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Beach Evolution in the Vicinity of a Submerged Bar

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ABSTRACT

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Two closely related processes which determine the evolution of the sea bottom topography with time are studied: the refraction of waves and the littoral sediment transport produced as a result of those waves. The initial sea bottom is an isolated shore-normal bar which smoothly spreads alongshore and offshore. Refraction is computed with standard numerical procedures and the littoral transport obtained through CERC's formula. The gradient of this littoral drift, together with the equation of sediment continuity, give the amount of sediment eroded or deposited. The subsequent change of bathymetry is forecasted. These cycles of refraction-littoral transport-change of bathymetry are repeated, and the evolution of the bar is followed in the course of time. An attempt is made to study the problem in its essentials, reducing the number of variables to a minimum and separately analysing each of them. Straight and non-straight coastlines are treated because of their opposite influence upon the beach evolution process.

ADDITIONAL INDEX WORDS: *Submerged bars, beach profile, wave refraction, coastal sediment transport, shoreline evolution, waves, refraction, cycles.*

INTRODUCTION

There is a great deal of interest among coastal engineers in studying problems on beach evolution through extensive use of numerical simulation techniques applied to the refraction of waves and the resulting littoral sediment transport.

However, beach evolution in nature is actually an extremely complicated process in which the variables involved are so hidden and linked together that a simultaneous control of them is impossible. Not only are real sea beds hard to deal with because of economical reasons but also hard to follow in the course of time.

Since the aim of this work is to analyse the problem of beach evolution in its fundamental features, the aforementioned difficulties have to be avoided somehow. This may be achieved by assuming a simple, controllable shoal, given by an analytic expression, over which refraction takes place. Subsequent changes of the sea bottom are then computed with a grid superimposed on the initial analytic bathymetry.

In so tackling the problem, it is possible to consider several variables, one at a time, in order to find out the individual influence they have on the process. Formally, this is better accomplished through dimensional analysis which, on the one hand, permits a reduction in the number of variables and, on the other hand, enables the results to be presented in a more systematic and compact way.

The standard numerical model for refraction considered herein is based on the ray theory and uses linear theory in deep and shallow water.

Some other restrictions were introduced into the model to avoid the presence of caustics for which ray theory does not seem to provide realistic solutions. Onshore-offshore transport was also neglected.

For such a limited frame of reference, this paper gives quantitative information on littoral sediment transport and, by using the continuity equation, shows the shape that the beach profile would attain to in the course of time.

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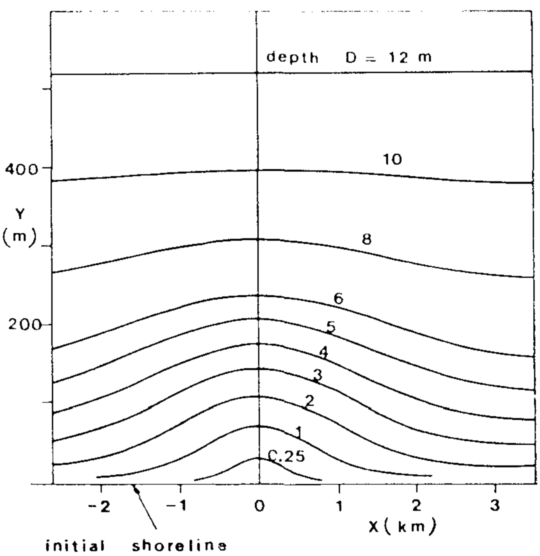

Figure 1. Initial analytic bathymetry over which refraction takes place.

BATHYMETRY

In order to build up a simple shoal the following Gaussian-like expression was used:

$$D = A\,(y - y_c)^B - SWL \tag{1}$$

where

$$y_c = 2\,\sigma_c \exp\left[-(y/2\sigma_c)^2\right] \sin(y/2\sigma_c) \exp\left[-(x/2\sigma)^2\right]$$

This equation represents a shore-perpendicular bar for which D is the water depth, x is the alongshore location, y is the on-offshore location, A and B are constants related to the sand grain diameter and beach profile (PERLIN and DEAN, 1983), σ and σ_c are parameters governing the alongshore and offshore bar's ranges, and SWL is the still water level.

Depth contours of this idealized shoal are illustrated in Figure 1 for σ = 1000 m, σ_c = 125 m, B = 0.57, A = 0.34 $m^{0.43}$, and SWL = 0. The scale of x is ten times that of y. In this example, the bar's influence is hardly noticeable 500 m away from the shore.

It should be carefully noted that a null value of SWL gives a straight shoreline because, in such a case, the depth contour for D = O will coincide with the x-axis. If SWL were greater than zero the coastline would show a smooth bend and would become convex to the sea. This merely reflects the fact that a positive value of SWL implies the lowering of the water level and in that case, one of the depth contours other than D = O will come to be the shoreline. In this paper SWL cannot take negative values.

As will be seen later on, the variable SWL, which is intended to simulate the effect of tides, has a strong influence upon the alongshore littoral transport.

Equation 1, complex as it may seem to be at first glance, is, however, easy to handle numerically. It has no discontinuities and represents a suitable disturbance of the sea bottom.

As was previously stated, successive wave fronts are allowed to refract over this bulge. These waves are slowed, shortened and steepened when they enter the shallow water region, and a point exists at which they become unstable and break, dissipating energy in this process and giving rise to a coastal sediment transport that modifies the original bathymetry.

REFRACTION NUMERICAL MODEL

The model considered herein is based upon the ray theory of refraction and uses linear theory everywhere (DEAN and DALRYMPLE, 1984).

The system of differential equations that governs wave refraction (GRISWOLD, 1963; DOBSON, 1967; SKOVGAARD and BERTELSEN, 1974) was solved by means of a fourth order Runge-Kutta method (BABUSKA *et al.*, 1966; CARNAHAN *et al.*, 1969), neglecting currents, bottom friction, percolation and wind effects. McCowan's criterion, H_b/D_b = 0.78, was used to determine the wave height, H_b, at the breaking depth, D_b, (GALVIN, 1972; WEGGEL, 1972).

Since the purpose of this work was to forecast bathymetry changes with time, refraction and sediment removal calculations had to be made with the aid of a grid superimposed on the shoal. A rectangular grid of 14000 m alongshore and 3000 m offshore, with a 10 × 10 m square mesh, proved to be useful in this case. The mesh size was adopted after an error analysis of the resulting littoral transport and was reduced to a minimum compatible with computer storage and time.

Another point that should be kept in mind when doing refraction calculations is that of the integration step along the orthogonals. It is possible to obtain good results by letting the integration step be proportional to the product of wave celerity and water depth. In this way, large integration steps are obtained in deep water, where curvature is negligible, and short steps in the vicinity of wave breaking, where larger curvatures are to be expected.

Much care has been taken in order to keep possible sources of numerical error under control. Apart from the length of the integration step and mesh size already mentioned, the deep water zone in which orthogonals start their path towards the shore has an influence. The usual procedure of considering deep water where D/L_o (L_o being the deep water wave length) equals 1/2 did not seem to be a suitable one in the type of study involved herein. Instead, a value of $D/L_o = 1$ appears more effective and does not represent a noticeable increase in computing time, provided the above criterion for the integration step is adopted.

SEDIMENT TRANSPORT

When breaking takes place, a consequent littoral sediment transport, Q, arises. The CERC's formula (CERC, 1984) has been used to calculate it:

$$Q = \frac{K}{16} \frac{1}{1 - p} \frac{1}{(\rho_s/\rho_w) - 1} C_{gb} H_b^2 \sin(2\,\alpha_b) \qquad (2)$$

in which C_{gb} is the wave group velocity at the breaker line, ρ_s (ρ_w) is the sand (water) density, p is the sand porosity, and α_b is the angle the wave front makes with the shoreline at breaking. The coefficient K was assumed to be 0.77 as recommended by the CERC.

Typical results for coastal sediment transport against longshore position are presented in Figures 2 and 3. Each of the open circles of these and following graphs of Q(x) represents a littoral drift computed with expression 2.

Both Figures correspond to the beginning of wave action (time = 0) and were obtained assuming that the wave rays start their path with a deep water wave angle, α_{dw}, of 270° with respect to the x-axis.

For the two Figures, the wave period and the deep water wave height are given by T = 7 s and H_0 = 1 m, respectively, and the bottom conditions by σ = 1147 m, σ_c = 287 m, A = 0.323 $m^{0.43}$ and B = 0.57.

In the case of a straight shoreline, SWL = 0, Figure 2 shows two extreme values of Q at each side of the bar's axis of symmetry. Between those extreme values, a nearly constant and negative $\partial Q/\partial x$ appears.

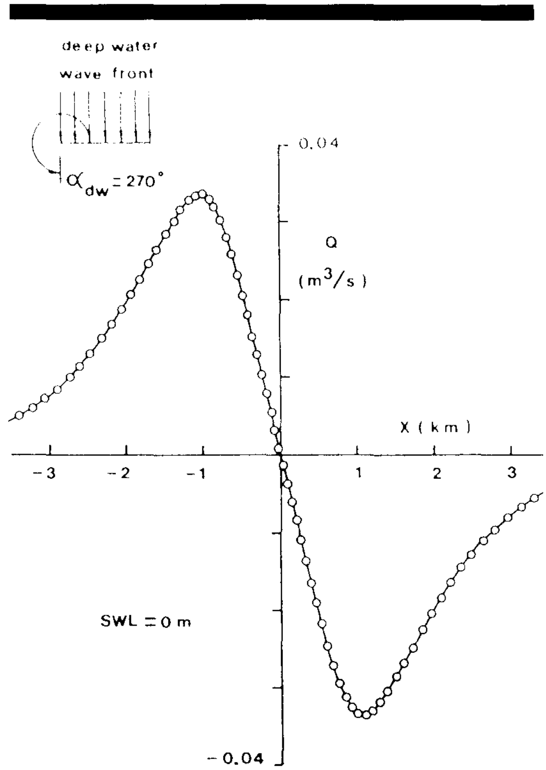

Figure 2. Initial littoral transport for a shore-normal wave attack and a straight shoreline. Notice that $\partial Q/\partial x < 0$ about x = 0, which means accretion there.

Figure 3 illustrates longshore transport for the same topography, but with a lowered water level. This was achieved with a positive value of SWL; the value being 2.55 m for one of the curves and 7.65 m for the other. Either bathymetry represents an initially cape-like shoreline. There are also two extreme values of Q at each side of the bar's axis, but now a positive $\partial Q/\partial x$ is present between them. The consequences of this complete inversion of $\partial Q/\partial x$ will be apparent once the continuity equation has been introduced, which is to be done next.

A schematic beach profile section defining a sediment control volume for a shore element Δx is illustrated in Figure 4. This profile should not be confused with a shore-parallel bar. Let Q_{in} and Q_{out} stand for the incoming and outcoming littoral transport, respectively. By sub-

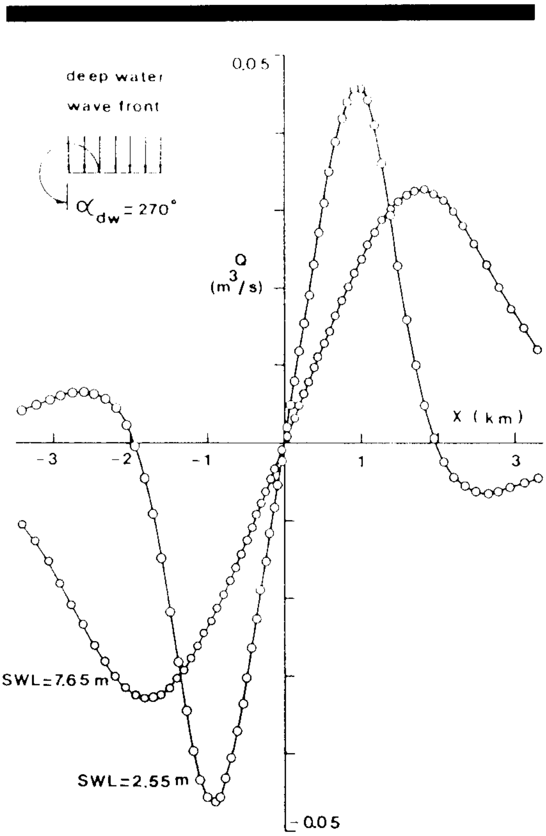

Figure 3. Initial littoral transport for a shore-normal wave attack and two different cape-like shorelines. Notice that $\partial Q/\partial x > 0$ about $x = 0$, which means erosion there.

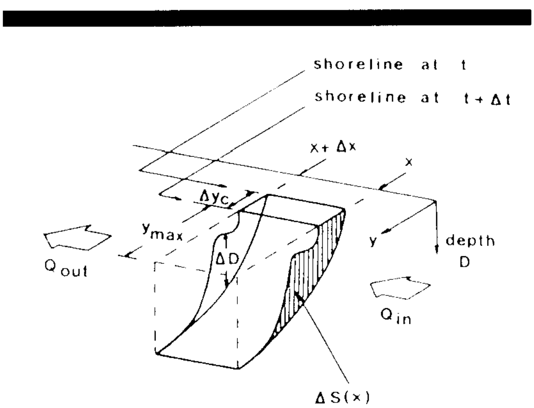

Figure 4. Change in beach profile and the continuity equation.

tracting Q_{out} from Q_{in}, a net sediment volume per unit time is obtained. This net volume can be thought of as coming from the product of the shore element Δx times an area $\Delta S(x,t)$, located at $x + \Delta x/2$, and representing the amount of sand per unit time and length put into motion in a time interval Δt. It is possible to set all this on mathematical grounds through the continuity equation. Assuming that there are neither sediment sources nor sinks, the following expression can be set on discrete terms:

$$\Delta S(x, t)/\Delta t + \Delta Q/\Delta x = 0 \tag{3}$$

Clearly, a positive value of $\Delta Q/\Delta x$ in (3) implies erosion and, conversely, a negative value, accretion. In either case a cross-shore change in depth, indicated by ΔD in Figure 4, will follow.

The possibility of coastline modification is also sketched in Figure 4. In regard to this, a hypothesis should be set concerning the value of the coastline change, Δy_c. The hypothesis could be, for instance, to make Δy_c proportional to $\Delta S(x, t)$ in sign and magnitude.

There is a region between the shoreline and somewhere near the breakers in which littoral sediment transport takes place, this region depending upon wave characteristics and water depth. As shown in Figure 4, we have supposed the existence of a y_{max} beyond which the profile is assumed not to change.

Returning to the influence of the still water level, SWL, it is now possible to see that for a straight shoreline, SWL = 0 in Figure 2, there will be an accretion process about the bar's axis of symmetry, at least at the beginning of wave action, because of the negative value of $\Delta Q/\Delta x$ there. On the contrary, if the shoreline is bent, SWL > 0 as in Figure 3, a positive value of $\Delta Q/\Delta x$ arises about the bar's axis, resulting in an erosion process instead.

These graphs of littoral transport against longshore position were all obtained with a deep water wave angle α_{dw} of 270°, but it is seen that if the deep water wave angle takes the symmetric values of 250° and 290°, as in the case of Figure 5 for SWL = 0, these graphs become shifted without affecting the kind of process undergone at a particular position, at least at the beginning of wave action. This is because erosion or accretion do not depend on Q directly but on $\Delta Q/\Delta x$ instead, and this ratio remains practically unchanged in this numerical model.

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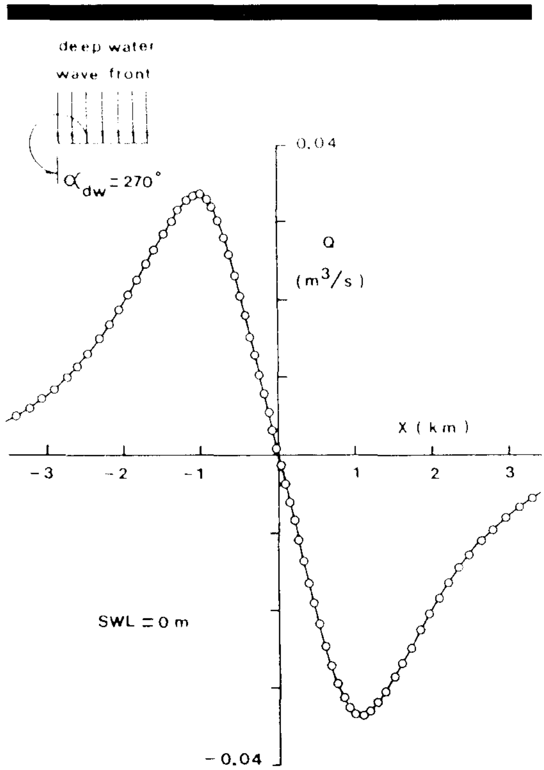

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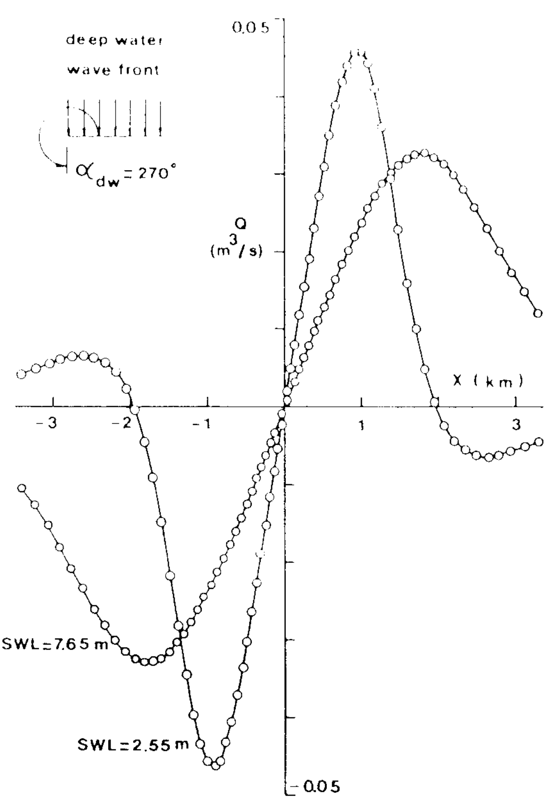

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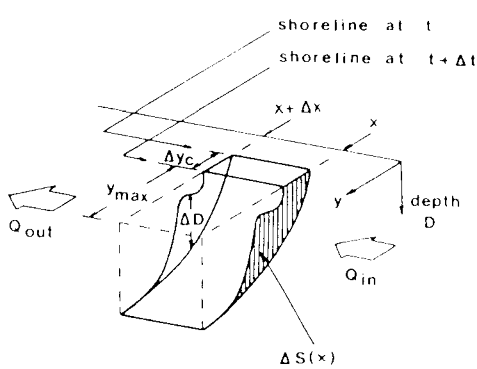

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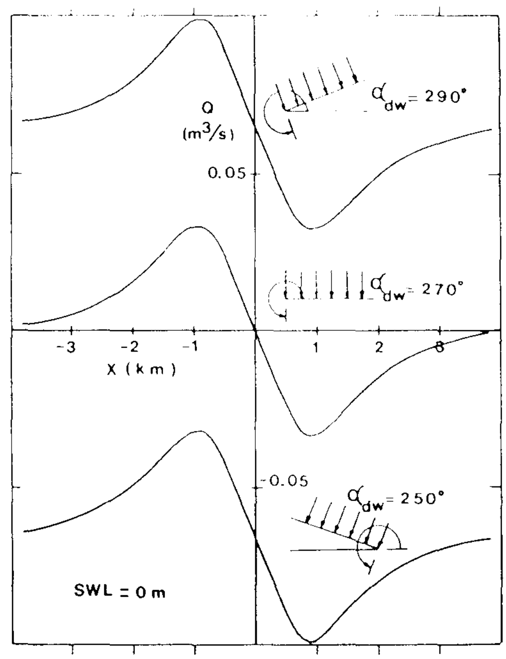

Figure 5. Initial littoral transport for different deep water wave front directions and a straight shoreline. Notice that $\partial Q/\partial x$ remains almost unchanged.

Since bathymetry changes ought to have forecasted from longshore transport curves, an attempt was first made to compute $\Delta Q/\Delta x$ directly from the results at many points along the coast represented by the open circles in Figures 2 and 3. However, this procedure is an important source of numerical instabilities, particularly if several calculations of refraction and sea bottom changes are performed. These instabilities were avoided by splitting the curve Q(x) into several sections and adjusting a polynomial to each segment of the function. In so doing, each sector was partly overlapped with the two neighbouring ones to avoid discontinuities. The gradient $\Delta Q/\Delta x$ was then taken from the polynomials and not from the numerical results directly.

The amount of sediment to be eroded or accreted at a particular shore position x, and over a time interval Δt, is obtained by calculating the derivative of the littoral transport with respect to x and multiplying it by the time interval Δt, as indicated by the continuity equation (3).

CROSS-SHORE SEDIMENT DISTRIBUTION FUNCTION

When dealing with the continuity equation it was seen that the difference between incoming and outcoming littoral sand drift is responsible for the amount of material eroded or accreted within a particular control volume (Figure 4). It was also pointed out that there would be a cross-shore change in bathymetry as a direct consequence of sediment removal. Now it is necessary to decide on how the sediment will be distributed once set into motion. The way sediment is distributed will certainly affect the bottom topography after a period of wave action. In order to see this influence, two different ways of distributing the sediment over the sea bottom were considered. Both sediment distribution functions should obey the following two boundary conditions:

(1) There are no depth changes at the shoreline at any time. The shoreline is allowed to move seaward, and to recede as well, but this degree of freedom will be considered later on.

(2) There are also no depth changes beyond a maximum distance y_{max} measured offshore.

Let $\Delta D(y)$ be the change in depth at a specific offshore position y, corresponding to fixed values of longshore position x and time t. Then it holds that:

$$\Delta S = \int_0^{y_{max}} \Delta D\,(y)\,dy \qquad (4)$$

where $\Delta D(y)$ represents the aforementioned distribution function (Figure 4).

Now it seems to be reasonable to suppose that if the wave front obliquity at breaking gives origin to a longshore velocity v (LONGUET-HIGGINS, 1970, 1972), then, the depth change $\Delta D(y)$ could be associated with such a longshore velocity. Hence, taking the LONGUET-HIGGINS' equations for v and using them to express the depth change $\Delta D(y)$, it is possible to write:

$$\Delta D(y) = \begin{cases} K[B_1(y/y_b)^{p_1} + A(y/y_b)] & 0 \leq y/y_b \leq 1 \\ K\,B_2(y/y_b)^{p_2} & 1 < y/y_b < \infty \end{cases} \qquad (5)$$

where y_b is the offshore breaking distance and

$B_1 = (p_2 - 1)\,A\,/\,(p_1 - p_2)$, $B_2 = (p_1 - 1)\,A/(p_1 - p_2)$

$A = (1 - 2.5\,P)^{-1}$

$p_1 = -3/4 + [(3/4)^2 + (1/P)]^{1/2}$

$p_2 = -3/4 - [(3/4)^2 + (1/P)]^{1/2}$

for $P \neq 0.4$, P being a nondimensional parameter related with the horizontal mixing (LONGUET-HIGGINS, 1970). Particular expressions for $P = 0.4$ can be found in LONGUET-HIGGINS' paper. The constant K may be determined from condition (4), yielding $K = (2 + P) \Delta S/y_b$. Notice that for this particular function the distance y_{max} is theoretically infinite.

The next distribution function proposed herein is different from the previous one and less involved. It simply states that $\Delta D(y)$ might obey a sinusoidal law:

$$\Delta D(y) = K_1 \sin^3(K_2\, y) \tag{6}$$

in which $K_1 = 3\pi \Delta S/(4 K_b\, y_b)$ comes from condition (4) and $K_2 = \pi / (K_b\, y_b)$, from the second boundary condition. The value of $K_b = y_{max} / y_b$ is to be chosen, and its influence tested, by numerical experiments.

An important point that should be stressed here is that either of the preceding distribution functions give accretion or erosion at a specific shore location x, but not both simultaneously.

Other investigators of beach evolution have also used simplified functions for sediment distribution. PELNARD-CONSIDÈRE (1956) and KOMAR (1977), for instance, assume a uniform distribution over the beach profile and use a simple constant for this function; the constant being a closure depth. PRICE (1972) assumes a triangular function which, when compared with those of PELNARD-CONSIDÈRE and KOMAR, simply changes the time scale by a constant.

SHORELINE CHANGE

The possibility of shoreline modification has been taken into account by means of a simple assumption.

$$y_c(x, t + \Delta t) = y_c\,(x, t) + c_{mc}\, \Delta S(x, t)$$

where y_c is the shore position at any time t and c_{mc} is a new constant which may be called the coast modification constant. The case of no shoreline change corresponds to $c_{mc} = 0$. No negative values of c_{mc} are allowed in this work.

COMPUTATIONAL PROCEDURE

The purpose of this paper is to forecast bathymetry changes in the course of time. Several steps were performed in order to achieve this for the particular topography considered herein.

The angle the wave rays make in deep water with respect to the x-axis, α_{dw}, was chosen to be 270° and held constant for any fixed value of wave period T and deep water wave height H_o. In this way, the initial graph of littoral transport Q against longshore position was obtained (Figures 2 and 3).

After multiplying $\Delta Q/\Delta x$ by the time interval Δt, the amount of sediment put into motion for a particular shore position is obtained. This amount of sediment is subsequently accreted or eroded according to either of the distribution functions already mentioned (Equations 5 or 6).

This procedure is repeated for each grid interval alongshore and, at last, the whole set of grid intersections will have a different depth because of sediment motion.

Over this modified topography a new refraction process takes place which, in turn, will be responsible for new depth changes, and so on.

Since the continuity equation represents an explicit scheme, it was necessary to test decreasing values of the time interval Δt until the whole process became independent of it. In this paper Δt was finally fixed in 6 hours.

RESULTS AND DISCUSSION

At the beginning of wave action (time = 0), the bottom topography is given by (1) and littoral drift by curves similar to those depicted in Figures 2 or 3. After a time t, the sea bottom will change and its final form will depend upon several variables which may be grouped as follows: the set of variables σ, σ_c, A, B, and SWL, which define the initial topography and the coastline shape; the variables H_o, T, and α_{dw}, which characterize sea conditions in deep water and, finally, the possibility of shoreline change, reflected by the constant c_{mc}. It should be noted that the way sediment is distributed after being removed by wave action also participates in the sea bottom evolution. This is controlled by the parameter P or by the constant K_b, according to the distribution function previously chosen.

If the π-theorem is applied to the $N = 12$ variables required to describe the present physical situation, it is seen that $M = 2$ dimensional categories are needed (length and time). Then the relationship among the N variables can be reduced to one comprising $N - R = 12\text{-}2 = 10$ non-dimensional products, $R = 2$ being the rank of the $N \times M$ dimensional matrix.

One of the choices for the relationship among the resulting numbers is:

$$\phi\,(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}) = 0$$

where

$\pi_1 = H_o/L_o$	$\pi_2 = t/T$
$\pi_3 = \sigma/L_o$	$\pi_4 = \sigma_c/L_o$
$\pi_5 = L_o/SWL$	$\pi_6 = c_{mc}L_o$
$\pi_7 = P$ (or y_{max}/L_o)	$\pi_8 = \alpha_{dw}$
$\pi_9 = B$	$\pi_{10} = L_o^{(1-B)}/A$

The initial bathymetry has been reduced to a simpler form; however, 10 variables seem to be a rather large number. Since the change of any of the above variables will modify the final sea bottom shape, it was decided to study two particular bathymetries during 10 days of wave action. One of these bathymetries has an initially straight shoreline and the other, a cape-like shoreline. This was achieved through the values $SWL = 0$ for the former ($L_o/SWL = \infty$), and $SWL = 2.18$ m for the latter ($L_o/SWL = 35$). The rest of the above-mentioned variables was kept constant. The purpose of this election is to show two entirely different patterns of bottom change. Both cases have been studied for $H_o = 1$ m, $T = 7$ s, $\sigma = 1147$ m, $\sigma_c = 287$ m, $c_{mc} = 0.196\ m^{-1}$, $B = 0.57$, $A = 0.323\ m^{0.43}$, $P = 0.1$, and $\alpha_{dw} = 3\pi/2$. The set of non-dimensional variables is: $H_o/L_o = 0.013$, $\sigma/L_o = 15$, $\sigma_c/L_o = 3.75$ (i.e. $\sigma/\sigma_c = 4$), $L_o^{(1-B)}/A = 15$, $c_{mc}L_o = 15$, and $t/T = 123429$.

Figure 6 shows depth contours before (full line) and after 10 days of wave action (dashed line) for the case of the initially straight shoreline. If one of the depth contours is taken into account, $D = 1$ m for instance, it is clearly seen that it moves seaward in the neighborhood of the bar's axis of symmetry, giving rise to an accretion zone; proceeding to the right, the contour line recedes to the coast, showing an erosion zone. At a certain distance, far from the bar's influence, the contour line remains unaltered. Each of the depth contours in Figure 6 shows the same behaviour. When the sea becomes sufficiently deep, $D = 5$ m for the present case, the bottom ceases to be influenced by wave action.

The upper sketch in Figure 8 shows the accretion zone about the bar's axis together with an

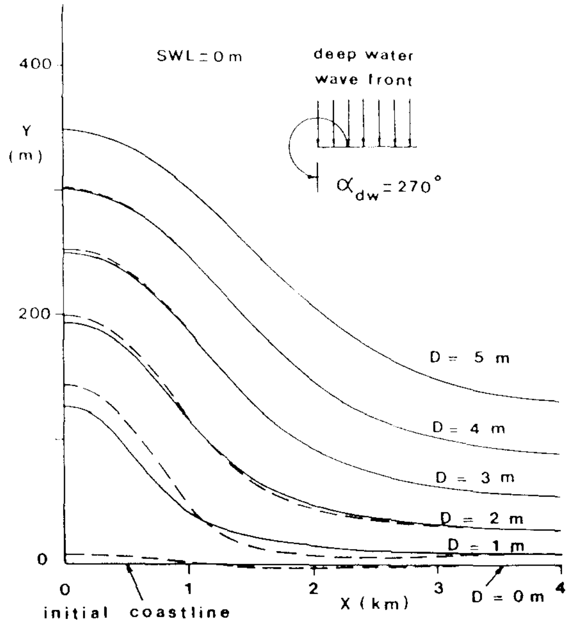

Figure 6. Depth contours for $t = 0$ (solid line) and after 10 days of shore-normal wave attack (dashed line) for an initially straight coastline. Notice accretion about $x = 0$.

adjacent erosion zone. An attempt has been made in this sketch to determine the region in which the bottom has changed in 10 days. The dotted line indicates a one meter displacement. In this way, the space between the coast and the dotted line represents, approximately, the region where wave energy removes sediments from the erosion to the accretion zones.

For the previously mentioned constants, an amount of 27500 m^3 is eroded at each side of the bar and deposited about the axis in 10 days.

An entirely different phenomenon happens if the coast is initially curved, as represented in Figure 7, where the water level has been lowered with $SWL = 2.18$ m over the same bathymetry. If any depth contour is considered with increasing positive values of x, it may be seen that near the bar's axis there is an erosion zone, then it follows accretion and, finally, a new and weak erosion appears. Bottom activity ceases for large x, where the bar is no longer appreciable.

The lower sketch in Figure 8 illustrates the two erosion zones with accretion in between, the dotted line limiting bottom changes according to the aforementioned criterion. The

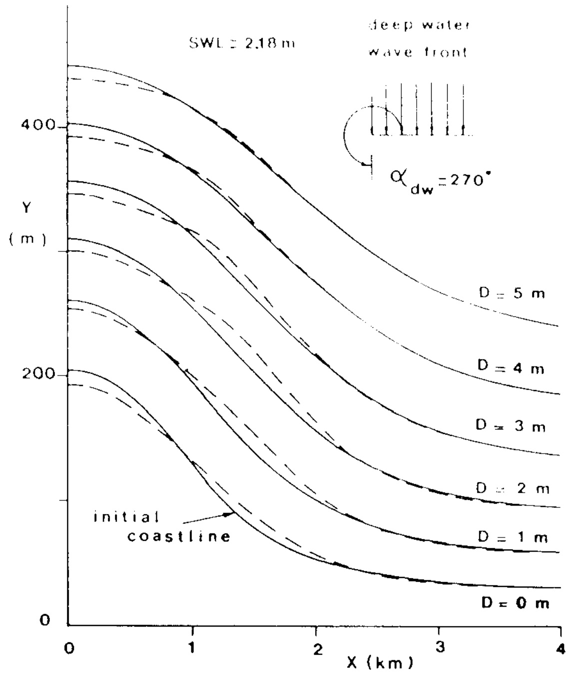

Figure 7. Depth contours for t = 0 (solid line) and after 10 days of shore-normal wave attack (dashed line) for an initially cape-like coastline. Notice erosion about x = 0.

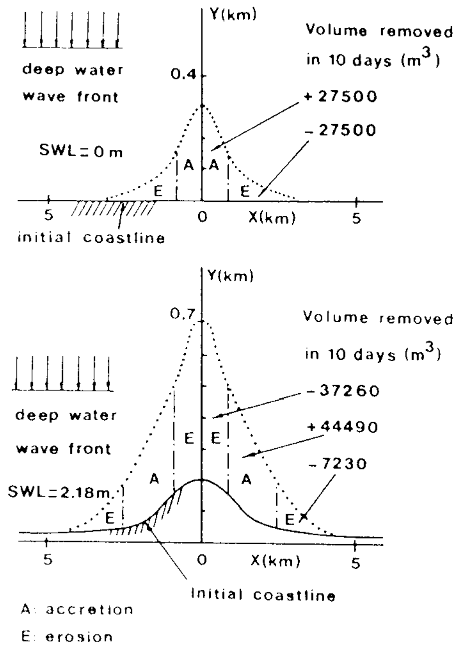

Figure 8. Shore-normal wave attack modifies depth contours mainly between the coastline and the dotted line. Notice inversion of accretion and erosion for straight and cape-like shorelines.

amount of material eroded for positive values of x is 37260 m^3 near the axis and 7230 m^3 far from it; the accreted material, 44490 m^3, is necessarily equal to the amount of eroded sediment, for there is a complete absence of sources or sinks.

Two features are worthwhile emphasizing from the examples presented herein. Although belonging to specific values of the multiple variables involved, they show the general trend demonstrated in this paper.

One of them is the increase in wave energy used in transferring sediments from one place to another when the water level is lowered. While 27500 m^3 of sediments are transported when the shoreline is initially straight, 44490 m^3 (almost 50% larger) are moved when the water level is lowered over the same bulge.

The other aspect that seems interesting is the space over the sea bottom in which this activity takes place; this space is limited by the coast and the dotted line. For SWL = 0 all activity ceases approximately 300 m away from the coast at x = 0, while for SWL = 2.18 there are still signs of bottom activity 500 m away from the shoreline.

Nevertheless, the most striking difference is the complete inversion of behaviour concerning the region in which accretion and erosion occur, a fact that would suggest a strong influence of tides upon the way a mound such as the one considered herein evolves in the course of time.

This phenomenon could have been forecasted by close inspection of Figures 2 and 3 in view of the drastic changes in $\Delta Q/\Delta x$ when $L_o/SWL = \infty$ or a finite number; in other words, when the initial shoreline is taken to be straight or bent. However, though predictable, this situation had to be confirmed with a procedure such as that studied here, in which successive refraction and sediment distribution cycles were set in a numerical algorithm.

It would remain an extensive study of the influence of all the non-dimensional variables. Preliminary analyses considering reasonable ranges of variation of $c_{mc}L_o$, $L_o^{(1-B)}/A$ and B have shown a minor influence upon the final sea bottom shape.

Even though the way sediment is distributed after it has been set into motion might seem to be of importance, numerical results have indicated that except for some minor changes the two sediment distribution functions proposed in this work give essentially the same results.

CONCLUSIONS

Beach evolution, by its very nature, is such an extremely complicated process that any attempt to study this problem in the case of a real seabed quickly collides with the difficulty of adequately separating the variables involved in order to analyse the influence each of them has on the whole process.

For that reason, an effort was made in this paper to tackle the problem in its essentials by means of a numerical model that assumes the existence of a simple, controllable shoal, given by an analytic expression, over which refraction takes place.

The shoal consists in a shore-perpendicular bar that smoothly spreads alongshore and offshore. Subsequent changes in the sea bottom were computed with the aid of a grid. The CERC's formula was used to calculate littoral sediment transport, avoiding the presence of caustics and neglecting onshore-offshore transport.

In regard to the shoreline, the model enables it to be straight or curved (cape-like) by simply modifying the still water level, which is of the utmost importance throughout the beach evolution process. In addition, the possibility of a shoreline change as time passes by has also been considered.

Once set into motion, sediments were redistributed by using two distribution functions of different kinds in order to see the relative significance of the way sediments are deposited or eroded. However, this has proved to be of minor importance.

In the case of a straight shoreline, a shore-normal wave attack lasting for ten days makes the bar to undergo sediment accretion about its axis of symmetry and, simultaneously, erosion on each of the nearby sides. As a result, the bar and its adjacent shoreline grow seaward at the expense of erosion on both sides.

When the shoreline is no longer straight but takes the form of a cape instead, the same shore-normal wave attack makes the bar undergo sediment erosion about its axis, accretion on each of the nearby sides and, finally, a new and weak erosion a little farther way. Under these circumstances, the bar and its adjacent shoreline tend to recede by nourishing both sides.

This complete inversion of behaviour concerning the regions where accretion and erosion take place, suggests a strong influence of tides upon the way a mound such as the one considered herein evolves in the course of time. Quantitative results in this work clearly show this possibility.

If the wave attack is no longer shore-normal but somewhat oblique instead, it is found that, at least at the beginning of wave action, the littoral transport curves become shifted without affecting the kind of process undergone at any particular position. This is because erosion or accretion do not depend on Q directly but on $\partial Q/\partial x$, and this gradient remains almost unchanged.

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RESUMEN

Se estudian dos procesos estrechamente vinculados con la evolución en el tiempo de la topografía del fondo del mar: la refracción de las olas y el consecuente transporte litoral de sedimentos. La topografía inicial consiste en una barra sumergida, normal a la costa y que se extiende de manera suave hacia ambos lados y mar adentro. Se calcula la refracción con los métodos numéricos habituales y se obtiene el transporte litoral a través de la formula del CERC. El gradiente de este transporte litoral y la ecuación de continuidad para sedimentos dan la cantidad de estos extraída o depositada, pudiéndose pronosticar así el consecuente cambio de la batimetría. Repitiendo estos ciclos de refracción, transporte litoral y cambio de la batimetría, es posible seguir la evolución de la barra con el transcurso del tiempo. Se intenta estudiar el problema en sus aspectos esenciales, reduciendo al mínimo el numero de variables y analizándolas separadamente. Debido a sus disimiles efectos sobre el proceso de evolución de la playa, se consideran costa rectas y curvilíneas.

ZUSAMMENFASSUNG

Zwei eng miteinander in Beziehung stehende Prozesse, die Entwicklung der Oberfläche des Meeresbodens in Küstennähe bestimmen, werden untersucht: die Wellenbrechung und der daraus resultierende küstennahe Sedimenttransport. Der ursprüngliche Meeresboden ist ein isoliertes Unterwasserriff, das sich gleichmäßig parallel zur Küste und meerwärts erstreckt. Die Wellenbrechung wird mit standardisierten numerischen Verfahren berechnet, der küstennahe Transport mit Hilfe der Formel von CERC. Der Gradient dieser litoralen Drift und die Nivellierung durch ständige Sedimentzufuhr geben die Menge an erodiertem oder akkumuliertem Sediment an. Die anschließende Veränderung der Meerestiefe wird vorhergesagt. Diese Zyklen aus Wellenbrechung, küstennahem Transport und Veränderung der Wassertiefe werden wiederholt simuliert und dabei die Entwicklung des Unterwasserriffs verfolgt. Es wird der Versuch unternommen, das Problem in seinen Grundzügen zu lösen, indem die Anzahl der Variablen auf ein Minimum reduziert und jede Variable einzeln analysiert wird. Ausgleichsküsten und unausgeglichene Küsten werden dabei untersucht, da sie gegensätzlichen Einfluß auf die Prozesse der Strandentwicklung haben.—*Helmut Brückner, Geographisches Institut, Universität Düsseldorf, F.R.G.*

RÉSUMÉ

Etudie deux processus qui déterminent étroitement l'évolution de la topographie des fonds avec le temps: la réfraction de la houle et le transport sédimentaire littoral induit par la houle. Le fond origine est une barre isolée perpendiculaire à la côte. Elle s'étale légèrement le long du littoral et vers le large. La réfraction est calculée selon une procédure standard et le transport littoral calculé selon la formule du CERC. Le gradient du courant littoral, combiné à l'équation de continuité du sédiment donne la quantité de sédiment érodée ou déposée. Ces cycles réfraction littorale transport et modification de la bathymétrie sont répétés. La modification de l'évolution de la barre est suivie dans le temps. Il est tenté de faire l'étude de ce problème en réduisant le nombre de variables au minimum et en les analysant séparément. Les côtes rectilignes et non rectilignes, dont les processus d'évolution sont dus à des influences opposées, sont traitées. *Catherine Bressolier (Géomorphologie EPHE, Montrouge, France).*