offshore of the cell where breaking occurs was identified using the breaking criterion Eqn. (5). If breaking was not predicted to occur in any cell along the row, then the cell adjacent to dry land was selected. Using the values of height, angle and depth in the selected cell, the height and angle at the breaking point was obtained by iteration of Snell's Law. The relevant equations were

$$\theta_b = \sin^{-1}[c_b \cdot \sin\theta_o / c_o] \tag{10a}$$

$$H_b = H_o[c_{go} \cdot \cos\theta_o / c_{gb} \cdot \cos\theta_b]^{1/2} \tag{10b}$$

$$d_b = H_b/0.8 \tag{10c}$$

$$c_o = \omega/k_o; \quad c_b = \omega/k_b \tag{10d}$$

Wave number k was obtained using the dispersion relation Eqn. (3). The subscripts o and b refer to values at the cell mid-point and at the break point respectively.

An iteration technique, which is constrained by the breaking criterion, provides the height and angle at the location where $H_b/d_b = 0.8$. In the first iteration, d_b is set to $H_o/0.8$. Then, θ_b and H_b are calculated using Eqns. (10a and 10b) after solving for c_b. With the new value of H_b, an updated value of d_b is calculated. The successive iterations proceed until convergence.

FINDING THE LONGSHORE TRANSPORT RATES

The following approach was adopted to find the one-dimensional longshore transport rates from the two-dimensional refraction model.

(1) Prior to commencement of the refraction simulation, a bathymetry contour was selected to represent the shoreline. This was usually a smoothed contour in the range $0 < d\ (m) < 0.5$. The contour was digitised at approximately 0.2• DY intervals, where DY was the cell width in the longshore direction and was typically 20–50 m. This contour accurately specified the actual length of the beach and provided a shoreline.

(2) To find the beach orientation along the contour, a best fit linear curve was placed through each data point and 2 adjacent data points on either side, although the number of adjacent points to include depended on the desired amount of smoothing. The beach orientation was calculated from the gradient of the best fit curve.

(3) During the model simulation, the cell in which breaking first occurred in each on/offshore-directed row was determined. The sub-grid scale iteration technique was then applied to find the actual breaking height and the angle relative to the beach orientation.

(4) The longshore transport rate was then calculated (Eqn. 7) and the result was assigned to the nearest location along the shoreline contour.

(5) Longshore transport rates at the intermediate points along the contour were obtained by linear interpolation between the known values.

(6) If a wind climate was to be modelled, Steps (1–5) were repeated for each wind strength and direction. The annual sediment transport rate was the sum of the rates for each input condition weighted by the probability of occurrence of each condition.

(7) The transport rates were smoothed with a 5-point binomial-weighted running mean to reduce unwanted grid scale variability. Net transport and the positive and negative components were separately treated and output as a function of distance along the shoreline.

FIELD SITES

Port Phillip Bay is an essentially fully enclosed water body in southern Australia where waves are locally generated across open water fetches of up to 50 km. The narrow entrance and irregular outline of the bay prevents external ocean wave energy from reaching most of the bay's interior.

In total, 6 locations on 5 beaches on the east and south coast of Port Phillip Bay were examined. These were Chelsea, Sandringham, Brighton, Elwood, Safety Beach and Rosebud (Figure 2). Chelsea is midway along on the sweeping beach between Frankston and Mordiallic denoted in this paper as the "eastern" beach (Figure 2). Sandringham, Brighton and Elwood (Figures 2 and 4) are situated on the suburban coastline immediately south-east of Melbourne (denoted here as the "Sandringham" coast). Safety Beach is adjacent to a headland at the northern end of a sweeping sandy beach (referred to as the "southern" beach in this paper) some 24 km long with Rosebud midway along the beach (Figure 2). The southern beach forms part of the south and south-east shoreline of Port Phillip Bay.

On the southern beach, the longshore transport was calculated at two locations: Site 1 at 200 m from the northern bounding headland, a location partially sheltered from wave attack, and Site 2 at Rosebud, a central, exposed section of the beach. Beach orientation was very different at the two sites, the offshore normal being directed towards 270°T and 344°T at Sites 1 and 2 respectively.

BIRD (1990) described the southern beach as a sandy coastline which has prograded to form a long gently-curving beach backed by parallel foredunes. He notes that the sediments consist of quartzose sand with some ferruginous gravel on the east coast of Port Phillip Bay. The eastern beach between Mordiallic and Frankston is described by Bird as the outcome of shoreward sweeping of quartzose sand and shelly material

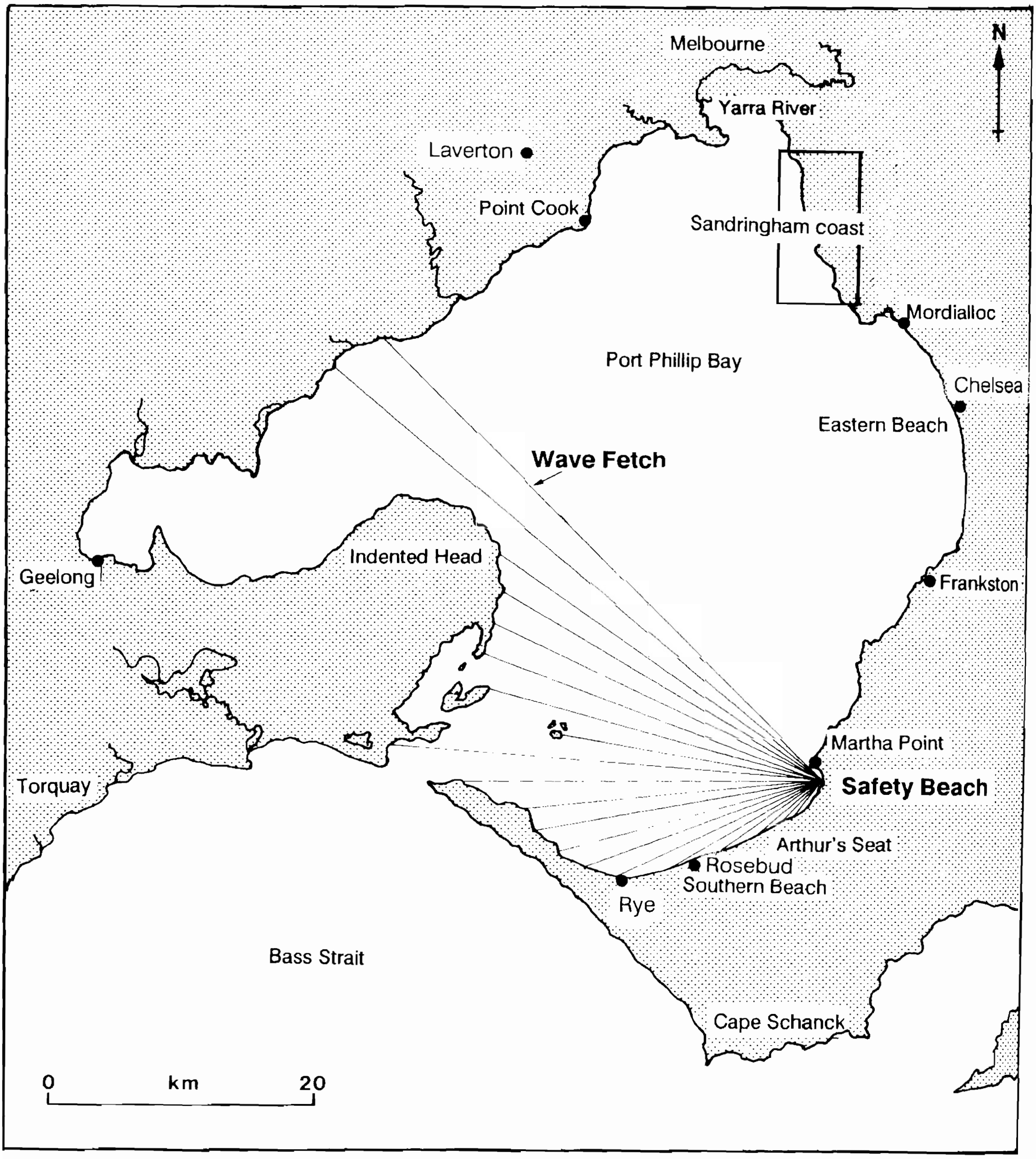

Figure 2. Port Phillip Bay showing Safety Beach and the wave fetch radials at 5° increments.

during and since the Holocene marine transgression.

A schematic diagram showing the longshore drifting of beach sediment in Port Phillip Bay is presented by BIRD (1990). On the southern beach, he indicates net southerly transport at the northern end and net easterly transport at the western end. We question the validity and compatibility of these net vectors.

FIELD INVESTIGATIONS

Field investigations at the northern end of the southern beach at Site 1 suggested a general stability of the region. Aerial photographs since 1977 showed that the beach position, although varying at short time scales, was essentially unchanged over the 23-year period. Wave over-topping occurs during storms and some shore protection (a rock wall above the natural high tide line) has been constructed for beach protection, but the aerial photographs indicated that the beach at the field site in the lee of the headland was resilient.

Observations of bedforms at Site 1 revealed an asymmetric bedform profile (directed shorewards) when observed during a period of low "swell." The asymmetry indicated a net shoreward sediment transport during the period of measurement, associated with the asymmetry of the wave orbital motion as the waves shoaled on the platform. The sediment, swept offshore during storms and deposited on the offshore platform, was being returned to the beach by the onshore-directed sediment transport under the wave crests.

The overall width of the sand platform was expected to be a function of the local wave climate (PICKRILL, 1983). Larger or longer period waves shoal in deeper water, making it possible for them to bring sediment shorewards from further offshore, and thereby maintain a wider sand platform. This mechanism would explain an observed widening of the platform on the more exposed locations south of Safety Beach where the wave heights are larger.

Although long term process could still be transporting sediment landward or seaward, the sand platform indicates an approximate dynamic balance in the on/offshore sediment transport, if the platform width and depth at the shelf break are in equilibrium with the prevailing wave conditions. Any artificial deepening of the platform would result in greater wave energy reaching the shoreline and erosion of the beach should occur. Thus, the platform behaves as a submerged breakwater which protects the beach from storm attack. The platform and beach therefore should not be considered in isolation, as indicated by numerous studies on natural ocean beaches which have noted transfer of sediment from the beach to the offshore bar during storms and the subsequent return of sands during the calmer "swell" conditions (*e.g.* KOMAR, 1976).

In the longshore direction, significant build-up or loss of sediment adjacent to the headland at Safety Beach was not evident in the aerial photographs. This suggested either (1) a neutral net longshore sediment transport rate or (2) a continuous movement of sand around the rocky headland, located 1,300 m offshore.

WAVE CONDITIONS AND EFFECTIVE FETCH

Locally-generated wave heights were obtained using the Sverdrup-Munk-Bretschneider (SMB) equations (SHORE PROTECTION MANUAL, 1975) and software of BLACK and HEALY (1981). Both the effective fetch and depth were calculated using the procedure recommended by the *Shore Protection Manual,*

$$X_{eff} = \sum X_i \cdot \cos \alpha_i / \sum \cos \alpha_i \qquad (18)$$

where α_i is the angle relative to the wind direction and X_i is the fetch (or depth) at that orientation.

Fetches

For the two sites on the southern beach, the fetches and depths were digitised manually at 5° increments from Port Phillip Bay Chart AUS 143 at 1:100,000. Because of the time required for this manual operation, a computerised system was subsequently developed which operated on a digitised outline of Port Phillip Bay taken from the same chart. The software calculated the fetches (at the same angle increments of 5°) by seeking the intersection of the fetch lines with the digitised coastline. This meant that the complete operation—fetch calculation, wave height and period calculation, and sediment transport—was automated, making it possible to rapidly assess the equilibrium orientation of a selection of beaches. As the typical depths of Port Phillip Bay have an insignificant effect in the SMB equations when the wave period is short, the depth was taken as the constant value of 10 m for all radials.

Winds used were measured by the MARINE MODELS LABORATORY (1986) at a nearby channel marker pile. The measurements, although only one year in duration, were selected in preference to longer records because they were recorded locally and because they were recorded over open water. The measured wind had an uninterrupted passage across open water in the dominant wave generating directions at the southern beach.

To investigate the influence of bay-scale wind variability, the results using the above winds were compared with results using winds recorded by the Bureau of Meteorology since September 1981 at Laverton airport at the north end of the bay (Figure 2). The Laverton record was longer and

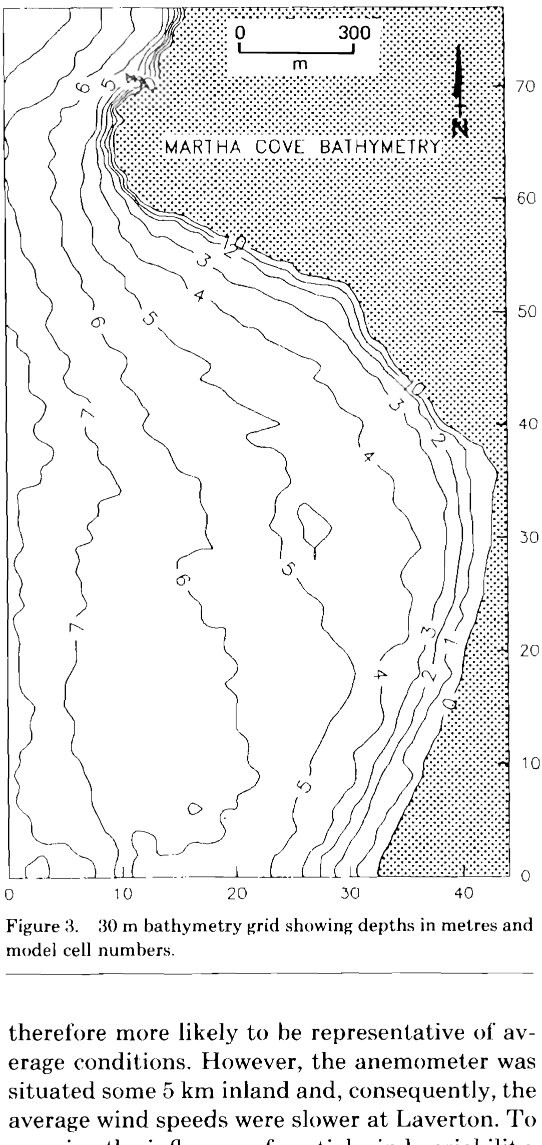

Figure 3. 30 m bathymetry grid showing depths in metres and model cell numbers.

therefore more likely to be representative of average conditions. However, the anemometer was situated some 5 km inland and, consequently, the average wind speeds were slower at Laverton. To examine the influence of spatial wind variability, the longshore sediment transport was calculated using both the southern and northern bay.

BATHYMETRY

At Site 1 in the lee of the headland, a 45 × 78 30 m-square cell bathymetry grid (Figure 3) was established from the "Port Phillip Safety Beach Model Study Survey" Sheets 1 and 2 (Port of Melbourne Authority). At 1:2,500, the measured depths were mostly less than 30 m apart and therefore a high quality representation was established. At the exposed Site 2, although rhythmic sandwaves were prevalent, the averaged longshore contours were mostly straight and parallel, so a Snell's Law approximation was applied, assuming a planar beach profile. The three additional sites on the Sandringham coast and the one on the eastern beach (Figures 2 and 4) were also treated by assuming a planar beach profile. Sediment transport rates were calculated at a point at the centre of each of these beaches (Figures 2 and 4).

SOUTHERN BEACH

At Site 1 in the lee of the headland, the wave heights corresponding to the annual wind rose were as large as 1.78 m, but the largest heights were less common than the smaller values due to the infrequency of the generating winds. The corresponding annual averaged sediment transport rates were 2.7×10^4 m^3 directed south and 3.4×10^4 m^3 directed north. The net transport was 7,396 ± 17,000 m^3/year, where the error bars assume a 50% error in the calculated transport rates.

The error of ±17,000 m^3/year is also equal to 1 standard deviation of the transport rates within 30 m of Site 1. This error is associated with a systematic change in longshore transport rate with distance from the site. As the site is situated in the shelter of a nearby headland, the degree of exposure changes quickly with distance along the beach. Notably, the fetches are calculated for Site 1 only and therefore the deviation in the values calculated away from the site are indicative of the sensitivity of the results.

While the net transport is non-zero, a zero net transport lies well within the error limits. The predicted net transport is in accordance with a net longshore transport rate of 7,500 m^3/year, obtained by analysing the accretion rate on the west side of the Rye pier (directed into the bay) in the 9 years after its construction (Jones, 1972).

The calculation of transported volume could be in error by as much as ±50%, due to factors such as inter-annual variability in the wind record, errors in the longshore transport equation coefficient, and errors in the SMB wave prediction method. However, any deviation in the absolute values would not be expected to alter the relative values of the southerly and northerly directed transport or alter the conclusion that net zero transport is well within the error bars.

The sediment transport rates at exposed Site 2 were much larger, being 11.3×10^4 m^3 to the east and 11.5×10^4 m^3 to the west. The net transport is therefore 2,000 m^3 to the west. Once again, the errors in the absolute values are possibly large. The plane beach assumption creates an additional error, causing the absolute values to be larger than those calculated if the wide flat platform is included in the bathymetry. However, the result once again indicates an almost neutral condition. In fact, a difference in beach orientation of less than 1° creates a neutral transport rate at this site.

While only two locations along the southern beach have been treated, the selected locations characterise the conditions that the beach is subjected to, in that the two sites have different degrees of exposure and the beach orientation at the two sites differs by 74°. The sites can be considered as reflecting overall conditions on the beach and thus it is reasonable to assume that the beach has essentially adopted a stable orientation throughout its length. The possible errors in the absolute transport rates do not detract from this conclusion.

Thus, the analysis indicates that this large beach in a nearly enclosed water body has adopted an orientation which is in equilibrium with (1) the shape of the bay as a whole which determines the effective fetch, (2) the prevailing weather patterns which form the locally-generated waves and (3) the sediment supply.

A significant net sediment transport along this beach would require a sediment supply to maintain the observed stability. The sediment volumes directed east and west at Site 2 are estimated to be larger than the volumes coming from locally eroding cliffs. The only remaining explanations are that (1) the sediment transport was in equilibrium and movement along the beach was approximately net zero or (2) that sediment was continuously entering or leaving Port Phillip Bay via this beach. There was no evidence of major accumulation or loss of sediment in the bay at modern time scales suggesting the alternative of a net zero transport, as shown by the numerical modelling.

BIRD (1990) noted that the beach has prograded since the later stages of the Holocene marine transgression (about the last 6,000 years). As the beach prograded, the equilibrium beach orientation could have become established in this period, while the beach responded to weather and sediment availability.

The pattern described here has similarities with the log spiral beach described by SILVESTER (1970). Log spiral beaches were found to be the equilibrium shape arising in response to waves refracting around a headland. However, in the Port Phillip Bay case the equilibrium condition is not dependent on a balance between wave refraction and net sediment transport. The balance is the result of an equilibrium sediment transport governed by the shape of Port Phillip Bay as a whole and the prevailing weather patterns.

There are two bedrock headlands along the southern beach. The first is Martha Point at the northern end of the beach and the second is Arthur's Seat some 6.5 km south. While the beach is located on the seaward tip of the latter headland, the beach intersects the Martha Point coastline some 1,300 m landward of the headland tip. For the beach to join the two headlands at their tip, its orientation would have to change by about 15°. This new alignment would not be in equilibrium with the prevailing wave patterns and would therefore be unable to occur in a sediment-limited environment without a continuous upstream supply of sand to maintain the unstable orientation. Thus, the point at which the beach intersects the Mount Martha headland can be seen as the position of natural stability.

BEASLEY (1969) found fragments of material from the Martha Point cliffs along the shoreline to the south. While this could be interpreted as reflecting net sediment transport to the south, it could equally well be evidence of oscillating (net zero) transport. The fragments moving south would mix with the local sands, causing a residue to remain when the net currents were directed northward at a later time.

OTHER LOCATIONS ON THE EAST COAST OF PORT PHILLIP BAY

If we apply to other bay segments the general principle that the stable beaches are in balance with the overall shape of the bay and the prevailing weather patterns some useful implications become evident. For this purpose, the four other east coast sites were examined (Table 1).

The sweeping shape and orientation of the eastern beach when viewed with respect to the shape of Port Phillip Bay suggests neutral longshore transport and, indeed, the sediment transport calculations confirmed this (Table 1). The Sandringham coastline, however, appears to be oriented too close to north/south to be in balance (Figure 2). The longest fetches on this coastline are from

Table 1. *Sediment transport rates. Positive rates are directed to the right of a shore-based observer looking seaward.*

Location	Negative transport ($m^3 \times 10^4$)	Positive transport ($m^3 \times 10^4$)	Net transport ($m^3 \times 10^4$)
Site 1	− 2.4	3.4	0.7
Site 2	−11.5	11.4	0.1
Eastern beach	−12.6	11.0	1.6
Sandringham	−11.0	11.5	0.5
Brighton	− 9.9	10.1	0.2
Elwood	− 8.2	11.2	3.0
Sandringham with Laverton winds	4.8	5.0	0.2

the south and net northerly transport would be expected. However, a closer inspection showed that the beaches along this coast have an alignment which is rotated some 10–15° anti-clockwise of the general alignment of the coastline (Figure 4). To create this rotation, the cliffs have eroded more at their southern end, creating a series of hooked headlands.

When this alignment was applied, the sediment transport calculations found net zero longshore transport on two of the three beaches examined (Table 1). Sandringham and Brighton beach were very nearly in equilibrium. Elwood beach appeared to have a net northerly transport. A further anti-clockwise rotation of 11° was needed to bring Elwood Beach into equilibrium, and no explanation for this is offered in this paper.

To specify the influence of bay-scale variations in the wind rose, the Sandringham beach case was re-examined using the Laverton airport wind rose (rather than the winds from the pile in southern Port Phillip Bay). The absolute transport rates were significantly less (Table 1), in accordance with the lower wind speeds recorded at Laverton, but the net transport was still essentially zero. Thus, the overall conclusion that the orientation of many of the sandy beaches throughout Port Phillip Bay was determined by the overall shape of the bay and the local weather patterns, was not altered using winds from northern Port Phillip Bay.

If sandy beaches occur where the equilibrium alignment develops, the rocky sections of the coast, conversely, may be indicative of orientations where net longshore currents are non-zero. The Sandringham coast is typified by alternating sandy beaches and exposed rocky shorelines, the latter having a general alignment rotated clockwise of the equilibrium alignment. This would explain the presence of the large number of rocky beaches and headlands along this coast and the more fragile nature of reclamation projects in this region compared with the reported success of reclamation on the southern beach (BIRD, 1990).

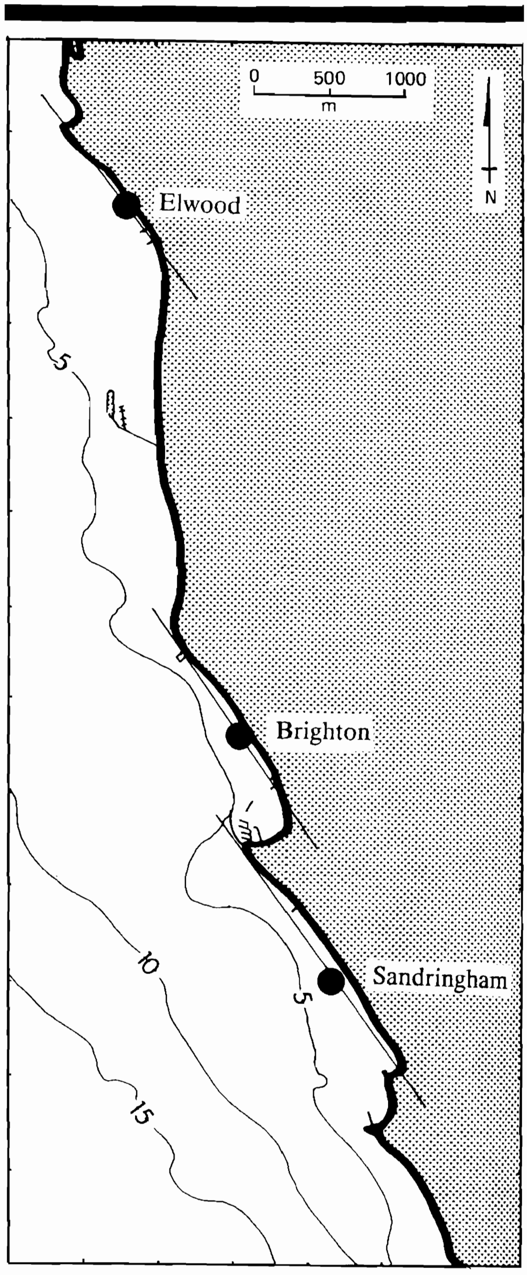

Figure 4. The "Sandringham" coast.

Due to seasonal and inter-annual variability in the weather, net southerly or net northerly transport occurs along the Sandringham coast (BIRD, 1990). Thus, sediment could be lost onto adjacent beaches in a stormy period. This was indicated by beach tracer experiments of BAKER (1963, 1964), conducted along the Sandringham coast. Material dumped on two Sandringham-coastline beaches was tracked for more than 3 months and large excursions of several kilometres were identified. However, while some material travelled considerable distances, much of the coarser fraction of the dumped material remained on the beach where the material was initially deposited. The coarser grain size was similar to the grain size of natural sediments. The result suggests that natural sorting of grain size occurs and that the material on these equilibrium beaches may have a grain size which reflects the energetics of the region, a factor working in unison with the larger beach orientation adjustments.

The two major beaches on the east and south coast, being in equilibrium, form a barrier to the passage of sand which isolates the Sandringham coast from the south. The only terrestrial supply of sediment from the north comes from the Yarra River which introduces only small volumes of material to Port Phillip Bay. Thus, cliff recession, identified earlier this century (MACKENZIE and MCCLELLAND, 1936), was evidently providing the only appreciable supply of sediment for the local beaches. Modern shore protection works reduced the cliff erosion and eliminated the potential for natural adjustment of beach orientation. With private property fronting much of the modern shoreline, beach re-alignment by renourishment may offer the most appropriate means to ensure a long-term stability of recreational beaches along this coastline. In the sediment-limited environment, coastal structures which store sediment (*e.g.* boat harbours) could cause sediment depletion on adjacent coastlines. Compensating renourishment may be required on beaches with these developments.

CONCLUSIONS

A wave refraction and sediment transport model, developed for application to natural bathymetries and weather conditions, indicated a net zero transport along a sweeping 24 km-long beach of Port Phillip Bay in southern Australia. Although the beach orientation changed by more than 74°, net zero transport was found to occur at a site sheltered by a headland at the northern end of the beach and at an exposed site to the south. The result indicated that the beach, formed in the later stages of the Holocene marine transgression, has established an equilibrium profile in balance with the overall shape of the bay and the prevailing weather patterns. The east coast of the bay was found to be similarly in balance on some sections. The Sandringham coastline at the northeast of Port Phillip Bay was mostly isolated from sediment supply, suggesting a need for conservation of the equilibrium alignments along this popular shoreline to ensure a sustainable stability in the region.

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APPENDIX: NUMERICAL SOLUTIONS

To solve for the height and angle, Eqns. (1 and 6) were written in the form

$$\frac{\partial A}{\partial x} - \frac{\partial B}{\partial y} = -F_D \tag{A1}$$

where $F_D = 0$ for the wave angles. This equation was solved using the space-centred mixed implicit scheme recommended by HARDY and KRAUS (1988). That is,

$$A_{i,j} = A_{i-1,j} + \Delta x\{(1 - a)[\partial B/\partial y]_{i-1,j} + a\,[\partial B/\partial y]_{i,j}\} - (F_{Di,j} + F_{Di-1,j})/2 \tag{A2}$$

where i is the on/offshore grid cell subscript (increasing shorewards) and j is directed longshore. The coefficient "a" was set to 0.5 which centres the y derivatives in the x direction. Finite difference approximations for the y derivatives are discussed below.

A smoothing factor (which mimics diffraction) was added in the form of an eddy viscosity term,

$$A'_{i,j} = A_{i,j} + N_A[A_{i-1,j} + A_{i,j-1} + A_{i,j+1} - 3.A_{i,j}] \tag{A3}$$

where N_A was a selected coefficient taken as 0.05. This adjustment occurred after each iteration at each cell.

In the model, the complete grid of angles is obtained first and the angles are then employed in the wave height solution. The model therefore does not account for wave set up and set down. In both cases, the solution marches forward from offshore, iterating each row until convergence, typically after 3–4 iterations. The angle and height at cell i,j are then obtained using

$$\theta_{i,j} = \sin^{-1}(A_{i,j}/k_{i,j})$$

$$H_{i,j} = [F_{i,j}/(0.125\,\rho g c_{gi,j})]^{1/2} \tag{A4}$$

ELLIPTICAL SHOAL

We sought a difficult two-dimensional test which included strong refraction, diffraction and shoaling to verify refraction model behaviour. For this purpose, the elliptical shoal modelled by HARDY and KRAUS (1988) was chosen (Figure 5). We modelled the largest wave height Case M1I considered by HARDY and KRAUS (1988), with H = 0.065 m and T = 1.3 s, to maximise the effect of wave shoaling, but these waves do not break on the shoal. The same test is described by BLACK and ROSENBERG (in press). In the present paper, the numerical techniques are examined in more detail.

Smoothing was required when solving for the angles and the coefficient $N_A = 0.05$ was selected to be the minimum value necessary to prevent the collapse of the numerical solution in the lee of the shoal (HARDY and KRAUS, 1988). This stablised the angles solution, with the result that the wave heights were also smoothed. Diffraction is a dominant process in the lee of the shoal and the model does not explicitly account for wave diffraction.

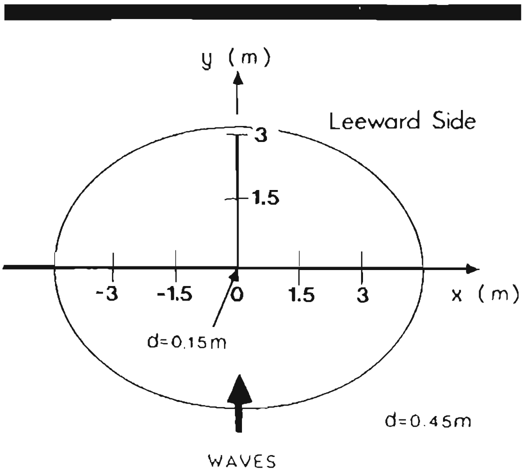

Figure 5. The elliptical shoal modelled by Hardy and Kraus (1988).

The results indicate that the smoothing scheme (Eqn. A3) provided an acceptable substitute for the full diffraction equations in this case at the measured locations. However, further leeward of the shoal, the solution was not as stable (PANCHANG *et al.*, 1987; HARDY and KRAUS, 1988).

NATURAL BATHYMETRY

Prior experience with natural bathymetry indicated that zig-zagging (oscillating height or angle at alternate grid cells) can occur over abruptly changing depth contours or adjacent to land (BLACK and HEALY, 1988) if the derivatives are poorly approximated. Thus, a number of y-derivative approximations were tested using the elliptical shoal and Safety Beach simulations for validation.

For application in the model, the following offset derivative approximations were centered by

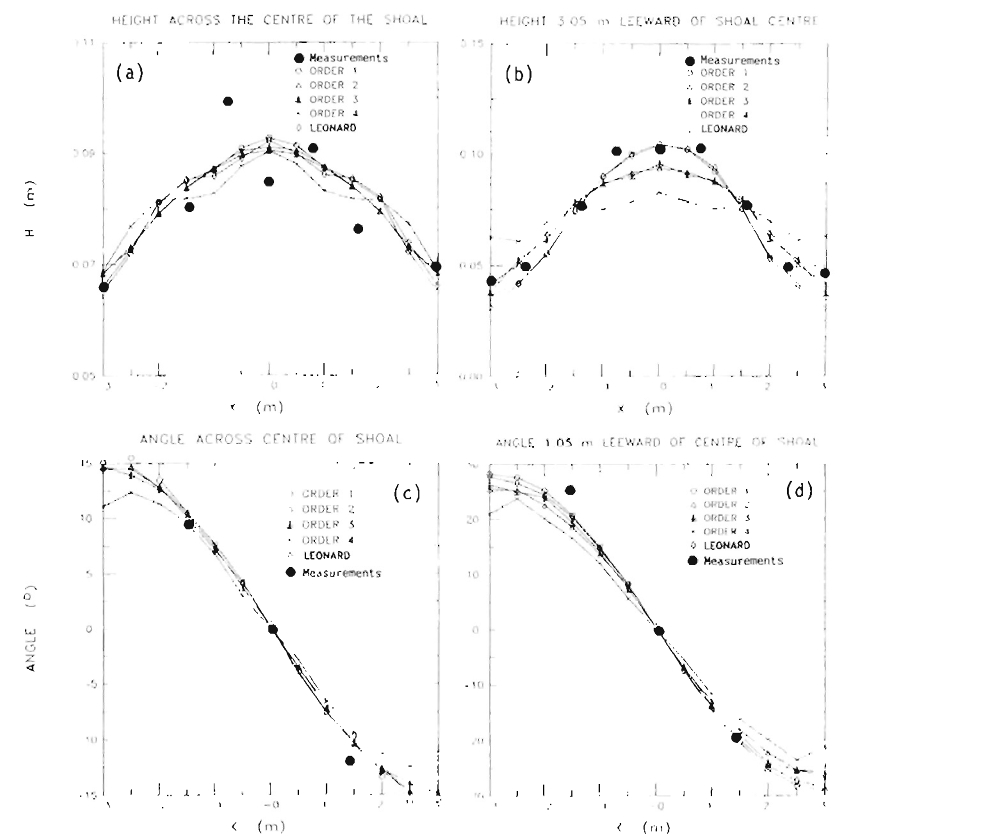

Figure 6. Comparison of wave height and angle predictions with measurements on an elliptical shoal using a variety of y-derivative approximations. (a) Wave height across the centre of the ellipsoid. (b) Wave height 3.05 m leeward of the centre of the ellipsoid. (c) Wave angle across the centre of the ellipsoid. (d) Wave angle 1.05 m leeward of the centre of the ellipsoid.

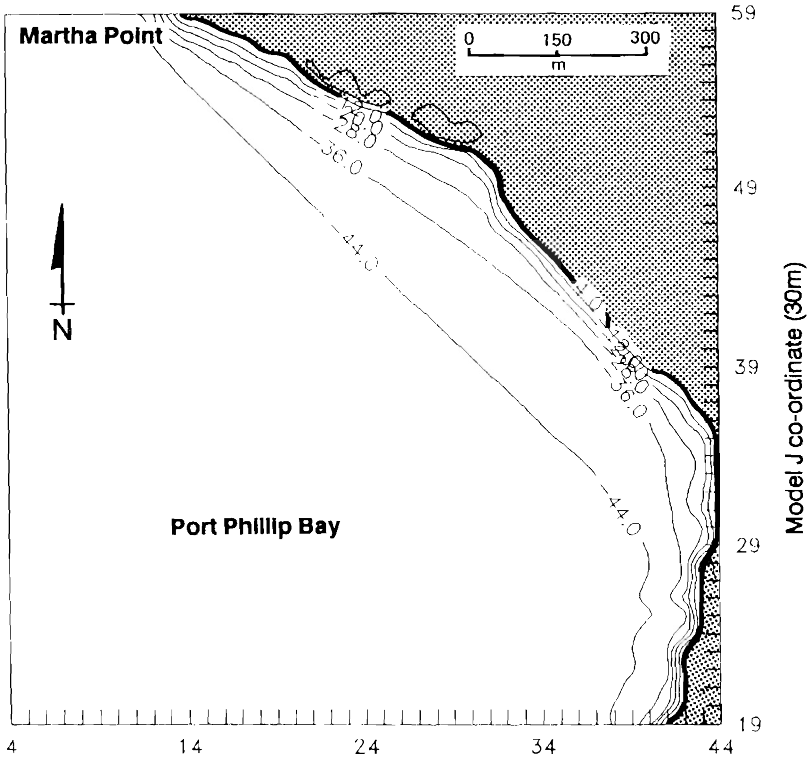

Figure 7. Wave angles in the lee of the headland (relative to due east) for an offshore height of 1 m and angle at 315°T.

calculating the offset derivative in both the positive and negative directions from the cell in question. The two approximations were then averaged to form the centered term. At land/sea boundaries the offset derivative only was applied. In addition, we examined the third-order approximation of LEONARD (1979) (Eqns. A9), once again calculating the approximation with both positive and negative offsets and averaging the two results. At land/sea boundaries in this case, a first order offset approximation (Eqn. A5) was used.

For the case of equally-spaced points with constant interval h, MILNE (1949, p. 96) proposed the following approximations for the offset finite difference approximations to the first derivative.

n = 1. Two points

$$y' = \frac{1}{h}(y_0 - y_1) \tag{A5}$$

n = 2. Three points

$$y' = \frac{1}{2h}(-y_0 + y_2) \tag{A6}$$

n = 3. Four points

$$y' = \frac{1}{6h}(-2y_0 - 3y_1 + 6y_2 - y_3) \tag{A7}$$

n = 4. Five points

$$y' = \frac{1}{12h}(y_0 - 8y_1 + 8y_3 - y_4) \tag{A8}$$

LEONARD (1979) suggested the 4 point schemes,

$$y' = (y_{-1} - y_1)/2h - (y_{-1} - 3y_0 + 3y_1 - y_2)/6h \tag{A9a}$$

$$y' = (y_{-1} - y_1)/2h - (y_{-2} - 3y_{-1} + 3y_0 - y_1)/6h \tag{A9b}$$

Figure 6 shows the heights and angles over the shoal obtained using the various approximations (Eqns. A5–8) and the Leonard approximation (Eqn. A9). There is considerable scatter in the measured angles and heights over the shoal and one cannot totally justify the selection of a particular result by comparison with the data. However, the higher order solutions appear to smooth the height variation in the x-direction, particularly leeward of the shoal (Figure 6). The smoothing causes the measured height to be underestimated.

The 3rd order Leonard approximation produces very similar results to those obtained with the centred second order derivative (the averaged form of Eqn. A5). BLACK and HEALY (1988) found that this approximation reduced zig-zagging on natural bathymetries. We simulated Safety Beach with the various methods and found that instabilities arose with the higher order approximations. With an offset derivative at the land/sea boundaries and a centred derivative elsewhere, severe zig-zagging occurred near the headland.

However, it was found that the zig-zagging could be virtually eliminated by using the 3rd order Leonard approximation. An example refraction simulation (offshore height of 1 m and angle of 315°T) using the 3rd order Leonard derivatives was zig-zag free. The angle contours (Figure 7) showed a smooth transition from $-45°$ to about $-10°$ in the lee of the headland. The height contours were equally well behaved. The Leonard approximation (Eqn. A9) appears to be far more robust than the alternatives tested and it is recommended that this approximation be utilised rather than the higher order or centered 2nd order derivatives, particularly over natural bathymetries.

□ RÉSUMÉ □

On a employé un modèle numérique de la réfraction des vagues et un modèle de transport sédimentaire parallèle è la côte pour étudier l'alignement de deux longues plages balayées et une série de plages de terre ferme dans une baie essentiellement fermée, Port Philippe, Australie. Le modèle, qui peut être appliqué à une bathymétrie irrégulière et des conditions de houle variables, indique qu'il y a, dans les conditions d'énergie de mer du vent qui ont dominé, un bilan sédimentaire quasi-nul le long de ces plages. Comme la baie est presque entièrement close, les résultats suggèrent que les plages ont atteint un alignement en équilibre avec la forme générale de la baie et les conditions climatiques. Cette analyse souligne l'importance de cet alignement des plages, dans un environnement à sédiments limités et la nécessité de se conformer cet alignement d'équilibre lorsque des plages artificielles sont créées dans cette baie ou d'autres semblables.—*Catherine Bousquet-Bressolier, Géomorphologie, E.P.H.E., Montrouge, France.*

□ ZUSAMMENFASSUNG □

Es wurde ein numerisches Modell entwickelt, welches die Wellenbrechung und den Sedimenttransport entlang der Küste berücksichtigt. Mit diesem Modell wurde die Ausrichtung von zwei geschwungenen Stränden sowie von weiteren Ständen an Landzungen in einer weitestgehend geschlossenen Bucht (Port Phillip Bay, Australien) untersucht. Dieses Modell, welches für die generelle Anwendung bei unregelmäßigen Tiefenverhältnissen und veränderlichen Wellen geeignet ist, zeigt an, daß entlang der untersuchten Strände der Netto-Sedimenttransport unter dem vorherrschenden windbedingten Wellenenergieklima gleich Null war. Da die Bucht fast vollständig geschlossen ist, legt dieses Ergebnis den Schluß nahe, daß die Anordnung bzw. Ausrichtung der Strände sich im Gleichgewichtszustand mit der Gesamtgestalt der Bucht und den lokalen Wetterbedingungen befindet. Die vorliegende Analyse macht deutlich, daß man bei der Gestaltung künstlicher Strände in dieser oder einer ähnlichen sedimentarmen Bucht berücksichtigen muß, daß die Anordnung der Strände mit der des Gleichgewichtszustandes übereinstimmt.—*Ulrich Radtke, Geographisches Institut, Universität Düsseldorf, F.R.G.*

□ RESUMEN □

Para analizar la alineación de dos playas y una serie de cabos costeros en una bahía encerrada, (Bahía de Port Phillip, Australia), se utilizó un modelo numérico de refracción de olas y de transporte de sedimentos a lo largo de la costa. El modelo que resulta apropiado para aplicarlo a batimetrías regulares y condiciones de olas variables, indicaba que el transporte neto de los sedimentos, en estas playas, era esencialmente nulo bajo las condiciones energéticas del régimen de olas de viento dominante. Dado que la bahía es casi totalmente cerrada, los resultados sugieren que las playas han logrado una alineación tal, que están en equilibrio con la forma total de la bahía y las condiciones meteorológicas locales. Los análisis señalan la importancia de la alineación de la playa en un ambiente sedimentario limitado y la necesidad de ajustarlas con la alineación de equilibrio, cuando se forman playas artificiales, tanto en esta bahía como en otras de condiciones similares.—*Néstor W. Lanfredi, CIC-UNLP, La Plata, Argentina.*