"F/ft. for frozen 25° Brix orange concentrate.

Work is continuing to determine a method of predicting freezing times of 42° Brix orange concentrate.

**Summary and Results**

The equations of unsteady state heating or cooling can be applied to 58.9° Brix orange concentrate for prediction of heating or cooling times. The average value of "k," the coefficient of thermal conductivity for 58.9° Brix concentrate, was found to be 0.17 B.t.u./hr./sq. ft./° F./ft. The same equations are applicable to 42° Brix orange concentrate above its freezing point. The value of "k" for 42° Brix concentrate was found to average 0.18 B.t.u./hr./sq. ft./° F./ft.

Values of "k" for 42° Brix orange concentrate during freezing and below the freezing point were calculated to be 0.04 B.t.u./hr./sq. ft./° F./ft. Since the method used does not take into account a change in phase, but is based on a homogeneous substance, this last figure can be applied only to the conditions of the experiment, i.e., natural convection.

**THE APPLICATION OF STATISTICAL QUALITY CONTROL TO FILLING MACHINE OPERATION**

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Our pursuit of statistical quality control started with the desire to know just how our filling machines were performing, and to be able to give our filling machine operators a little information that they didn't already know. Avoidable over-fill is an economic loss in any case. An extreme example is that of 6-ounce orange concentrate, which represents an expensive product in a large number of small units. In a plant producing 10,000 cases per day, each gram, or each 32/ounce of unnecessary overfill, together with $50,000 worth of cans and cartons, represents $400.00 worth of production. With the can filling operation, as with any repetitive process there are minor variations or chance causes which prevent each unit from being exactly like the last. These acts of chance the statisticians call "unassignable causes" and they result in the units falling into a distribution pattern which is called "the normal distribution curve." (See Fig. 1.)

In view of these unavoidable variations, the first question to be answered is "In fairness to the consumer what should the minimum filling limit be?" The maximum will then be determined by the accuracy of the filling machine. Minimal standards that have been established in Court are that a pack should average above the amount declared on the label, and that no single unit should be unreasonably light. This would mean that if the spread between the average can and the lightest can were not unreasonable, the pack could theoretically be made with 49% underweight and 51% overweight. We have adopted 1% as a reasonably light limitation, but in the interest of quality and providing a safety factor we are following the government Grade A practice of limiting lightweights to 1 in 6, or 16% lightweight and 84% overweight.

To summarize, then, it is our endeavor to keep the fill as low as possible and yet stay above the following limits:

1. That no can shall be lighter than 99% of the amount stated on the label, and
2. That no more than 16% of the cans shall be lighter than the amount stated on the label.

In order to meet these limits knowledge is required concerning both the average fill and the range of fill from the lightest to the heaviest can that is being incurred by unassignable causes. By the use of the control chart method, this information can be derived from the minimum of data. Five random cans drawn from each filler line two or three times per hour are usually sufficient. Each group of 5 cans is considered as a sample. The average weight of the 5 cans minus the tare weight and the range in weight between the lightest and the heaviest of the 5 cans are recorded. Since individual can weights are not required, we use a double pan balance to determine the total weight of the 5 cans and multiply that by .2 for the average. The heaviest and lightest cans are then quickly picked out by comparing them against each other on the pans, and the
FIG. 1
NORMAL DISTRIBUTION CURVES

FIG. 2
FREQUENCY TABULATION

FIG. 3
CONTROL LIMITS FOR AVERAGES
OF 5 (X) AND INDIVIDUALS (X)
SHOWING THE LOCATION OF THE
X LOWER LIMIT WITH RESPECT
TO THE LABELED FILL.

INITIAL SPECIFICATION UPPER LIMIT = SPEC. LL + 2(0.577R) + ARBITRARY 0.2 fl. oz.

X UPPER CONTROL LIMIT = \( \bar{X} + 0.577R \) = 46.51

X LOWER CONTROL LIMIT = \( \bar{X} - 0.577R \) = 45.93

SPECIFICATION LOWER LIMIT (FOR X) = LABEL - 0.147R = 45.92

FIG. 4
A TYPICAL
CONTROL
CHART
The weight required to balance the lightest against the heaviest is the range.

The nomenclature used in referring to these values is as follows:

- $X$ equals the net weight of any individual can.
- $\bar{X}$ equals the average net weight of any 5-can sample.
- $\bar{X}$ equals the average of a series of $\bar{X}$ values, and as can be seen represents the average net weight of all the individual cans contained in the series.
- $R$ equals the range between the heaviest and the lightest cans in any 5-can sample.
- $\bar{R}$ equals the average of a series of $R$ values.

(A reasonably valid average is usually based on not less than 25 $R$ values and may be based on as many as desired, as long as all values represent samples which were drawn from production manufactured under the same conditions.

Having once established the foregoing minimum limits for individual cans (X values) the first things to be determined are the limits for the average weights of 5 cans ($\bar{X}$ values) which will assure the X values being within their limits. This involves the collection and examination of what is called "preliminary data." The filler is set close to the desired operating point, and allowed to run as steadily as possible. 25 samples are then collected and weighed as outlined. These may be collected oftener than two or three times per hour, in order to save time, but they should be taken over a period of at least an hour or two so as to more nearly represent operating conditions. From the 25 $\bar{X}$ values and $R$ values thus determined, the averages, $\bar{X}$ and $\bar{R}$ are extracted. $\bar{X}$ represents the location or average of the distribution being produced, and can be changed higher or lower by raising or lowering the setting of the filling machine. $R$ is a measure of the spread of the distribution, and thus will indicate how far above the minimum the average must be in order that the lightest can will not be less than the minimum. Being a measure of the spread, $R$ becomes greater as the filler wears out and becomes erratic in operation. Conversely, by overhaul of the filler or by purchase of a more accurate one, $R$ becomes smaller.

If the filler was in statistical control the values for $R$ and $\bar{X}$ are representative and valid, and it will be found that none of the $R$ values will have exceeded 2.11 $R$ and as will be shown later, none of the $\bar{X}$ values will have deviated more than plus or minus .577 $\bar{R}$ from $\bar{X}$. If some of the $R$ or $\bar{X}$ values do not fall within these limits, and if for mechanical reasons it is known that they are not representative of the process, those values may be discarded and the averages $\bar{X}$ and $\bar{R}$ redetermined. For example, if one filler valve were set substantially different from the rest, an unusually large range, not representative of the process, might result, and the average of the 5 cans, $\bar{X}$ would not be representative even though it might fall within its limits. If the filler were stopped, and several cans filled to the Brim, $\bar{X}$ would fall out of limits and the range would be out unless 5 of the heavy cans were included in the sample. Neither the range nor the average, $\bar{X}$ would be representative of the process.

On the other hand, if the main filler adjustment were to slowly shift higher during the run, a few $\bar{X}$ values at the beginning of the run would later be found below limits, while those at the end of the run would be above limits. All the $R$ values collected during the run would be unaffected, however, and they will be representative of what the process could have been had the filler been held in adjustment.

In cases where the filler is in serious need of adjustment before it can be brought into control, a frequency tabulation (Fig. 2) of the weight of about 250 consecutive cans together with observation of the filler will often uncover the trouble. Two large peaks in the distribution of a frequency tabulation would indicate, for example, that several valves were set differently from the majority. A small peak set off to one side, or several small peaks in the main distribution would indicate one or two valves set differently from the majority.

A further refinement is to mark about 250 covers in rotation with the filler valve number and make a frequency tabulation for each valve. Frequency tabulations can be made very quickly using, for example, a Speedway over-and-under balance, with one person weighing and calling out the readings while another person records them.

Once a representative value for $\bar{R}$ is determined, the characteristics of the filler are known and specifications can be set. The sta-
statistical basis for the control chart method is the fact that a known percentage of the units making up the normal distribution are contained in any measured distance from the average, if the measured distance from the average is expressed as units of standard deviation. The standard deviation is referred to as Sigma ($\sigma$) and is a dependent characteristic which can be calculated by lengthy means for any distribution. It is the square root of the average of the squares of the deviation of each unit in the distribution from the average of all units in the distribution. In general terms, $\sigma$ is a weighted average of the deviations of each unit from the average unit, so weighted as to give more influence to the units further from the average. The average range ($\bar{R}$) is a function of $\sigma$ and offers a short cut for determining it. When using samples of five it has been found that $\bar{R} \times .430 = \sigma$. From a table of areas under the normal curve it is found that 99.7%, or practically all of the area on both sides of a normal curve fall within limits of plus or minus three $\sigma$ from the average, and that 32% of the area on one side of the curve, or 16% of the total area falls outside the limit of one $\sigma$ from the average.

One additional relationship to mention is that if the dotted curve in Fig. 1 represents the normal distribution curve for the fill of a group of individual cans, the solid curve will represent the distribution for the average fill of samples of 5 cans drawn at random from the same group. The curves are related because they represent fill weights taken from the same group (or universe) of cans and the relation is proportional to the square root of the number of cans which the averages include (in this case 5).

For example, the spread that contains a certain percentage of the area of the distribution for the individuals is the square root of 5 times the spread required to contain the same percentage of the area of the distribution for the averages of 5. This simply means that $\sigma$ for the distribution of the averages of 5 equals $\sigma$ for the distribution of the individuals divided by the square root of 5. Turning the distribution curves from Fig. 1 on their sides as in Fig. 3 and using the values mentioned above the control chart for $\bar{X}$ values can be illustrated graphically with the 16% and 99.7% upper and lower limits expressed in terms of $\bar{R}$.

Referring to Fig. 3 and to the original limitations that (1) no can be more than 1% light; and (2) no more than 16% of the cans be light, it can be seen that both of these limitations do not apply at once. If, for example, the filler is a poor one, and $\bar{R}$ is very high, thus giving a wide spread, the first limitation will be exceeded before the second, and cans 1% light or more will be produced before 16% of the cans are light. Conversely, if the filler is in good condition and $\bar{R}$ is low, more than 16% of the cans will be light before any one of them is 1% light. If $.86 \bar{R}$ is greater than 1% of the fill stated on the label, the first limitation applies; if it is less then the second applies. The lower limits for $\bar{X}$ to satisfy the first limitation is then: the label fill minus 1% of the label fill plus 0.713 $\bar{R}$. To satisfy the second limitation it is: the label fill minus .860 $\bar{R}$ plus 0.713 $\bar{R}$ or the label fill minus 0.147 $\bar{R}$. The upper limit for $\bar{X}$ in both cases is the lower limit plus twice .557 $\bar{R}$. A typical control chart for $\bar{X}$ and $\bar{R}$ values with 46-ounce cans is shown in Fig. 4. Since $\bar{R}$ reflects the mechanical condition of the filler and should change slowly if at all, it is determined only twice a week from the preceding half week's data and a line representing it is drawn in on the range chart as a basis for the coming half week.

It should be mentioned here that at higher speeds some fillers may become less accurate, while others become less accurate at lower speeds. This, of course, will result in a different range. However, a reasonable variation in the filler speed may not affect the range although it may affect the average.

Next, the upper limit of the range, 2.11 x $\bar{R}$ is drawn on the range chart. Then the lower specification limit (the label minus 1% plus .713 $\bar{R}$ or the label minus .147 $\bar{R}$ as the case may be) is drawn on the chart for $\bar{X}$.

Initially we set an upper specification limit for $\bar{X}$ wider or greater than twice .577 $\bar{R}$ to avoid drawing too fine a bead on the filling machine operator. As the inspector determines and records his $\bar{X}$ and $\bar{R}$ values he calls for higher or lower fill as required to keep the $\bar{X}$ value within the initial limits. The only exception to this is in the event that $\bar{R}$ falls
above its limit of 2.11 \( \bar{R} \). In this case, even though the accompanying \( \bar{X} \) value is out of limits, it is not likely that the average fill of the machine has shifted, but rather that some determinable cause has resulted in an unrepresentative can. It is the purpose of the control chart to disclose these misdeeds, and if close control is to be achieved, their causes must be determined and brought under control.

Since with different inspectors and operators different results may be obtained, the actual \( \bar{X} \) is determined and drawn on the chart at the end of each shift, based on that shift's values. Above and below this a distance of .577 \( \bar{R} \) the control limits within which the filling machine is capable of operating are drawn. If all the \( \bar{X} \) values fall within this line or very close to them a good job of operating the filler is being accomplished.

Once the operator and inspector have the system smoothed out to the point that the \( \bar{X} \) values at the end of the shift fall reasonably well within the control limits of the shift average, \( \bar{X} \), plus or minus .577 \( \bar{R} \) then the upper specification limit can be lowered closer to the goal of twice .577 \( \bar{R} \) above the lower specification limit. The number of points falling within the control limits is a measure of how near to the best possible limits around a given average the filler is being operated. The closeness of the lower control limit to the lower specification limit is a measure of how close the actual average is being held to the minimum possible average.

Although this information is easier to visualize when presented as a graph, the values and specifications may be kept and operated in tabular form.

A difficulty which we encountered on our piston type fillers was that occasionally very high ranges would be obtained. This was quickly attributed to the occasional necessity of having to stop the filler while cans were under it. In this case the valves would drain and underfill the cans coming through on the next revolution. Urging the operator to throw out these cans did not help the matter, and although we only found an occasional sample of that nature, that was only because we were taking samples occasionally. Since these cans were extremely light, a magnetic separator which would lift them off of an inclined belt was finally found to be the answer.

On our 46-ounce filler it has been found that daily attention to the valves is necessary. After a few days' operation they begin to leak. This results in occasional high ranges and numerous \( \bar{X} \) values out of limits. With no maintenance \( \bar{R} \) values of .8 fluid ounces were found at the end of the week as compared with values of .4 fluid ounces at the beginning of the week. Comparable results were obtained on the piston type filler after two months' operation at 400 cans per minute. These examples illustrate that if the occasion arises that the operator can no longer meet the limits, it will be found that the \( \bar{R} \) has increased and new limits are indicated, or repairs to the filler to bring the \( \bar{R} \) back to the original level should be undertaken.

One thing we have found to be of practical necessity is a series of charts converting the net weights into fluid ounces, because it is difficult to visualize how full a 6-ounce can is when it has 214 grams in it, or how full a 46-ounce can is when it has 3.146 pounds in it.

In conclusion, I would like to state that the control chart is an aid to precise adjustment and maintenance of a filling machine. As such it has enabled us to keep the average fill of 6-ounce cans within .9 gram of 6 ounces without more than 16% of the cans being less than 6 ounces, and it has indicated when such control was no longer possible and overhaul became necessary.

Three practical references on this subject are: