

An Integrated Numerical Approach for Coastal Engineering Problems

A. S.-Arcilla

University of Catalunya
Escuela Técnica Superior
Ingenieros de Caminos
Canales y Puertos.
Jordi Girona Salgado, 31
08034 Barcelona, Spain



ABSTRACT

ARCILLA, A.S., 1989. An integrated numerical approach for coastal engineering problems. *Journal of Coastal Research*, 5(3), 603-616. Charlottesville (Virginia). ISSN 0749-0208.

This paper presents a system that is capable of generating numerical models for water and sediment motion, in which the level of precision is defined by the accuracy of initial and boundary data. This provides a simulation in which the cost is concordant not only with the amount and precision of available input information but also with the general objectives of the project.

The system is divided into three units: (i) wave/current propagation, (ii) nearshore circulation and (iii) shore-line evolution. Unit (i) is composed of two models of which the first is based on linear (sinusoidal) theory and valid only for wave refraction. The second model uses a rather general set of mass and momentum conservation laws and is able to reproduce wave and current propagation, including their mutual interactions together with the effects of refraction, diffraction and reflection. An original absorbing-reflecting boundary condition is also presented. This condition, apart from being more accurate for 2D flow than most of the state-of-the-art conditions, is also more general, allowing a unified treatment for all boundaries.

Unit (ii) is made up of two modules. The first, based on analytical solutions for wave-decay and longshore flow, is used for alongshore uniform problems. The second is based on mass and momentum conservation equations (including radiation stresses), using vertical and across-shore integration. This technique, developed here to allow an efficient calculation of set-up and transverse and alongshore velocities for a varying coast-line, provides hydrodynamic information with the precision level required by bulk formulations for sediment transport. These formulations are the basis of unit (iii) which, at the moment includes only a 1-line shoreline evolution model. This model is able to reproduce the effect of the most usual coastal works (perpendicular groynes, detached breakwaters, etc.).

ADDITIONAL INDEX WORDS: *Numerical models, boundary conditions, waves, sediment transport, shoreline evolution, breaker zone, hydrodynamics.*

INTRODUCTION

The analysis of coastal engineering problems (associated with water and sediment motion) usually requires numerical or hydraulic, reduced scale, models. The former are complex and sophisticated due to the existence of unknown boundaries, high turbulence and stochastic forcing terms. The latter are very much limited because of scale effects and distorted sediment characteristics. Furthermore, measurements in the breaker zone (in which the most significant processes take place) are scarce, expensive and not very accurate. This generally precludes an accurate calibration of the more sophisticated models.

However, most coastal problems are associated with large costs (including environmental impact) and therefore require quantitative engineering solutions. This explains why in many of the numerical simulations of water and sediment motion in the surf zone coexist rather sophisticated models (*e.g.* hydrodynamics) with semi-empirical formulations (*e.g.* sediment transport).

This paper presents an integrated numerical model for water and sediment motion in which the level of precision is defined by the accuracy of initial and boundary data. This provides a simulation in which the cost is closely related not only to the amount and precision of the input values but also to the general objectives

of the project. The proposed model is divided into the following routines:

- (i) Wave/current propagation
- (ii) Nearshore circulation
- (iii) Shore-line evolution

Most of the included models are rather well-known and will not, thus, be treated in detail here. The emphasis will be on two new items of the integrated model:

- (i) Boundary conditions for the wave/current propagation module.
- (ii) Across-shore integrated nearshore circulation module.

WAVE/CURRENT PROPAGATION MODULE

This module is made up of two models:

- (a) Standard wave-ray model, which is automatically selected for the cases in which only wave refraction (eventually also bottom friction) is important.
- (b) Wave propagation model based on the Boussinesq approximation for the mass and momentum conservation laws (e.g. (S.-ARCILLA and MONSÓ, 1986)). This model is automatically selected for the cases in which reflection/diffraction effects are important.

Hydrodynamic Model

The **b** model uses a set of non-linear mass and momentum conservation laws, similar to the ones proposed by other authors (ABBOTT, 1981), (S.-ARCILLA and MONSÓ, 1985):

$$\frac{\partial \eta}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0 \text{ (mass equation)}$$

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x} \left(\frac{p^2}{H} \right) + \frac{\partial}{\partial y} \left(\frac{pq}{H} \right) + gH \frac{\partial \eta}{\partial x} = F_x + G_x + E_x \text{ (x-momentum equation)}$$

in which

- t: time
- x,y : horizontal coordinates
- η : free-surface elevation
- p,q: x and y components of the mass flux vector
- H = h + η : total water depth (h : still water depth)

F_x : term including various external acting forces (Coriolis, wind stress, bottom friction and atmospheric pressure effects) (S.-ARCILLA *et al.* 1986)

G_x : term including third-order derivatives arising from the vertical acceleration included in the momentum equations (and which, in turn, induces a non-hydrostatic pressure distribution and a linear variation with depth of the vertical velocity) (S.-ARCILLA *et al.*, 1986).

E_x : term including second order derivatives associated to turbulence effects, modeled as a function of an eddy viscosity coefficient, (S.-ARCILLA *et al.*, 1986).

The y-momentum equation is entirely similar.

These equations are discretized with an Abbott type finite differences scheme. The centred and implicit algorithmic equations obtained are solved by means of an alternating-direction double sweep technique. The algorithmic structure, though non-linear in the problem variables (η , p, q) is linear in the variables at the unknown time-level, with a second order truncation error (S.-ARCILLA *et al.*, 1986).

- (i) x-sweeping

$$Ap_{j,k}^{n+1} + BH_{j+1,k}^{n+1/2} + CH_{j-1,k}^{n+1/2} = D$$

$$EH_{j,k}^{n+1/2} + Fp_{j+1,k}^{n+1} + Gp_{j-1,k}^{n+1} = I$$

- (ii) y-sweeping

$$Rq_{j,k}^{n+1} + SH_{j,k+1}^{n+1} + UH_{j,k-1}^{n+1} = V$$

$$WH_{j,k}^{n+1} + Xq_{j,k}^{n+1} + Yq_{j,k}^{n+1} = Z$$

in which A, B, C, D, E, F, G, I, R, S, U, V, W, X, Y, Z are functions of various parameters and the problem variables at known (past) time levels. These coefficients include the non-linear part of the equations at various time levels in such a way that the truncation error associated to the finite differences discretization is always of order $O(\Delta S^2, \Delta t^2, \Delta S, \Delta t)$, (with ΔS being some representative mesh size) or smaller.

Horizontal sweepings are performed first, providing values for p^{n+1} and $H^{n+1/2}$. Vertical

sweepings are carried out afterwards and are used to estimate q^{n+1} and H^{n+1} .

Once horizontal and vertical sweepings are completed the problem variables (H, p, q) are known at time level $n + 1$ (starting from level n) in a staggered mesh as shown in Figure 1.

The non linear version of the discretized equations require, however, the three variables (H, p, q) at each node point. This necessitates an additional interpolation throughout the domain to supplement the values obtained from the sweeping operations. The order of the interpolation must be connected to the truncation error of the discretized scheme. The overall truncation error so obtained for the model is of second order, both in space- and time-increments.

The resulting non-linear scheme is consistent and stable except for large Courant numbers, due to the presence of some explicitly estimated values to elude iterations in the solution procedure. This, however, does not severely restrict the model applicability for short (wind) waves cases.

Absorbing-Reflecting Boundary Conditions

The main points which, in practical engineering, define the model applicability are computer requirements and boundary modules

available. The first point has been satisfactorily solved (at least from the standpoint of practical applications) with the selected algorithmic structure and double sweeping technique. This, together with a staggered information storage and a numerical grid generation module (S.-ARCILLA *et al.*, 1986) provide a very efficient wave propagation model.

With respect to the second point, the number of boundary conditions (B.C.) required by the hyperbolic equations presented in the previous section is determined by the number of characteristic lines (1-D) or surfaces (2-D) defined in the considered area (DAUBERT and GRAFFE, 1967). These conditions must, also, define a well-posed problem and allow the exit of waves generated in the domain through the "open" boundaries (*i.e.* lines separating the domain from other sea areas). For practical application, also partially reflecting B.C. are required, to simulate partial reflection effects as, *e.g.*, those encountered in breakwaters, *etc.*

The existing non-reflecting B.C. are based on the Riemann invariants for the 1D, linear, hyperbolic equations (VERBOOM, 1983). These conditions, however, are not locally a good approximation to the physical problem when waves are not planar and/or are obliquely incident to the boundary. The induced reflection is not present in the physical problem even though far from the boundary the model results

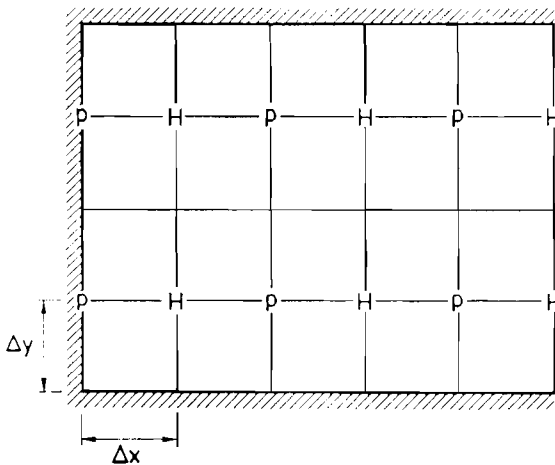


Figure 1. Staggered spatial disposition of the problem variables (H, p, q) after horizontal and vertical sweepings have been performed. This information is then completed by quadratic interpolation.

may be acceptable from an engineering standpoint.

The new B.C. module presented here permits not only a reduction of the degree of reflection in "open" boundaries but also allows a unified treatment of conditions with variable reflection. This means that the same equations can be employed for a reflecting vertical wall, for a partially reflecting breakwater or for a dissipative beach. This, in turn, means less boundary modules for a specific application of the model, with the subsequent savings in pre-processing time.

The proposed B.C. is based on a linear superposition of the in- and out-going mass-flux vectors at the boundary. This may be written as:

$$\vec{P}_r = C\eta_j \vec{U}_j + C\eta_s \vec{U}_s$$

in which:

j, s : sub-index denoting in-going or out-going waves, respectively.

\vec{U} : unit vector in the direction of wave propagation.

C : wave celerity.

The free-surface elevation at some generic boundary point, η_r , may be, likewise, written as:

$$\eta_r = \eta_j + \eta_s$$

Considering that

$$\vec{P}_r = (p, q)$$

$$\vec{U}_j = (\cos\theta_j, \sin\theta_j), \vec{U}_s = (\cos\theta_s, \sin\theta_s)$$

it is easy to obtain two equations for every stretch of discretized boundary:

$$p = C\eta_j \cos\theta_j + C\eta_s \cos\theta_s$$

$$q = C\eta_j \sin\theta_j + C\eta_s \sin\theta_s$$

$$\eta_r = \eta_j + \eta_s$$

The problem so defined has five unknowns at the boundary: $p, q, \eta, \eta_s, \theta_s$. Two of the three remaining equations required are the mass and (corresponding) momentum conservation laws.

To calculate θ_s (out-going wave direction) an additional equation must be invoked:

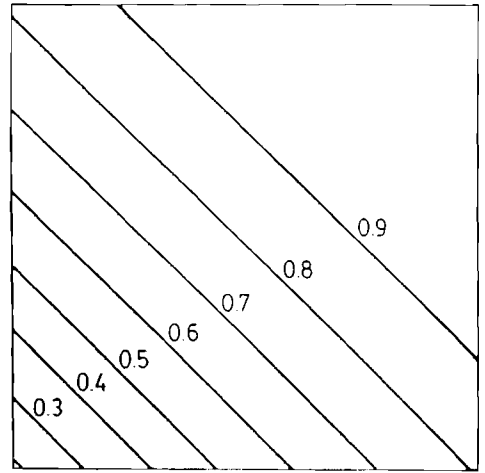


Figure 2. Contour lines of the disturbed free-surface for a square domain, with four open sides and oblique wave incidence, to illustrate the performance of the absorbing-reflecting condition. (a) $n = 20$ points per wave length, (b) $n = 100$ points per wave length.

$$\frac{\partial \Theta_s}{\partial n_B} = 0$$

in which n_B is the unit, normal to the boundary, vector.

This approach to the problem means an improvement over the conventional weakly-reflecting boundary condition, based on the 1-D Riemann invariants. Furthermore, the conventional solution usually requires the use of linearised conservation equations in a finite width strip, parallel to the boundary. The proposed solution, on the other hand, only needs linearised superposition at the boundary line.

The model has been calibrated with cnoidal waves, which are the analytical solution of the employed equations when no extra terms of type F_x, E_x are considered (S.-ARCILLA *et al.*, 1986). After this, the model was run for a square domain with four open sides, in which the direction of wave incidence coincides with one of the diagonals (Figure 2). This case is particularly illustrative of the performance of the new absorbing-reflecting B.C. Figure 2 shows contour lines of the disturbed free-surface which are nearly parallel and oriented perpendicularly to the diagonal. This confirms the accuracy of the proposed B.C.

Application to Coastal Hydrodynamics

To illustrate the model output for a practical coastal engineering problem the program was applied to the analysis of long-wave propagation and resonance in the Bay of Mahon, Minorca (Balearic Is., Spain).

The test run was done with a long, regular wave train, since the main aim of the study was the occurrence of resonance in some of the inner basins of the bay. The wave period and amplitude were, respectively, 240 secs. and 1.0 m.

The results shown were calculated by a routine primarily designed for irregular waves and, thus, correspond to the ratio of the variance (a measure of the wave energy) at the considered point over the variance at the bay entrance. The numerical results correspond to 360 points per wave length and 24 time steps per wave period which means a high level of accuracy. The time of run was calculated so that at least 10 complete waves reach the farthermost end of the bay. This is to give statistical significance to the calculated variance.

Figure 3 shows the velocity field at time $t = 7.500$ secs. due to the long wave propagation. The arrows indicate direction and modulus of the induced velocity.

Figure 4 depicts the contours of iso-agitation associated to this same long wave propagation problem. The agitation coefficient has been

defined as the afore mentioned ratio of variances.

The run was performed with the newly developed reflecting-absorbing B.C. This means a more realistic simulation of the long-wave problem in the Bay due mainly to the occurrence of seiching ("rissagas" in the local vernacular) for which multiple reflections play an important role.

NEARSHORE CIRCULATION

Nearshore circulation, from an engineering point of view, is usually required to nourish a shore-line evolution model. The basic input are wave conditions near the breaker-line, obtained either from direct measurements or, more often, from deep-water wave data propagated with a model such as the ones described in section 2. Once the nearshore circulation has been determined, sediment transport and shore-line evolution can be evaluated. The final aim is, thus, shore-line evolution which is normally calculated with 1- or 2-line evolution models, based on bulk formulations for sediment transport. This means that the only hydrodynamic information required by these simplified models is an average or characteristic longshore and transverse current velocities. With this, both the alongshore and transverse sediment transport may be evaluated.

The hydrodynamic model developed fulfills

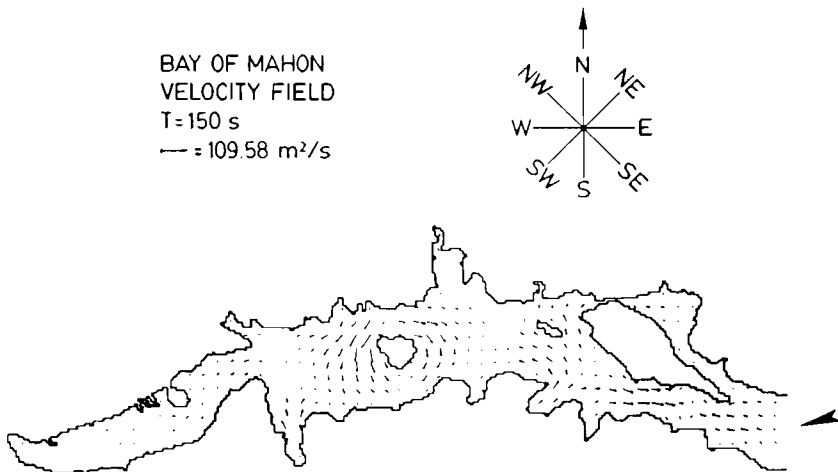


Figure 3. Velocity field at $t = 7.500$ sec. induced by long-wave propagation into the Mahon Bay (Spain).

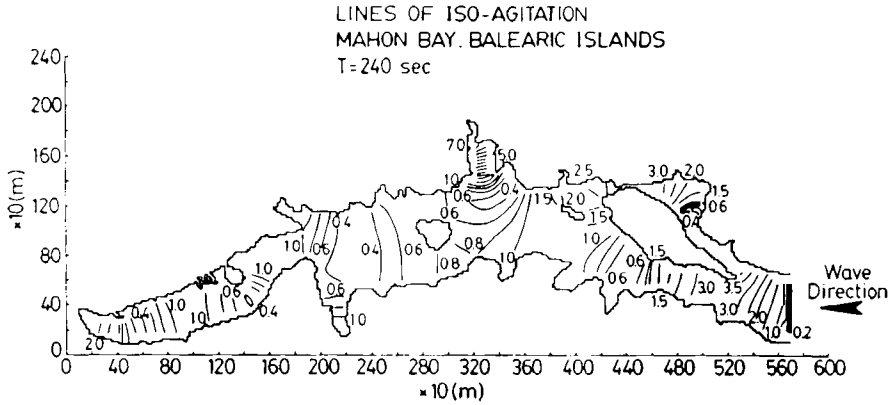


Figure 4. Contours of iso-agitation for a long-wave propagation problem in the Mahon Bay. (Spain).

these requirements, providing the alongshore and transverse velocities at each profile together with the corresponding set-up. It also considers alongshore variability, nearly always present in coasts with a high number of groynes, revetments, etc.

Hydrodynamic Model

The model is based on the mass and momentum conservation equations vertically integrated and time-averaged, for an incompressible quasi-horizontal flow (e.g. PHILLIPS, 1977).

$$\frac{\partial \bar{\eta}}{\partial t} + \frac{\partial \bar{p}}{\partial x} + \frac{\partial \bar{q}}{\partial y} = 0 \quad (\text{mass})$$

$$\frac{\partial \bar{p}}{\partial t} + \frac{\partial}{\partial x}(\bar{p}\bar{U}) + \frac{\partial}{\partial y}(\bar{p}\bar{V}) + \frac{\partial}{\partial x} [S_{xx} - T_{xx}] + \frac{\partial}{\partial y} [S_{xy} - T_{xy}] = \bar{P}d \frac{\partial h}{\partial x} - g(h + \eta) \frac{\partial \bar{\eta}}{\partial x} + \tau_x^F - \tau_x^B \quad (\text{x-momentum})$$

denotes time averaging and \bar{U} , \bar{V} are the x and y components of the averaged velocity, given by:

$$\bar{U} = \frac{\bar{p}}{\bar{\eta} + h}, \quad \bar{V} = \frac{\bar{q}}{\bar{\eta} + h}$$

The total fluid velocity is, thus:

$$u(x, y, z, t) = \bar{U}(x, y, t) + \hat{u}(x, y, z, t)$$

with \hat{u} the fluctuating component (and similarly for the v).

h : water depth measured from the still water surface.

$\bar{P}d$: average dynamic pressure on the bottom.

$$\bar{P}d = P_{total} - g(h + \bar{\eta})$$

S_{ij} : (ij) component of the radiation stress tensor, representing excess momentum fluxes associated to the oscillatory motion.

T_{ij} : (ij) component of the viscous stress tensor, representing excess momentum fluxes associated mainly to the turbulent motion.

τ_x^F : x-component of the tangential stress at the free-surface.

τ_x^B : x-component of the tangential stress at the bottom.

Assuming the mean flow is stationary, and thus there are no stresses acting on the free surface (usually of a transient nature) and that turbulent and oscillatory motions are uncoupled, the equations can be simplified (MEI, 1983). Furthermore, the S_{ij} terms are usually evaluated from sinusoidal theory (LONGUET-HIGGINS and STEWART, 1962) while T_{ij} terms are normally expressed as a function of the mean flow variables (\bar{U}, \bar{V}) by means of the closure hypothesis of Boussinesq (e.g. VON SCHWIND, 1980).

The bottom friction term, τ_x^B , is written as a friction coefficient, C_r , times the square of the total (mean + oscillatory) velocity. Assuming

mean velocities are smaller than the amplitude of the oscillatory motion, the following expressions may be obtained (LIU and DALRYMPLE, 1978), (KRAUS and SASAKI, 1979), (S.-ARCILLA *et al.*, 1986):

$$\tau_x^B = Cf \frac{y}{\pi} \sqrt{g(h + \bar{\eta})} [U(1 + \cos^2\alpha) - V\sin\alpha\cos\alpha]$$

in which:

γ : breaker index defined as the ratio of wave height to water depth.

α : angle of wave incidence (angle between crests and shoreline general orientation).

The problem is, thus, governed by three equations (depth and time-averaged mass and momentum conservation laws) and is written in terms of three unknowns: the two horizontal velocities or mass fluxes (\bar{U}, \bar{V}) or (\bar{p}, \bar{q}) and the mean water surface ($\bar{\eta}$) due to the presence of breaking waves. The model also requires, as an additional input, a wave-height decay law, which is calculated externally using the model proposed by (GUZA and THORNTON, 1985).

Numerical Solution

The resulting system of 3 coupled non-linear partial differential equations must be solved numerically together with appropriate boundary conditions.

Since the hydrodynamic information from this model is going to be used to evaluate sediment transport with bulk formations (*e.g.* C.E.R.C. formula) only across-shore average values of the three unknowns will be calculated. This means that just the across shore mean of \bar{U} , \bar{V} , and $\bar{\eta}$ will be obtained. This allows, in spite of the complexity of the equations, a very efficient solution procedure which can be used even in rather small personal computers.

Considering an alongshore varying coast-line (Figure 5), the solution procedure can be schematized as follows:

Divide the nearshore area into finite elements stretching from shore- to breaker-line and with aspect ratios near 1.0.

Prescribe the across-shore variation of the

three unknowns in terms of linear shape functions.

With this, all x-derivatives (x: local transverse axis, Figure 5) in the conservation equations can be explicitly evaluated. Up to this point, the model requires as input the wave conditions at the mean breaker-line (wave height, period, direction and set-down).

The resulting equations are only a function of the cross-shore averages of \bar{U} , \bar{V} and $\bar{\eta}$ (u_m, v_m and η_m) and their alongshore derivatives ($\partial/\partial y$):

$$u_m \frac{(h_b + \eta_m)}{x_b} \beta_{1x} + \frac{\partial[(h_m + \eta_m)v_m]}{\partial y} = 0 \text{ (mass)}$$

$$u_m^2 \frac{(h_b + \eta_b)}{x_b} \beta_{2x} + \frac{\partial[(h_m + \eta_m)u_m v_m]}{\partial y} + \frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - A_H \frac{\partial^2[(h_m + \eta_m)u_m]}{\partial y^2} + g(h_m + \eta_m) \frac{\eta_b - \eta_m}{x_m} + \tau_x^B = 0$$

(x-momentum)

in which

- A_H : eddy viscosity coefficient.
- β_{1x}, β_{2x} : coefficients obtained from the prescribed shape functions for the across-shore variation of the unknowns \bar{U} , \bar{V} , $\bar{\eta}$.
- x_b : distance from breaker- to shore-line of which only the initial value (corresponding to the still water level) is known.
- x_m : distance from the central node of each element to the shore line.
- η_m : mean water level (measured from the still water level) at the central node.
- h_m : water depth at the central node.

(the y-momentum equation is entirely similar)

These equations are, next, discretized in the alongshore direction by means of a second-order, centred finite-differences scheme. The algorithmic structure obtained only uses values of the unknowns at the central node of each element, expressed in the local axes of each element (Figure 5). This lumping of the scheme provides excellent stability and accuracy properties. Further information on the derivation and characteristics of these equations may be found in (S.-ARCILLA *et al.*, 1986).

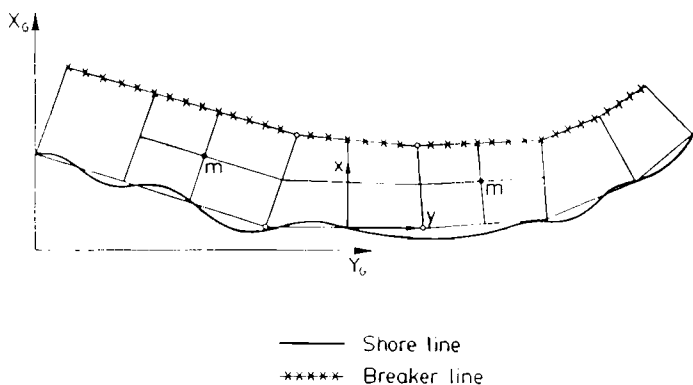


Figure 5. Schematization of an alongshore varying coast-line discretized into finite elements.

The resulting system of $3N$ discretized equations (3 equations applied to the central node of each element, with N being the number of elements) together with the $3N$ unknowns (U_m , V_m , η_m at each central node) can be solved. To minimize the number of iterations, due to the non-linearity of the equations, the problem was subdivided into three steps, inspired on the analytical solution for uniform beaches:

Step 1: Starting from known (hypothesized) values for U_{mi} and η_{mi} ($i = 1, \dots, N$) V_{mi} is estimated with the alongshore (y-direction) momentum equation. The starting values for U_m and η_m are obtained from the solution for uniform beach and normal wave incidence.

Step 2: Using the assumed values for η_{mi} and the calculated values for V_{mi} (step 1), U_{mi} ($i = 1, \dots, N$) is estimated with the mass conservation equation.

Step 3: Using the calculated values for V_{mi} (step 1) and U_{mi} (step 2), η_{mi} ($i = 1, \dots, N$) is estimated with the transverse (x-direction) momentum equation.

The process is now repeated, using always the two most recent values of the two "frozen" variables to estimate a new value of the third unknown. The formal structure of the resulting discrete equations defines the number and type of B.C. necessary to have a well defined problem. The structure of the equations, derived in (S.-ARCILLA *et al.*, 1986), is:

(i) mass equation:

$$a_1 U_m + b_1 = 0$$

(ii) x-momentum equation:

$$a_2 + \frac{d(b_2 \eta_m)}{dy} + \frac{d^2(c_2 \eta_m)}{dy^2} + e_2 \eta_m + f_2 \eta_m^2 = 0$$

(iii) y-momentum equation:

$$a_3 V_m + \frac{d(b_3 V_m^2)}{dy} + \frac{d^2(c_3 V_m)}{dy^2} + e_3 = 0$$

in which a_i , b_i , c_i , e_i , f_i are variable coefficients, functions of the two "frozen" variables in each equation (that explains why only one unknown appears explicitly in each equation).

It should be remembered that these expressions are the result of discretizing the original conservation laws with a prescribed transverse variation for all three unknowns in terms of given shape functions. Furthermore, various B.C. have been already imposed at the breaker line (see the preceding sub-section).

From this formal structure for the conservation equations it is easy to obtain the number and type of required B.C. The y-momentum equation, which is first solved with the proposed iteration scheme, permits two B.C. provided the second-order derivative term (arising from the lateral turbulent mixing) is comparable to the rest of terms. If this is not so, we have in fact a first-order differential equation which only accepts one B.C. Since in alongshore varying coasts it is usual to know V_m at both boundaries (e.g. $V_m = 0$ at the two limiting groynes or natural barriers of a pocket beach) it is there-

fore important to maintain a true second-order differential equation. This requires that the order of magnitude of the second-order derivative (lateral mixing term) be comparable to the rest of terms.

Performing an order of magnitude analysis of the various terms of the discretized equations, it is easy to obtain that all are of the same order, except the turbulent mixing term which is A_H/x_c times smaller (x_c : characteristic across-shore distance). The only possibility to retain this second-order derivative term is to use an eddy viscosity coefficient of order:

$$A_H = \hat{A}_H x_c \text{ with}$$

$$\hat{A}_H = O(1)$$

This solution, which allows to maintain two B.C. for the unknowns V_m and η_m , has been introduced from purely numerical considerations. It is, however, easy to find that it possesses a physical meaning as well. The reason is the additional circulation induced by the two lateral boundaries with respect to the longshore-uniform problem. In this latter case $A_H = O(1)$ and it seems, thus, reasonable to assume for a non-uniform coast an A_H value modified by the length of the induced additional circulation. Since this length appears to be of order x_c (e.g. GOURLAY, 1976) it is physically consistent to use an eddy viscosity coefficient so distorted (the extra length included is duly considered in the equations to avoid trouble with the dimensions).

The x-momentum equation, solved to obtain η_m , shows the same formal structure and will not be further considered here. It is, however, important to mention that the model uses Neumann B.C. for η_m (the reason being that boundary values for η_m are not usually available):

$$\frac{\partial \eta_m}{\partial n} = 0$$

in which n is the normal to the boundary.

The last equation to be considered is the mass conservation law. Due to its particular structure no alongshore boundary conditions are allowed (transverse conditions have been already prescribed via the shape functions and the values given at the breaker line). This situation presents practical advantages because U_{mi} , $i = 1$ and N , (values of the cross-shore

velocity at the central node of the two boundary elements) are either unknown or hard to evaluate in most coastal problems.

The numerical solution is, then, obtained using the iterative approach described. The model now works with an error of 10^{-5} for the three variables, U_m , V_m , η_m with the required number of iterations being usually less than 5.

Calibration

The model has been first calibrated with the existing analytical solution for longshore uniform beaches (e.g. BOWEN, 1969; LONGUET-HIGGINS, 1970). The only free parameters of the model are C_f (bottom friction coefficient), \hat{A}_H (eddy viscosity coefficient) and γ (ratio of breaker wave height to water depth). Values for γ are usually obtained from measurements which leaves the model with two free parameters (C_f and \hat{A}_H) as is the case for most analytical models for the longshore current velocity. The model appears to work satisfactorily for all test problems run, showing a very low sensitivity to the actual size of the elements used to discretize the surf zone, provided the aspect ratio is not too different from 1.0. All these test runs were performed with data from Leadbetter beach (WU *et al.*, 1980) using for C_f the values proposed in the original paper. To obtain the measured longshore current velocity the only free parameter was, then, \hat{A}_H . The required values were always $\hat{A}_H \approx O(1)$, as expected, showing deviations from 0.5 to 1.5.

After this, the model was calibrated with data obtained from hydraulic tests for an alongshore varying beach (GOURLAY, 1976), (GOURLAY, 1978). The beach considered (Figure 6) has two distinct parts: a straight zone exposed to the incident waves and a circular zone, sheltered by a perpendicular groyne. The shoreline is nearly parallel to the breaker line (after being affected by refraction and diffraction). This test case, therefore, basically reproduces the nearshore circulation induced by an alongshore gradient of wave height. The breaker line was obtained from the measurements.

Wave-heights were recorded with capacity sensors while the velocity field (Figure 7) was evaluated using horizontal photographs (with floating tracers added to the water) and pitot tube recorders (GOURLAY, 1976).

The resulting circulation pattern, extremely

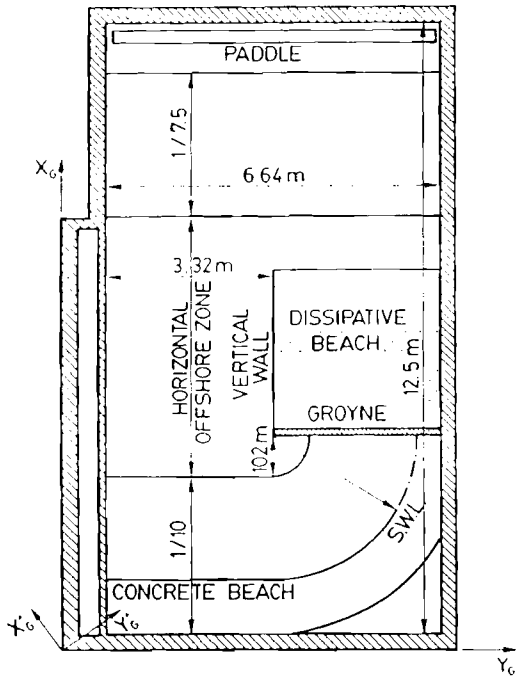


Figure 6. Schematization of the reduced-scale model test done by Gourlay to obtain the nearshore circulation for an alongshore varying coastline (Gourlay, 1976).

complex as may be seen from the figure, extends well beyond the breaker line due to the two lateral boundaries (groyne and model wall). Since the numerical model only considers the surf

zone circulation these outer currents must be subtracted from the surf zone measurements. This correction introduces a further uncertainty in the calibration which must be considered, in spite of the goodness of the fit obtained, as only qualitative. Furthermore, the difficulties associated to this correction only allow for a calibration of the longshore current velocity, v_m , precluding any operations with the transverse velocity, u_m and the mean level variation, η_m . The latter two variables, therefore, have only been calibrated in a very qualitative sense.

The surf-zone has been discretized with three mesh sizes originating 28, 16 and 12 elements, respectively. This has permitted an analysis of the sensitivity of the model to the element shape and size. The lower side of the elements, corresponding to the shore-line discretization, is shown in Figure 8 for the cases of 28 and 12 elements. The line has been rotated 35° , with respect to the hydraulic model, to elude angles greater than 90° between the shore-line and the coordinate axes (to avoid some trigonometric difficulties in the equations).

The overall circulation pattern obtained from the hydraulic test is correctly reproduced by the numerical model (longshore current flowing towards decreasing wave heights and existence of an eddy near the groyne which implies a negative longshore velocity and a limiting point of zero velocity). A more detailed comparison of the longshore current velocity appears in Fig-

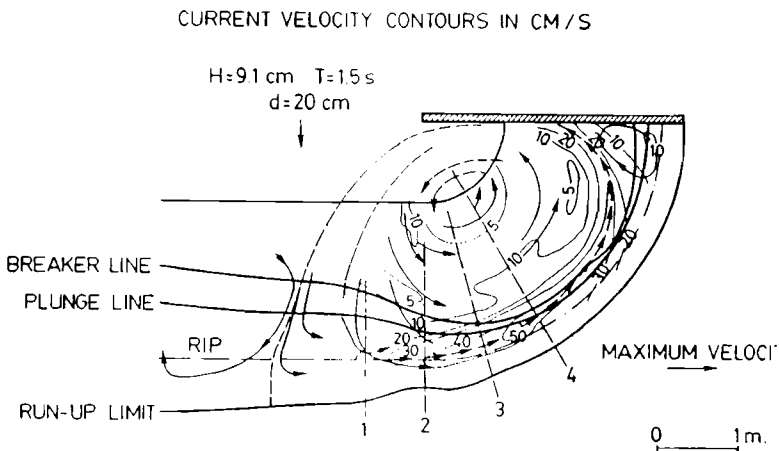


Figure 7. Contours of equal current velocity and lines of breaking, plunging and run-up for the hydraulic model shown in Figure 6 (Gourlay, 1976).

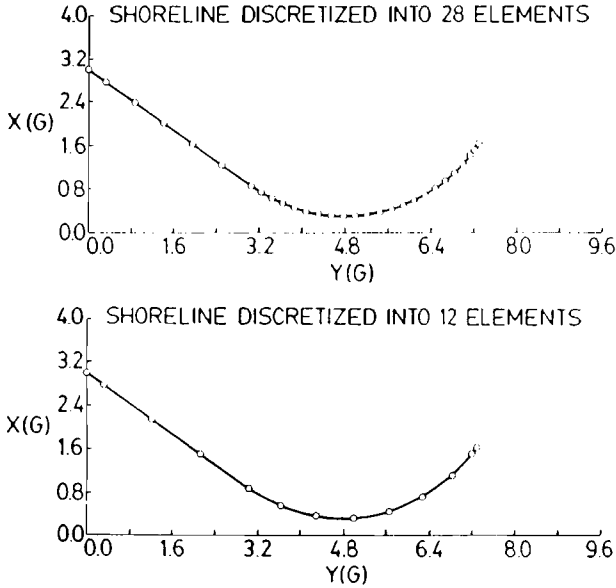


Figure 8. Schematization of the beach discretization (with a): 28 elements, and (b): 12 elements) performed for the problem shown in Figure 6. Only the lower sides of the four-sided elements are shown.

ure 9. These velocities have been calculated with $C_f = 0.01$ and $\bar{A}_H = 0.7$, which are very reasonable values for this particular problem.

The obtained fit is surprisingly good, particularly for the 28 element discretization, but must be considered only as qualitative, due to all uncertainties involved (correction due to the outer circulation, plunging breakers for which the assumed relationship $H = \gamma h$ is doubtful, etc.). It is worthwhile mentioning the limited effect of C_f on the velocity values. This appears to be the case for currents generated by an alongshore variability, in which convection and longshore gradient terms are essential. For the case of a uniform beach, in which the currents are essentially generated by the oblique incidence, the main retarding term is bottom friction, which also retains its importance in the general (intermediate) problem.

The numerical model has been, therefore, calibrated with two cases (longshore uniform beach and beach everywhere parallel to the breaker line) corresponding to two distinct generation mechanisms for nearshore circulation, viz. oblique incidence and alongshore gradient of wave-height. Since the calibration has proved to be satisfactory for these two problems

it is reasonable to expect the model will also be adequate for a general intermediate case, though no calibration has been performed due to the lack of data.

The hydrodynamic information so obtained can then be used as input for a 1- or 2- line model of shoreline evolution (ARCILLA and VIDAOR, 1986). These two modules (hydrodynamic + shore-line evolution) have been applied to a coast-line with extreme alongshore variability due to the high number of groynes, revetments, etc. built. This coast is located north of Barcelona (Spain) in the Mediterranean Sea. A sample result of the predicted evolution is shown in Figure 10. The information required as input by the model was obtained from a 1-year field campaign which included wave-heights, longshore currents and aerial photographs of the shore-line.

CONCLUSIONS

The main advantage of the presented numerical model for coastal hydrodynamics is that it automatically selects the most appropriate equations for each case. This means that, for a uniform coastline with no obstacles

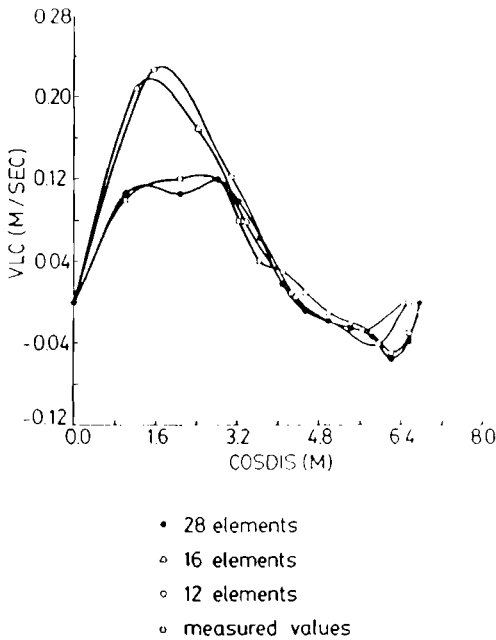


Figure 9. Longshore current (VLC axis) variation along the shoreline (COSDIS axis) of the problem shown in Figure 6. The numerically obtained results with 28, 16 and 12 elements are compared with measured values from the reduced-scale test.

(islands, groynes, etc.), the linear refraction model plus some analytical formulation for the longshore current are automatically activated. On the other hand, for a coast with reefs and islands together with an alongshore variability due to headlands (natural or artificial), groynes or even some harbour, the more complex Bousinesq model is selected. The surf zone circulation is afterwards obtained with the nearshore circulation model presented.

The conclusion is that, for each particular application, the optimum model (from the point of view of accuracy and cost) is selected based on a simple input code associated to the available information and the aims of the project. This, furthermore, implies that, in most cases, the hydrodynamic problem and the associated shoreline evolution can be solved with a personal computer at a reasonable cost.

The presented models also include two improvements, advantageous both from a theoretical and a practical standpoint. The first is the proposed absorbing-reflecting B.C., which is

more accurate and versatile than most state-of-the-art B.C. for this problem. The second is the across-shore integrated nearshore circulation model. The improvement in this case is of a more practical nature, for it allows the resolution of a very complex physical problem with high speed and low cost and a level of accuracy appropriate to the precision of the standard bulk formulations for sediment transport.

ACKNOWLEDGEMENTS

Most of the results presented here are due to the research work carried out by Dr. Monsó, Ing. Sierra and Ing. Vidaor. The manuscript was typed by N. Negre. The financial support for the theoretical models was provided mainly by the Spanish Ministry of Public Works. The field campaign was sustained by the Generalitat of Catalonia.

LITERATURE CITED

- ABBOTT, M. B., MC.COWAN, A. and WARREN, I. R., 1981. Numerical Modelling of Free-Surface Flows that are 2-D in plan. In: *Transport Models for Inland and Coastal Waters*. New York: Academic Press.
- ARCILLA, A. S.- and MONSO, J. L., 1985. Modelado Numérico del Flujo en Zonas Costeras. Ed. Dirección General de Puertos y Costas. Pub. n° 7 M.O.P.U. *Programa de Clima Marítimo*.
- ARCILLA, A. S.- and MONSO, J. L., 1986. Numerical Modeling of Coastal Flow, Envirosoft 86. (Los Angeles). *Computational Mechanics Publications*.
- ARCILLA, A. S.- and MONSO, J. L., 1986. Numerical Grid Generation in Coastal Hydrodynamics. *International Conference on Numerical Grid Generation*. Landshut (Germany).
- ARCILLA, A. S.- and VIDAOR, A., 1986. Análisis Hidrodinámico de la Zona de Rompientes. Servicio de Publicaciones del M.O.P.U. Madrid.
- ARCILLA, A. S.- and VIDAOR A., 1987. Impacto de Obras de Defensa de Costas empleando un Modelo de evolución costera. *Revista de Obras Públicas*. Madrid.
- BOWEN, A. J., 1969. The Generation of Longshore Currents on a Plane Beach. *Journal of Maritime Research*, 27.
- DAUBERT, A., and GRAFFE, O., 1967. Quelques aspects des Écoulements presque Horizontaux a Deux Dimensions en Plan et Non Permanents. *La Houille Blanche*. Vol. 22 n° 8.
- GOURLAY, M. R., 1976. Non-Uniform Alongshore Currents. *Int. Conference on Coastal Eng.* ASCE.
- GOURLAY, M. R., 1978. Wave set-up and Wave Generated Currents in the Lee of a Breakwater or Headland. *International Conference on Coastal Engineering*. ASCE.

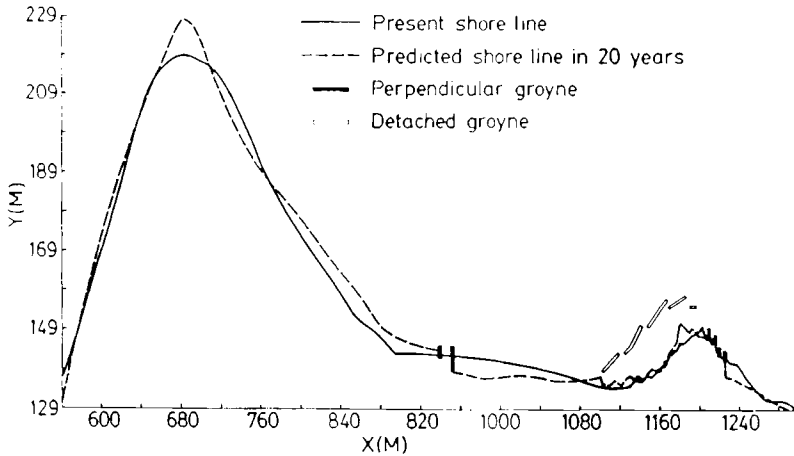


Figure 10. Sample result of shore-line evolution for a coast with pronounced alongshore variations due to perpendicular and detached groynes. This stretch of coast is located north of Barcelona (Spain).

GUZA, R. T. and THORNTON, E. B., 1985. Surf Zone Longshore Currents and Random Waves: Models and Field data. *Jnl. Phys. Ocean.*

LONGUETT-HIGGINS, M. S. and STEWART, R. W. 1962. Radiation Stresses and Mass Transport in Gravity Waves with Applications to Surf Beats. *Journal of Fluid Mechanics.*

MEI, C. C., 1983. *The Applied Dynamics of Ocean Surface Waves, Wiley Interscience.*

PHILLIPS, O. M., 1977. *Dynamics of the upper Ocean. Cambridge Univ. Press.*

VERBOOM, G. K., STELLING, G. S. and OFFICER, M. J. 1983. Boundary Conditions for the Shallow Water Equations. *Eng. Applications of Comp. Hydraulics I.* Pitman.

VON SCHWIND, J. J., 1980. *Geophysical Fluid Dynamics for Oceanographers. Prentice Hall, N. J.*

WU, C. S. and LIU, P. L. F., 1985. Finite Element Modelling of Nonlinear Coastal Currents. *J. Waterway, Port, Coastal and Oceanic Engineering. Vol. III, 2.* ASCE.

[] RESUMEN []

Este artículo presenta un sistema, capaz de generar modelos numéricos para el movimiento de agua y sedimento, en los que el nivel de precisión está definido por la exactitud de los datos iniciales y de contorno. Esto proporciona una simulación en la que el coste concuerda no sólo con la cantidad y precisión de la información disponible como input sino también con los objetivos generales del proyecto.

El sistema está dividido en tres unidades: (i) Propagación de olas/corrientes, (ii) Circulación en zona de rompientes y (iii) Evolución de la línea de orilla. La unidad (i) está compuesta por dos modelos, de los que el primero está basado en la teoría sinusoidal (lineal) y es sólo válido para refracción del oleaje. El segundo modelo utiliza una versión bastante general de las ecuaciones de conservación de masa y momentum y es capaz de reproducir la propagación de olas y corrientes, su interacción y los efectos de refracción, difracción y reflexión. También se presenta una condición de contorno original tipo absorbente-reflejante. Esta condición, además de ser más precisa para flujo 2D que las usuales, es también más general, permitiendo un tratamiento unificado para todo tipo de contornos.

La unidad (ii) está formada por dos módulos. El primero está basado en soluciones analíticas para la disminución de la altura de ola y el flujo longitudinal y se utiliza para problemas longitudinalmente uniformes. El segundo se basa en las ecuaciones de conservación de masa y momentum (incluyendo tensiones de radiación) y está integrado vertical y transversalmente. Esta técnica, desarrollada aquí para permitir un cálculo eficiente del set-up y las velocidades longitudinales y transversales para una costa variable, proporciona una información hidrodinámica con el nivel de precisión requerido por las formulaciones globales de transporte de sedimentos. Estas formulaciones son la base de la unidad (iii) que actualmente sólo incluye un modelo de 1-línea de evolución costera. Este modelo puede reproducir el efecto de las obras y estructuras costeras más usuales (espigones perpendiculares, sumergidos, etc.).

[] ZUSAMMENFASSUNG []

In diesem Aufsatz wird ein System vorgestellt, welches in der Lage ist numerische Modelle für Wasser- und Sedimentbewegung zu erstellen, in denen der Präzisionsgrad definiert ist durch die Exaktheit der initialen und Grenzdaten. Dies erlaubt eine Simulation in der die Kosten nicht nur der Höhe und der Präzision der verfügbaren Input-Information entsprechen, sondern auch mit den allgemeinen Zielsetzungen des Projekts in Übereinstimmung stehen.

Das System ist in drei Einheiten unterteilt: (1) Wellen-/Strömungsausbreitung, (2) Zirkulation im küstennahen Bereich, (3)

Entwicklung der Küstenlinie. Einheit (1) besteht aus zwei Modellen von denen das erste auf einer linearen (sinusförmigen) Theorie basiert und nur gültig ist für die Wellenrefraktion. Das zweite Modell benutzt relativ allgemeine Gesetze zur Erhaltung des Massen und Momentensatzes und ist in der Lage Wellen- und Strömungsausbreitung zu reproduzieren. Des Modell beinhaltet weiterhin ihre wechselseitigen Interaktionen zusammen mit den Auswirkungen der Refraktion, der Diffraktion und Reflektion. Erstmals wird eine Grenzbedingung für Absorption und Reflektion vorgestellt. Diese Bedingung, wie auch die—im Gegensatz zu anderen jüngeren Bedingungen vergleichbarer Modelle—genauere Bestimmung des zweidimensionalen Flusses, ist auch mehr allgemeiner Natur und erlaubt eine vereinheitlichte Behandlung für alle Grenzbedingungen.

Einheit (2) besteht aus zwei Teilen. Der erste basiert auf einer analytischen Lösung für Gleichungen des Wellenzerfalls- und küstenparalleler Strömungen und wird für Küsten benutzt, die ein einheitliches Profil besitzen. Der zweite Teil basiert auf Massen und Momenten-Erhaltungsgleichungen (incl. radialem Druck bzw. Beanspruchung) und benutzt eine Integration, die vertikal sowie rechtwinklig zur Küste ausgerichtet ist. Diese Technik wurde entwickelt um eine effiziente Berechnung der Anfangsbedingungen und der rechtwinkligen bzw. küstenparallelen Geschwindigkeiten für wechselnde Küstenlinien zu ermöglichen. Weiterhin liefert diese Technik Information zur Hydrodynamik mit einer Genauigkeit, die zur Berechnung des Sedimenttransportes notwendig ist. Die Formeln der Berechnungen stellen die Basis der Einheit (3) dar, welche zur Zeit nur ein lineares Modell zur Küstenlinienentstehung beinhaltet. Dieses Modell kann den Effekt der häufigsten Küstenverbauungen reproduzieren.—Ulrich Radtke, *Geographisches Institut, Universität Düsseldorf, F.R.G.*

□ RÉSUMÉ | |

Le système présenté est capable de générer des modèles numériques du mouvement de l'eau et de particules, dont le niveau de précision est défini par la concordance avec les données initiales et environnantes. Cette simulation a un coût en rapport, non seulement avec le nombre et la précision des informations entrées, mais aussi avec les objectifs généraux du projet. Le système est divisé en trois unités; 1) propagation de la houle—courants; 2) circulation littorale; 3) évolution du littoral. L'unité 1 est composée de deux modèles; le premier est basé sur une théorie linéaire (sinusoïdale) et n'est valable que pour la réfraction des vagues; le second utilise un ensemble de lois assez générales sur la conservation des masses et des moments, et est capable de reproduire la propagation des houles et courants, y compris leurs interactions mutuelles avec les effets de la réfraction, de la diffraction et de la réflexion. Une condition de milieu particulier d'absorption-réflexion est aussi présentée. En dehors du fait que cette condition est plus exacte pour un flux bidimensionnel que la plupart des conditions types, elle est aussi plus générale et permet une unification du traitement pour tous les milieux. L'unité 2 se compose de deux modules. Le premier, à base de solutions analytiques de la décroissance des vagues et du flux parallèle à la côte, est employé pour les problèmes uniformes parallèles à la côte. Le second module repose sur les équations de conservation des masses et des moments (incluant les "forces rayonnantes"), et intègre les mouvements verticaux et transversaux à la plage. Cette technique, développée ici afin de permettre le calcul efficient des vitesses parallèles et transverses à la côte pour un littoral varié, fournit une information hydrodynamique avec la précision nécessaire aux formulations des charges solides pour le transport sédimentaire. Ces formulations sont les bases de l'unité 3, qui, pour le moment ne comprend qu'un modèle d'évolution de côte linéaire. Ce dernier modèle est apte à reproduire l'effet de la plupart des ouvrages côtiers (épis perpendiculaires, brise lames etc...).—Catherine Bressolier, *Laboratoire de Géomorphologie EPHE, Montrouge, France.*