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Beach Profile Analysis by Empiric Orthogonal Functions

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ABSTRACT

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A set of soundings from a beach profile surveyed during 1980 and part of 1981-1982 in the locality of Pinamar, Province of Buenos Aires, Argentina, has been statistically analysed by means of Empirical Orthogonal Functions in order to isolate their spatial and temporal variations. The first three modes of organization can explain 99.17% of the variations of the profile. It is shown that the second and third temporal autofunctions have, respectively, a significant correlation with the variation of the energy level and with the direction of wave incidence. Analyses indicate that the second mode is associated with the transport of sediment normal to the shore, while the third mode is linked to the longshore transport. The second autofunction also suggests the presence of hinge points, at depth of 0.5 m and 2 m relative to mean sea level, and through which accretional and erosional processes occur out of phase.

ADDITIONAL INDEX WORDS: *Beach profile, sand bar, waves, sediment transport, field experiments, Eigenvalues, Argentine coast.*

INTRODUCTION

As part of the Program on Coastal Observations performed by the Argentine Naval Hydrographic Service, beach and nearshore profiles were measured during, 1980, 1981 and 1982 alongside the fishing pier of Pinamar, located on the central sector of the coast of the Province of Buenos Aires, Argentina (Figure 1).

It was assumed that it is possible to resolve the movement of the beach sand into two directions, one along shore and the other normal to the shore. These two modes reflect the effects of the longshore currents and temporal changes in the height and direction of the waves.

The change in the profile of a beach and nearshore is an important aspect in the variability of the coastal environment. One approach to the analysis of these changes is the generation of a set of empiric autofunctions, starting from the observations themselves. The autofunctions generated by the data quantify the considerable variation shown by the profile configuration. WINANT *et al.* (1975) first applied this methodology to profiles and defined typical functions associated with coastal processes. In a later paper, WINANT and AUBREY (1976) studied the stability and the sensitivity of the method, as applied to beach profiles in southern California.

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In the present study the relationship between the behaviour of the nearshore and the variations in the intensity and direction of the waves in the area of Pinamar are analysed, in an attempt to clarify the prevailing causes inducing transport of sediments normal to the shore and alongshore.

FIELD METHODS

The fishing pier of Pinamar is 156 m long and was used as a fixed platform for measurement of the tide, waves, coastal currents and bottom profiles in the proximities of the surf zone.

The typical nearshore profile has a gentle slope of 1:33. The average diameter of beach is 0.25 mm. Profiles were first measured on a weekly basis, but they were later undertaken every fifteen days. Profiles were measured normal to the shore at 12.5 m intervals while the

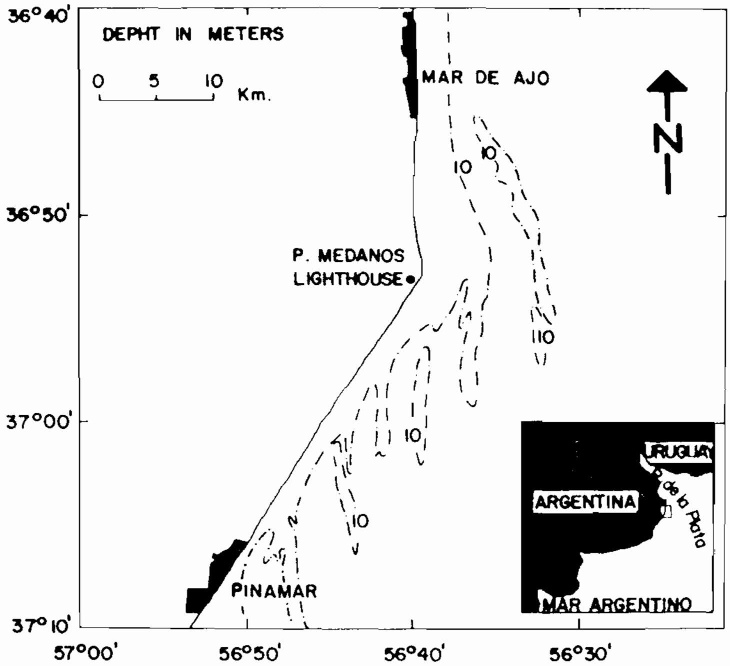

Figure 1. Location of Pinamar, Argentina.

remaining observations (wave height, wave direction and longshore currents) were performed twice a day.

METHODS OF ANALYSIS

Profile data were statistically analyzed using Empirical Orthogonal Functions (EOF), which give a representation of depth d (x, t) as a linear combination of the products of the time and the distance normal to the shore functions.

Soundings are represented by $d(x_i, t_k)$ where x_i is the distance (referred to a fixed point) with i varying from 1 to N (total number of measuring points) and t_k is the time index, with k from 1 to K (the total number of recording times). Data are represented by:

$$d(x_i,t_k) = \sum_{n=1}^{N} E_n(t_k)\, P_n(x_i)$$

where $E_n(t_k)$ and $P_n(x_i)$ are, respectively, the time and the spatial autofunctions.

The autofunctions (modes) $P_n(x_i)$ result from the diagonalization of the symmetrical correlation matrix.

$$R_{ij} = \frac{1}{K}\sum_{k=1}^{K} d(x_i,t_k)\, d(x_j,t_K)$$

The structure of the autofunctions is defined by the data set, and does not assume a priori any particular functional form. Each mode has an associated eigen-value λ_n with which the n^{th} mode contributes to the total mean square value.

The time functions are found from the relation

$$E_n(t_k) = \sum_{i=1}^{N} d(x_i,t_k)\, P_n(x_i)$$

Both sets of autofunctions satisfy orthogonality relationships.

$$\sum_{k=1}^{N} P_n(x_i)\, P_m(x_i) = \delta_{nm}$$

$$\sum_{k=1}^{K} E_n(t_k)\, E_m(t_k) = \lambda_n\, \delta_{nm}$$

A detailed presentation of the method can be found in KENDAL and STUART (1968).

As a consequence of the present analysis, the empirical prediction model of the beach profile changes provides three modes of organization related to the spatial-temporal variations of the bed forms.

STATISTICAL RESULTS FROM THE PROFILES

Empirical Orthogonal Functions were computed for the survey performed alongside the Pinamar pier (Figure 2). As expected, the autofunction with the largest eigen-value, accounting for 99.17% of the total of the mean square value, shows little variation in time (Figure 3a) and, hence, the first mode represents essentially the average profile. The time dependent E_1 of the first autofunction shows a much smaller variance than the square of its mean, which in turn justifies the denomination of the first mode as average profile.

Based on a linear fitting by least squares, the E_1 component does not significantly represent any trend, indicating that over the length covered by the profile and for the observed time interval, there is neither erosion nor net accumulation of sediments. This fact may suggest the existence of a beach practically in a state of equilibrium.

The second mode has positive values over a wide range of distances, 50 m to 120 m, and negative values for distances longer than 120 m and shorter than 50 m. The temporal variations of the E_2 component (Figure 3b), associated with the second mode, show a trend obviously suggesting relationship to some oceanic variable related to the transport of sediments.

To clarify the causes of such changes in the profile, the possible connection between the time component E_2 and the magnitude of the wave height was investigated. Figure 4a shows the time variations in the height of the breakers. The measurements were visually performed from the beach, at a short distance from the pier. From Figures 3b and 4a, it may be seen that a reasonable correlation between E_2 and the height of the waves does exist.

For example, during the months of March, April and until mid-June 1980, when the

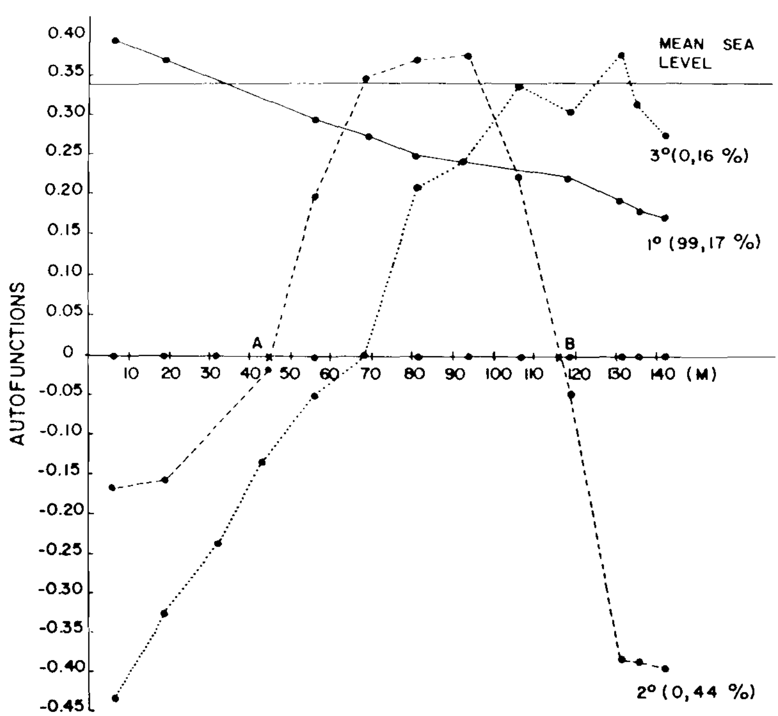

Figure 2. First three autofunctions of the beach profiles, Pinamar pier.

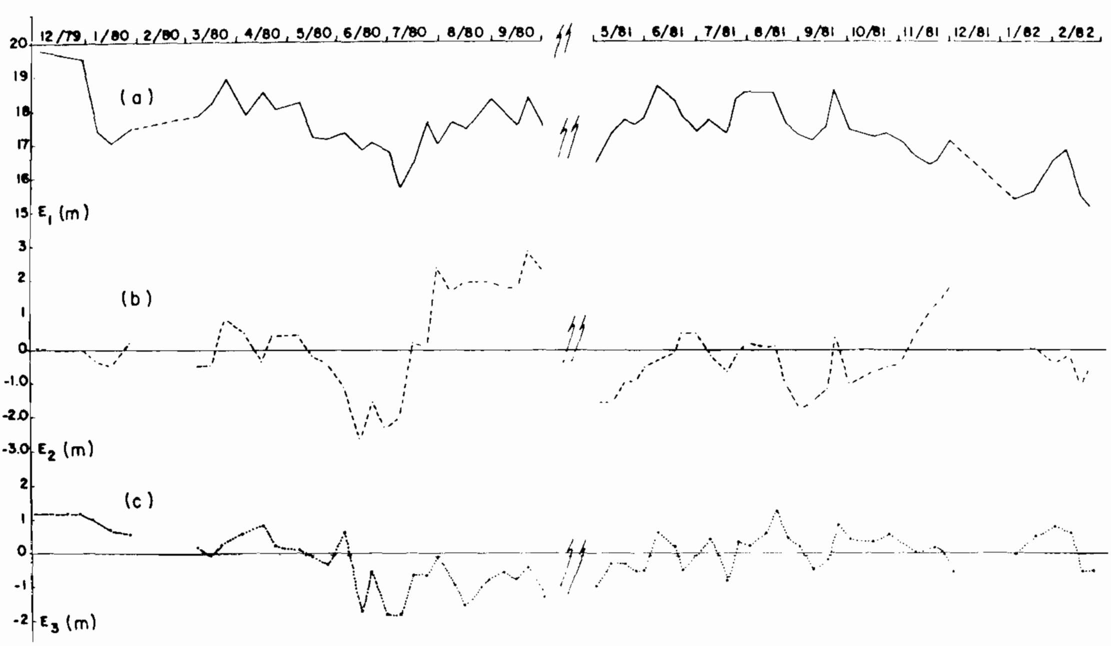

Figure 3a, 3b and 3c. Temporal autofunctions. First three modes of organization.

energy was high, the normal wave period been about 5 to 7 s, the E_2 function has a decreasing trend. This fact, together with the form of the second mode, indicates transport of sediment from the middle zone of the profile towards deeper areas, with the consequent formation of a bar in deeper water. This behaviour is characteristic of the winter season, when waves have a high energy content. At the same time, there also exists, although to a lesser degree, a shoreward transport. This process will be considered later in this paper.

From mid-June until the end of September 1980, when the energy level had considerably decreased, the E_2 component shows an increasing trend, until maximum positive values are attained. This situation leads to the erosion of the bar. Erosion occurred at much faster rate than that of bar formation, after which the level of the profile in that region stays almost stationary during low wave energy conditions.

From May, onwards until the end of 1981, similar trends occurred. With large waves the E_2 component decreased and, vice versa, with smaller waves it increased. These trends, together with the form of the second mode, indicate the formation and erosion of a bar at depths of the order of 3.5 m relative to mean sea level.

Consequently, it can be concluded that the second autofunction (54% of the mean square value, without considering the first mode), is associated with the profile changes due to the transport of sediments normal to the shore and caused by the change in wave energy.

In the section of the profile closer to the shore, the possibility exists that the variations in the profile might be due, not only to the transport of sediments normal to the shore, but also to transport parallel to the shore. In general, such a parallel transport of sediments is due to the obliquity in the incidence of the wavefronts with respect to the shoreline, thus generating longshore littoral currents. Figure 4b shows the time variation of the direction of incidence.

If we compare the temporal component E_3 (Figure 3c) with the variations of the direction, it can be observed that a certain degree of correlation exists between them. For example, from May to August 1981, the E_3 component tends to increase, as the waves tend to approach more consistently from the south. On the con-

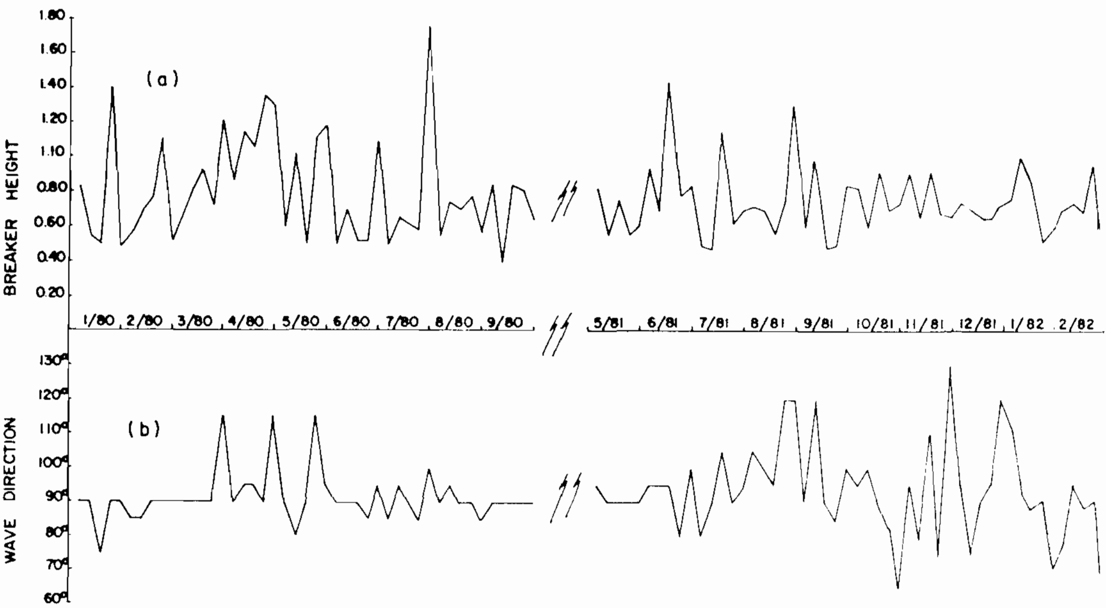

Figure 4a and 4b. Daily breaker height and wave directions at Pinamar.

trary, in September and October 1981, when the waves begin to approach from the north, the E_3 component tends to increase.

In the nearshore area, an increase or decrease of E_3 implies erosion or accumulation of sediments. This suggests the possibility that the third autofunction (19% of the mean square value, explained without considering the first mode) is related to the longshore transport of sediments.

The mean square value of the second mode provided three times more explanation than the third mode implying that, in general, the process associated with variations of transport normal to the shore is more important than that associated with the longshore transport.

This indicates that the effect produced by the variation in the energy level of the wave regime, prevails over that caused by the direction of approach. Some doubts might arise, however, regarding the interpretation and the associations ascribed to the second and third modes. If we compare Figure 2 with Figure 9 of WINANT *et al.* (1975) it might be thought that the third mode as presented here, could represent the formation of the continuous structure called berm-bar. This appear unlikely as it would imply that the berm-bar is being formed by a component which is contributing three times less "energy" than that of the second mode.

It would also be difficult to justify the view that the larger variations shown by the second mode in the deeper zones of the profile could be produced by longshore currents. Besides the formation of a berm-bar, which must be associated with wave energy, rather than with the direction of wave apporach and it is not clear that any relationship between the E_3 component (Figure 3c) and the height of the breakers exists (Figure 4a).

This line of reasoning leads to the solution of a problem previously mentioned, that is, if the second mode explains the formation of the bar, associated with high wave energy levels, how can it be explained that at the same time an accumulation of sediments is taking place, although to a lesser degree, in the nearshore zone when there should be active an erosion process? To clarify this point the ideas of WINANT and AUBREY (1976) should be considered.

The "Zero-Crossing" of the second mode suggest the possible existence of two "hinge" points, A and B (Figure 2). Each, divides zones of accumulation from areas of erosion, as shown by the switch in the sign of the autofunction. At a hinge point, the change in the level of sediments is not necessarily small, since the remaining organization modes contribute their share, that is to say a hinge is not a mode, with no vertical movement. The hinge points occur at depths in the order of 0.5 m and 2.0 m, relative to mean sea level.

During April 1980, the period of high waves the E_2 component was positive, indicating an erosion process in the nearshore area. At the same time (in phase) these high energies transport sediments through hinge point B, shoreward. During the following months, May and June, 1980, a decay in the wave activity resulted in the return of sediment from the zone between both hinge points, where they had accumulated during the previous month, spreading now partly towards deeper areas and partly towards the beach. The shoreward transport might be due to the action, itself, of the waves and the offshore transport could be the product of the influence of the ever increasing gravitational forces generated by the increase of the profile slope in the neighbourhood of the B hinge point. That is to say, forces on hinge point B acting on the sediments and induced by the action of the waves on the bottom cannot balance gravitational forces, resulting in a transport of sediments towards greater depths in the search of a new more stable slope of equilibrium and in balance with the energy level of the acting waves.

WINANT and AUBREY (1976), when studying beach profiles in southern California, reached a similar process sequence associated with the wave regime. It is likely that a denser and more systematical observation program on the area of the hinge points will be necessary to clarify and quantify the possible sequence of processes.

CONCLUSIONS

A set of beach profiles was analyzed by using Empiric Orthogonal Functions (EOF) which provide quantitative results showing the spatial and temporal changes of the profiles. In order to establish the predominant causes of these variations, the relationships existing between the temporal autofunctions and the magnitude and direction of the incident waves was investigated. It was found that a 99.17% of the mean square value of the profile can be explained by the first three autofunctions (organization modes). The mode with the largest contribution represents the average profile function, with a small temporal variation and without a definite tendency. This would indicate a beach in a state of equilibrium during the observation period.

Due to correlations with wave energy levels, the second mode would be representative of the formation and erosion of a bar at the offshore end of the studied profile. This mode also suggests the presence of two hinge points, through which the processes of accumulation and erosion occur out of phase, thus indicating a possible interplay between the wave induced forces and the gravitational forces on the seabed slopes. The third mode is apparently related to the direction of wave incidence, suggesting a transport of sediments alongshore.

Since the percentage of the explained variance (with respect to the first mode) by the second mode is three times larger than that explained by the third mode, the dominant cause of the profile variation is due to the wave energy level, not to its direction. This does not necessarily indicate that the main volume of transported sediment is being moved in a direction normal to the shore, since a longshore transport of sediments of stationary characteristics in time, might exist. With only one line of profiles it is not possible to resolve volumetrically the two contributions.

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□ RESUMEN □

Un conjunto de sondajes de un perfil de playa realizados durante 1980 y parte de 1981–1982, en la localidad de Pinamar, Provincia de Buenos Aires, Argentina, se ha analizado estadísticamente por medio de Funciones Empíricas Ortogonales que permiten separar sus variaciones espaciales y temporales. El 99,17% de la variación del perfil es explicado por sus tres primeros modos de organización. Se muestra que la segunda: y tercera autofunciones temporales poseen una correlación significativa con las variaciones del nivel energético y dirección de incidencia de las olas, respectivamente. Ello indicaría que el segundo modo está asociado al transporte de sedimentos perpendicular a la costa y el tercer modo al transporte a lo largo de la misma. La segunda autofunción también sugiere la presencia de puntos de pivotes (en profundidades de 0,5 m y 2 m, respecto del nivel medio del mar), a través de los cuales, los procesos de acumulación y erosión ocurren fuera de fase.

□ ZUSAMMENFASSUNG □

Eine Reihe von Sondierungen eines Strandprofils wurden 1980 und teilweise 1981–82 in Pinamar in der Provinz von Buenos Aires, Argentinien, durchgeführt und dann statistisch mit Hilfe der empirischen orthogonalen Funktionen ausgewertet, um ihre räumlichen und zeitlichen Variationen zu trennen. Die ersten drei Anordnungsmuster können 99,17 % der Veränderungen des Profils erklären. Es wird gezeigt, daß die zweite und dritte zeitliche Autofunktion jeweils signifikant mit den Änderungen des Energieniveaus bzw. des Einfallswinkel der Wellen korrelieren. Analysen belegen, daß das zweite Anordnungsmuster mit dem normalen, senkrecht zur Küste verlaufenden Sedimenttransport zusammenhägt, während das dritte auf küstenparallele Strömung zurückzuführen ist. Die zweite Autofunktion läßt außerdem auf Angelpunkte in 0,5 m und 2 m Meerestiefe schließen, durch welche Akkumulations- und Erosionsprozesse phasenweise gesteuert werden.—*Helmut Brückner, Geographisches Institut, Universität Düsseldorf, F.R.G.*