Spatial Modeling of Mud Thickness and Mud Weights (1988-2006), Lake Okeechobee

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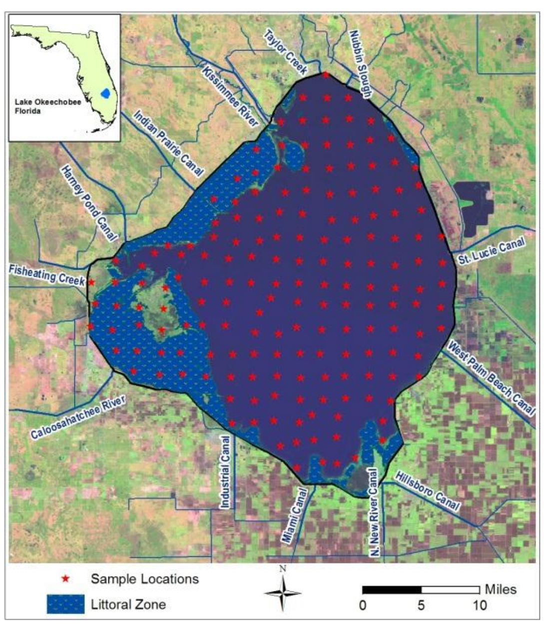
1. Introduction

Lake Okeechobee is large and shallow with an area of approximately 1730 km^2 and an average depth of 2.7 m (Figure 1). It is the largest freshwater lake in the southern United States. The name Okeechobee comes from the Hitchiti words oki (water) and chubi (big). Lake Okeechobee formed out of the ocean about 6,000 years ago when the waters receded (Brooks, 1974). The floor of the lake is a limestone basin, and its water is turbid from mud sediments that underlay a large portion of the lake.

Over 44% (as of 2006) of Lake Okeechobee is underlain with P enriched sediments (Yan and James, 2007). The lake sediment has been studied infrequently since the late 1960s. Fifteen sediment samples were analyzed during 1969-1972 by the U.S. Geological Survey in cooperation with the Central and Southern Florida Flood Control District (the original name of the SFWMD) (Joyner, 1974). In 1988, a comprehensive survey reported that the upper 10 cm of mud sediments within the lake contained an estimated 28,600 metric tons of P (Reddy *et al.,* 1995). An additional survey in 1998 made similar estimates of sediment P within Lake Okeechobee (Fisher *et al.,* 2001). The distributions of these sediments, and their compositions and changes over time provide insights into the historical conditions of the lake and a better understanding of sediment-water interactions; which have a large effect on the lake environment.

Figure 1: Major tributaries and sediment sample locations, Lake Okeechobee.



Source: Author

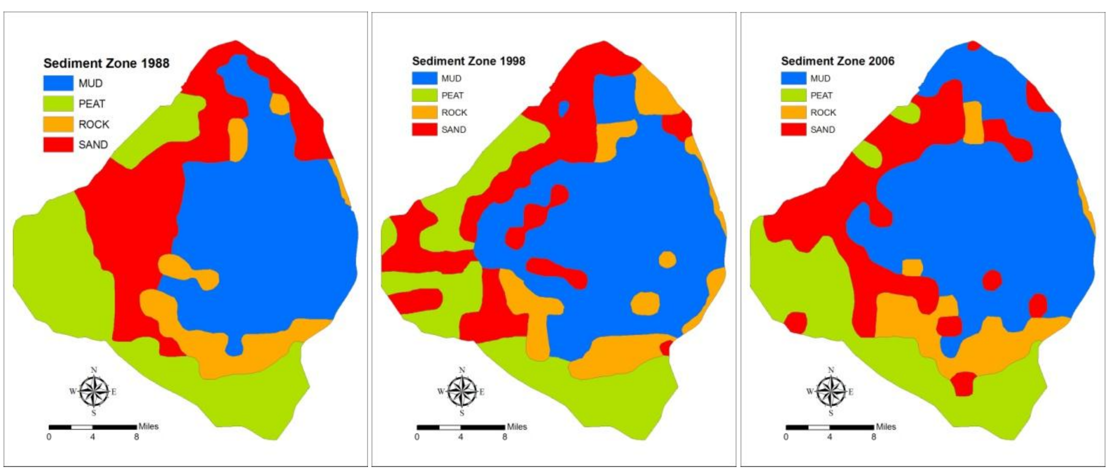
Hurricanes (especially during 2004-2005) have affected Lake Okeechobee's ecosystem causing sediment re-suspension from wind generated waves, which in turn caused dramatic changes of water quality (Havens *et al.*, 2001; James *et al.*, 2008). The re-suspension of sediments facilitated their redistribution throughout the lake, increased the active layer thickness, and reduced the cohesiveness of the surface layers of sediments (James *et al.*, 2008; Jin *et al.*, 2011). All of these effects resulted in poorer water quality and increased turbidity throughout the lake.

The aim of this research was to develop optimal spatial models to describe the mud distributions and estimate the mud weights in Lake Okeechobee over-time (1988-2006). In particular, their spatial changes were examined to identify the potential impacts of extreme environmental forcing during 1998 and 2005 hurricane seasons.

2. Data Description

Three sediment surveys using similar techniques and the same sampling locations were conducted in 1988 (Reddy *et al.*, 1995), 1998 (Fisher *et al.*, 2001) and 2006 (BEM & University of Florida, 2007) (Figure 1). The last survey was undertaken after the 2004 and 2005 hurricane events. Bottom sediment type, mud thickness, mud density, and other variables were measured at all sites. Four sediment zones were identified in the lake: Mud, Peat, Sand and Rock. Sediment zone boundaries were delineated based on sampling site description, site locations, elevation data and high resolution aerial-photography (Figure 2). From 1988 to 1998, there were no major changes in the rock and sand zones, but the peat zone declined from 27.5% to 22% of the lake area; from 1998 to 2006, the mud zone increased from 38.5% to 46%; showing the greatest change (Figure 2).

Figure 2: Sediment zones distribution from 1988 to 2006, Lake Okeechobee



Source: Author

Mud thickness varied both on a temporal and spatial scale. The maximum mud thickness in 1988, 1998 and 2006 was 66, 74 and 51 cm respectively and the mean thickness declined from 12.47 cm in 1988 to 8.27 cm in 2006 (Table 1). Spatially, there was a first order trend of mud thickness in an East/West direction and a strong second order trend in a North/South direction (Figure 3).

Table 1: Summaries of mud thickness and bulk density

	1988		1998		2006	
	Mud thickness (cm)	**Density (g/cm³)**	**Mud thickness (cm)**	**Density (g/cm³)**	**Mud thickness (cm)**	**Density (g/cm³)**
Min.	0.00	0.01	0.00	0.01	0.00	0.04
1st Qu.	0.00	0.11	0.00	0.14	0.00	0.16
Median	0.00	0.15	0.00	0.23	1.00	0.25
Mean	12.47	0.38	11.17	0.53	8.27	0.58
3rd Qu.	18.88	0.42	13.75	0.92	14.75	1.07
Max.	66.00	2.49	74.00	1.67	51.00	2.06
Skewness	1.41	1.93	1.66	0.89	1.43	0.90
Kurtosis	3.68	6.13	4.51	2.24	4.05	2.37

Source: Author

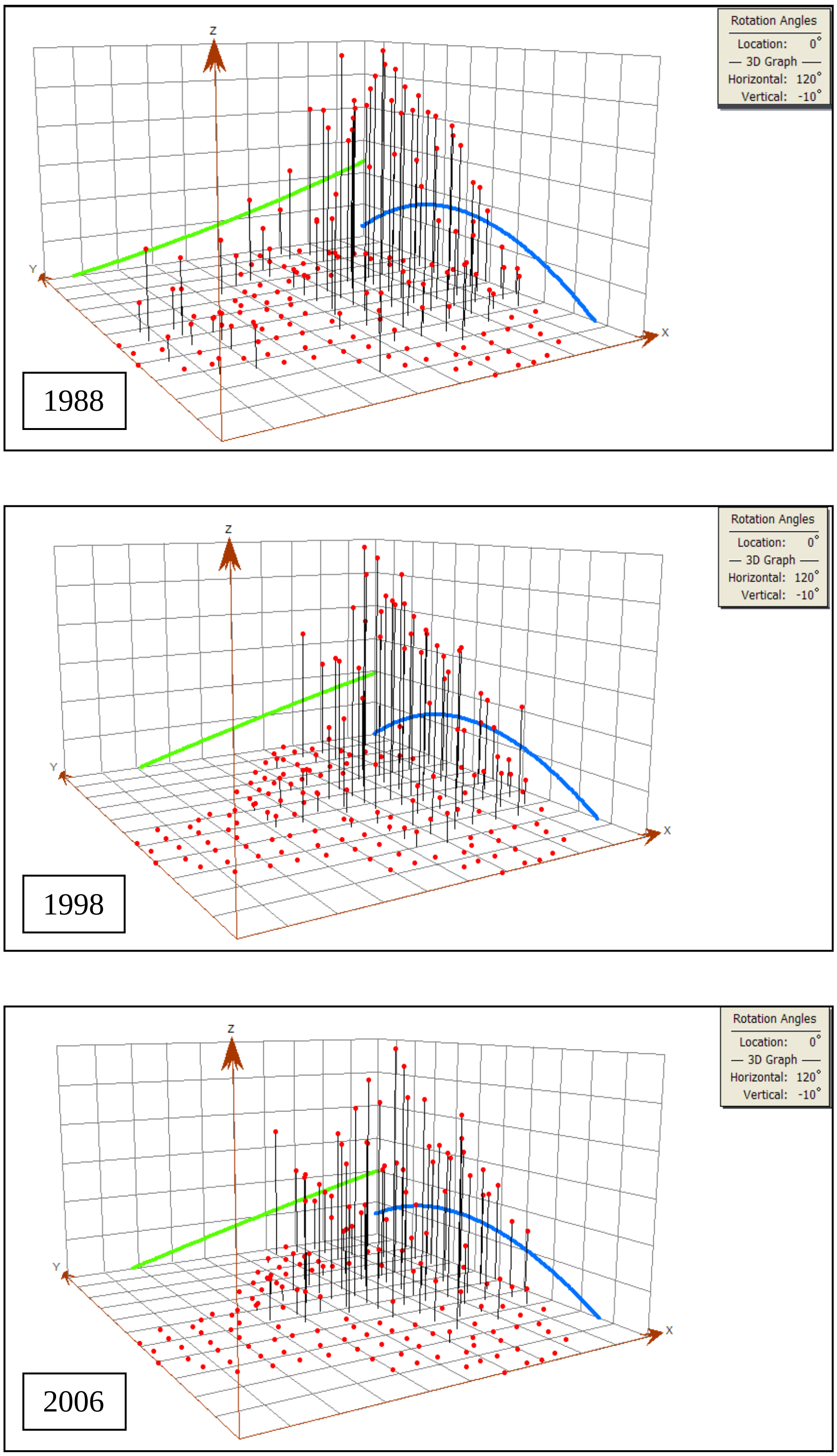
3. Methodology

Kriging has been used as a synonym for geostatistical interpolation for many years (Burrough and McDonnell, 1998). The original idea came from the mining engineer D. G. Kridge and the statistician H. S. Sichel. Kridge published this technique in 1951, but the mathematician G. Matheron derived the formulas and established the linear geostatistics (Cressie, 1990; Webster and Oliver, 2001). The major contribution from Matheron (1962) and Gandin (1963) was the development of the semi-variance ($\lambda(\mathrm{h})$):

$$\lambda(\mathrm{h}) = \frac{1}{2}\sum_{i=0}^{n}[(\mathrm{z}(\mathrm{s_i}) - \mathrm{z}(\mathrm{s_i} + \mathrm{h}))^2] \quad (1)$$

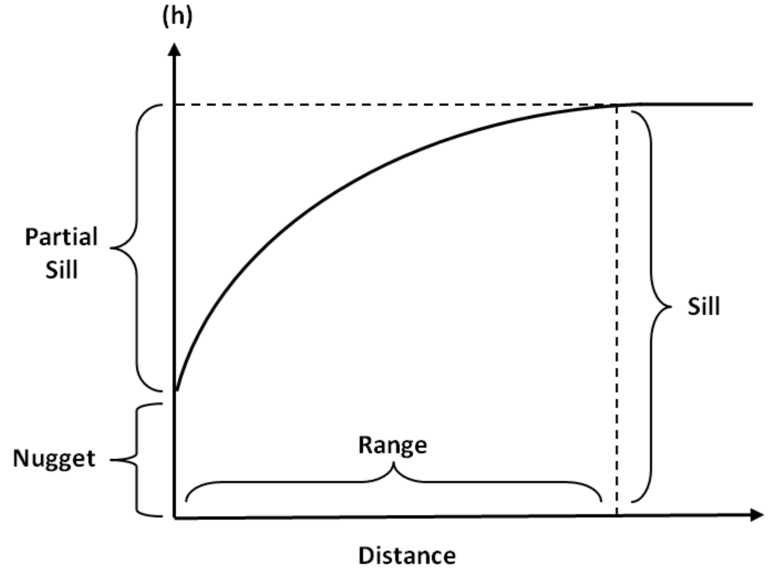
where z (s_i) is the value of the target variable at sampled location *i* and *z* (s_i +h*)* is the value of the variable at distance *h from* s_i. If there are *n* point observations, there will be n*(n-1)/2 pairs for which a semi-variance can be calculated. All semi-variance pairs can be plotted against a standard distance or lag to create a standard experimental variogram (Figure 4). The sill is where the fitted curve levels off at large lag. It implies that at these large lag values there is no spatial dependence between the data points. Range is the distance value at the sill. This is a critical measurement of the variogram because it describes how inter-site differences are spatially dependent. Within the range, closer sites are more similar to each other. The range also defines the size the search window should be for weighted moving average interpolation. The nugget is the positive $\lambda(\mathrm{h})$ value when $h \rightarrow 0$. It is the estimate of the residual and spatially unrelated noise. This is also the variance of measurements. The variogram provides a quantitative description of the regionalized variation. Next the variogram is fitted to a standard variogram model such as linear, spherical, exponential, circular, Gaussian, Bessel, or power (Isaaks and Srivastava, 1989; Goovaerts, 1997). The variogram provides useful information for interpolation, optimizing sampling and determining spatial patterns (Figure 4). It also offers a measure of associated uncertainty, i.e. the estimated variance of the prediction error for a given model.

Figure 3: Trend analyses of mud thickness data
(X, Y and Z represent E, N and vertical direction, respectively; red dots are samples; green and blue lines are trends)



Source: Author

Figure 4: A Variogram and its parameters



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Kriging methods provide great flexibility for interpolations and different sub-methods were developed such as simple kriging, indicator kriging, universal kriging, etc. All these methods produce smoothly varying surfaces accompanied by an estimated variance surface. Simple Kriging (SK) assumes that the data have a known mean value throughout the study area and exhibit both first and second order stationarity. These assumptions are overly restrictive for most problems and hence this method is rarely used. De-trending and z-score (normal) transformation may help to remove some of these problems. Ordinary Kriging (OK) and its variants have more relaxed assumptions than Simple Kriging.

Ordinary Kriging assumes second-order stationarity with an unknown mean. In Ordinary Kriging the expected value of the random function is locally re-estimated from local data, while the covariance model is kept stationary:

$$Z(s) = \mu + \varepsilon'(s) \quad (2)$$

where μ is the constant stationary function (global mean) and $\varepsilon'(s)$ is the spatially correlated stochastic part of variation. The prediction at location S_0 is a weighted average:

$$\hat{z}_{\mathrm{OK}}(s_0) = \sum_{i=0}^{n} w_i(s_i) \cdot \mathcal{Z}(s_i) \quad (3)$$

where w_i is the kriging weight at location s_i, $\mathcal{Z}(s_i)$ is the observation at location s_i. In a way, kriging can be seen as a sophisticated inverse distance interpolation scheme, with the weights based on the spatial autocorrelation structure. Co-kriging allows samples of an ancillary variable (also called the co-variable), besides the target value of interest, to predict the target value at un-sampled locations. With Co-kriging the estimated value at an un-sampled location is a linear weighted sum of all of the variables being examined (i.e. two or more). The co-variables may be measured at the same points as the target (co-located samples), at other points, or both. Co-kriging is the extension of the kriging paradigm to estimate one attribute using a data set that contains observations related to other attributes (Goovaerts, 1997). For more detailed discussions, refer to the geostatistics textbook by Isaaks and Srivastava (1989).

4. Mud Thickness Model Validation Results

Training and testing data sets were created randomly using GIS Geostatistical Analyst (GA) extension (ESRI, 2001). Models were calibrated and validated for accuracy with the training and testing data sets, respectively. A spherical semivarigram model was fitted for each mud thickness data set. The following parameters were selected based on data exploration and interactive visual check using ArcGIS Geostatistical Analyst: lag size 8000 ft, lag number 9, range 50000/45000 ft, and the second-trend removal.

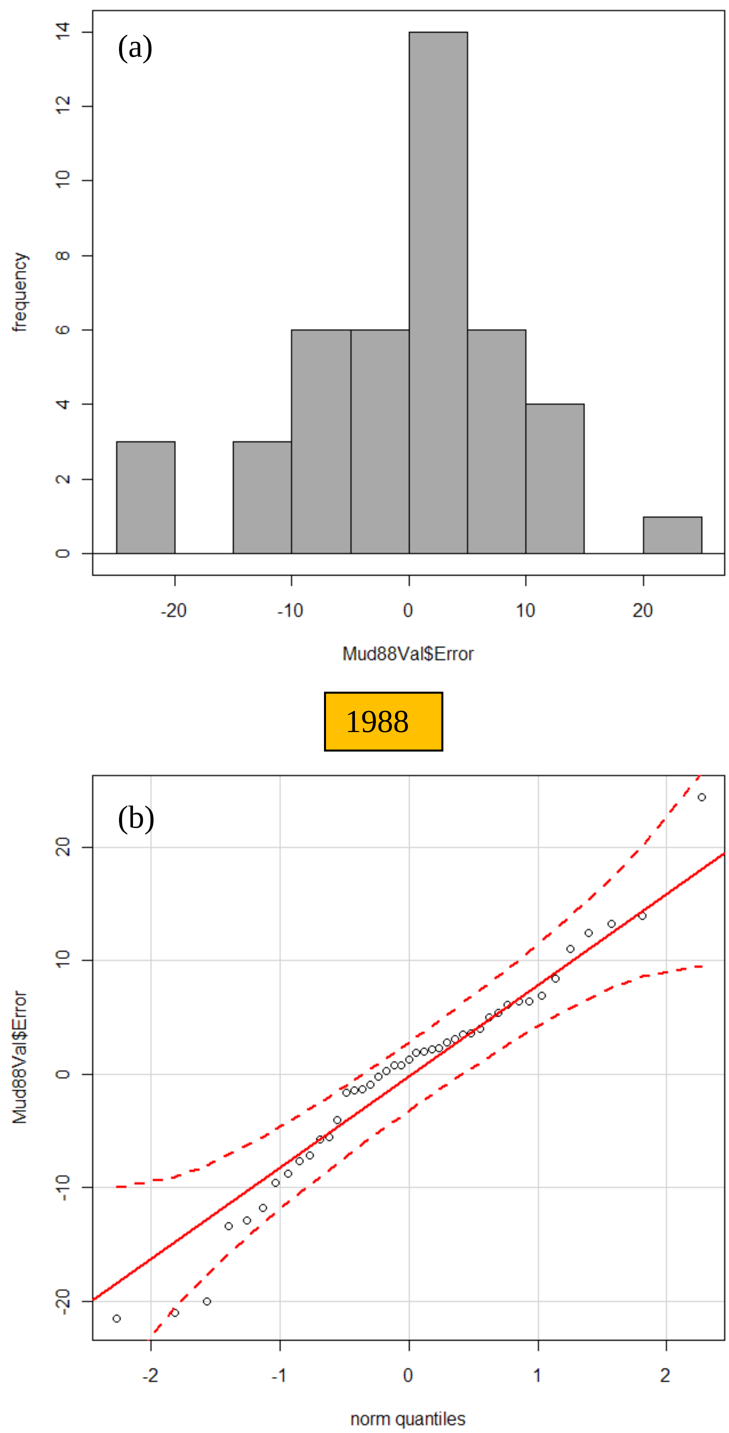
Table 2: Calibration and validation results of mud thickness data (cm)

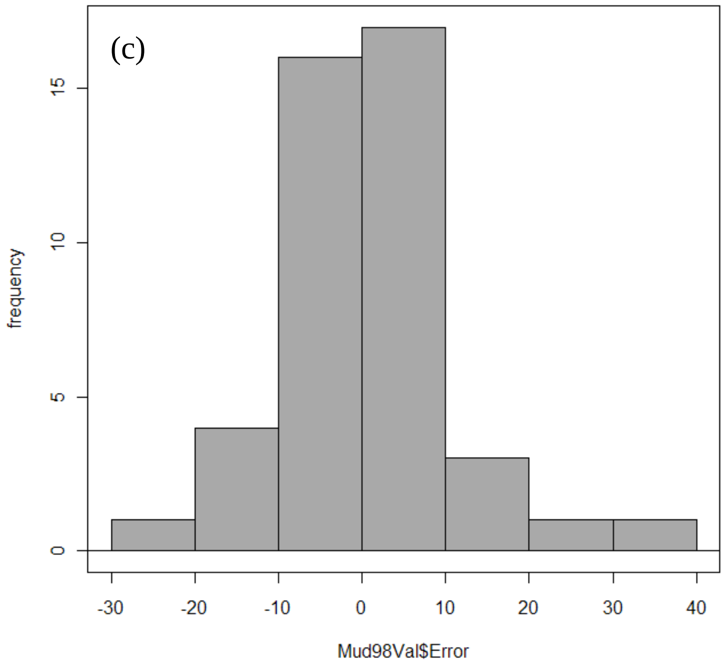
	Errors	**2006**	**1998**	**1988**
Calibration	Mean	0.0195	0.0601	-0.0862
	Root-Mean-Square	8.6650	9.5020	9.7910
	Average Standard Error	8.3010	11.0000	10.4400
	Mean Standardized	0.0019	0.0014	-0.0050
	Root-Mean-Square Standardized	1.0510	0.8653	0.9190
Validation	Mean	-0.1243	0.1014	-0.0638
	Root-Mean-Square	7.8210	10.2400	9.3870
	Average Standard Error	8.2920	11.1600	10.6500
	Mean Standardized	-0.0272	0.0119	-0.0088
	Root-Mean-Square Standardized	0.9470	0.9270	0.9224

Source: Author

The validation errors were determined (Figures 5 a, c and e) and Quantile-Comparisons were plotted (Figures 5 b, d and f) using R software (http://www.r-project.org/). All the errors were normally distributed for the three mud thickness data sets. In the normal quantile plots, which plot the empirical quantiles of the errors (Y-axis) against theoretical quantiles (X-axis) of a comparison normal distribution, the 1988 mud thickness has errors limited to the 95% confidence envelopes; some of the validation sites have errors outside of the envelope for 1998 and 2006 data sets, which show under-estimates (positive outliers) and over-estimates (negative outliers). The scatter plots of the measured and predicted values of the validation sites show reasonable goodness of fit with determination of coefficients (R2) of 0.7893, 0.6904 and 0.6529 for 1988, 1998 and 2006 mud thickness, respectively (Figure 6). All the calibration errors are a little lower than the validation errors, partly due to smaller number of validation sites (Table 2).

Figure 5: Validation error distributions of the mud thickness data




(c)
frequency
0
5
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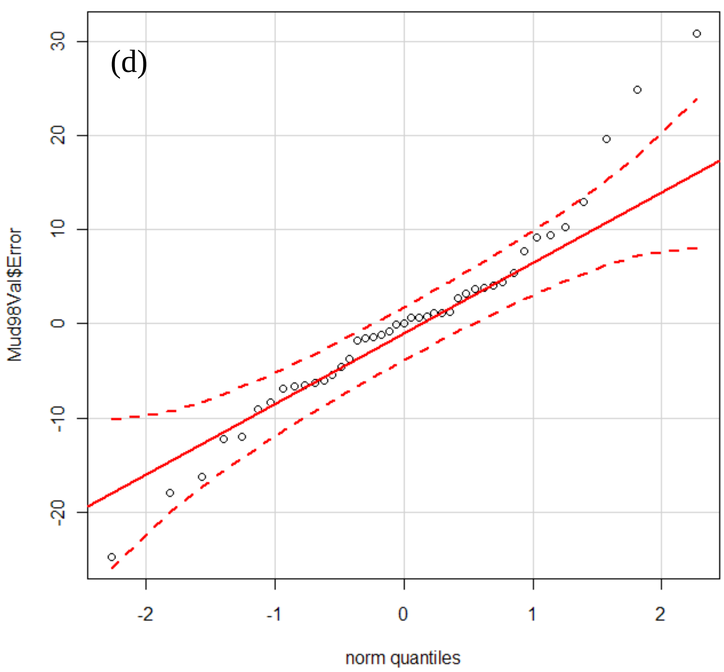
1998


(d)
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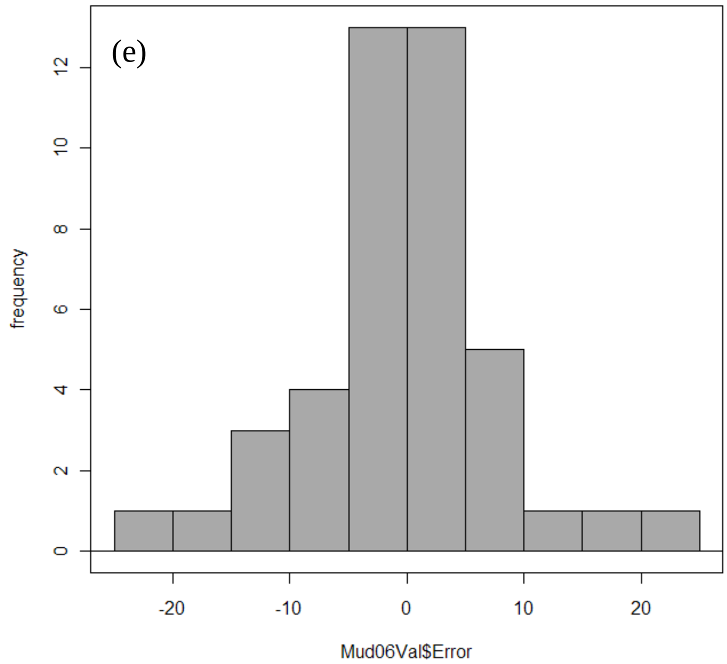

2006

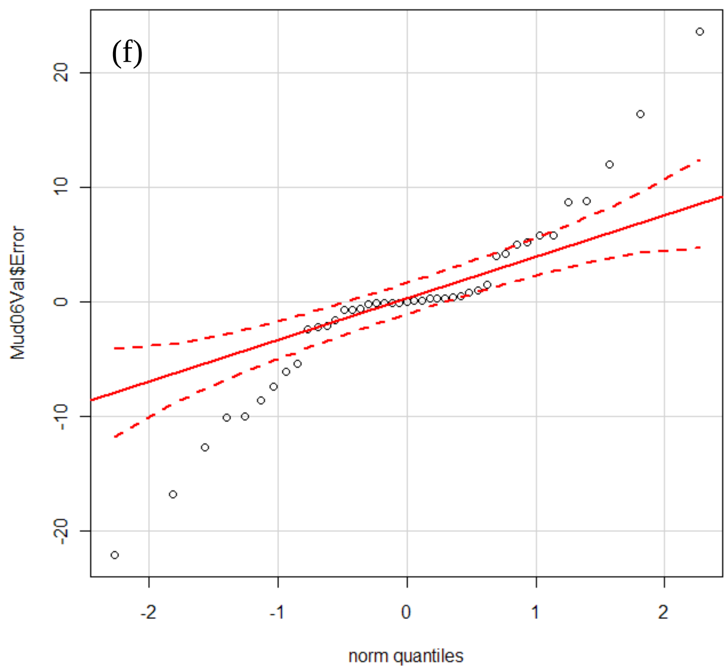

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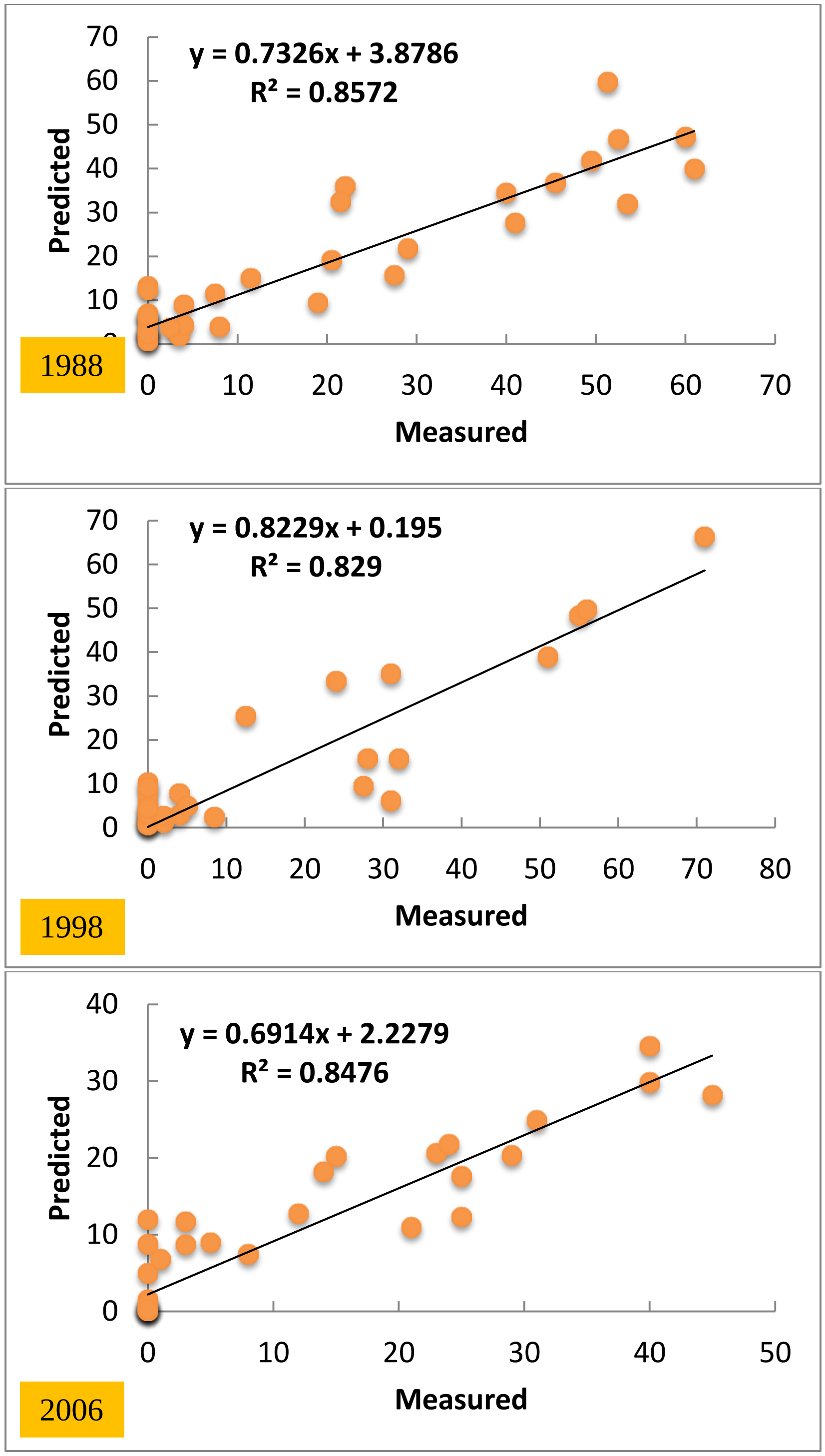
Figure 6: Scatter plots of the mud validation data



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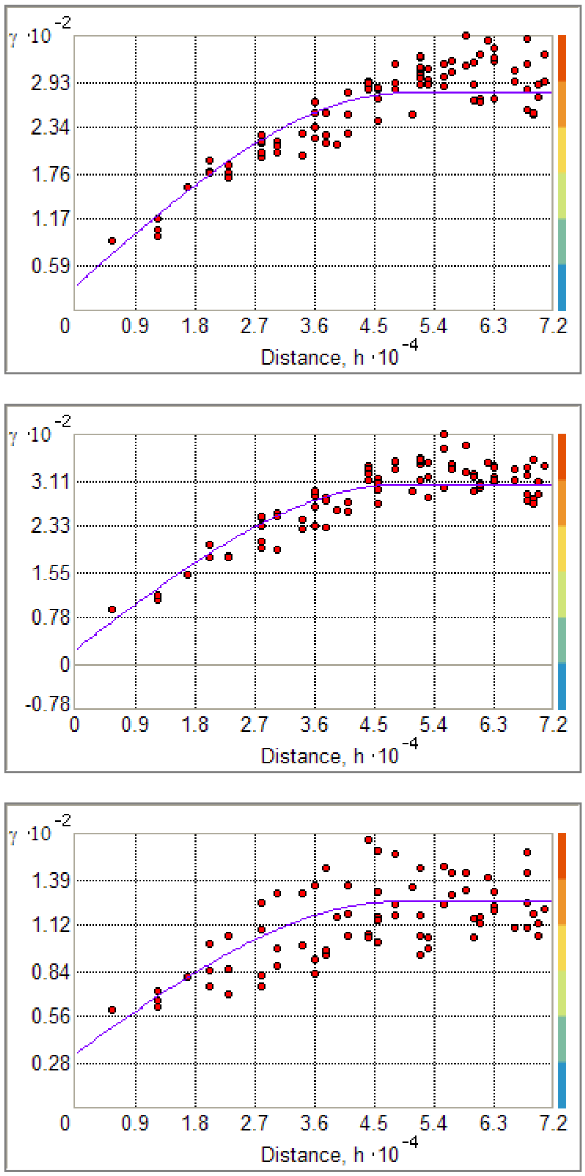
5. Spatial Changes of Mud Thickness Over-time

5.1 Validated Kriging Models and Mud Thickness Mapping

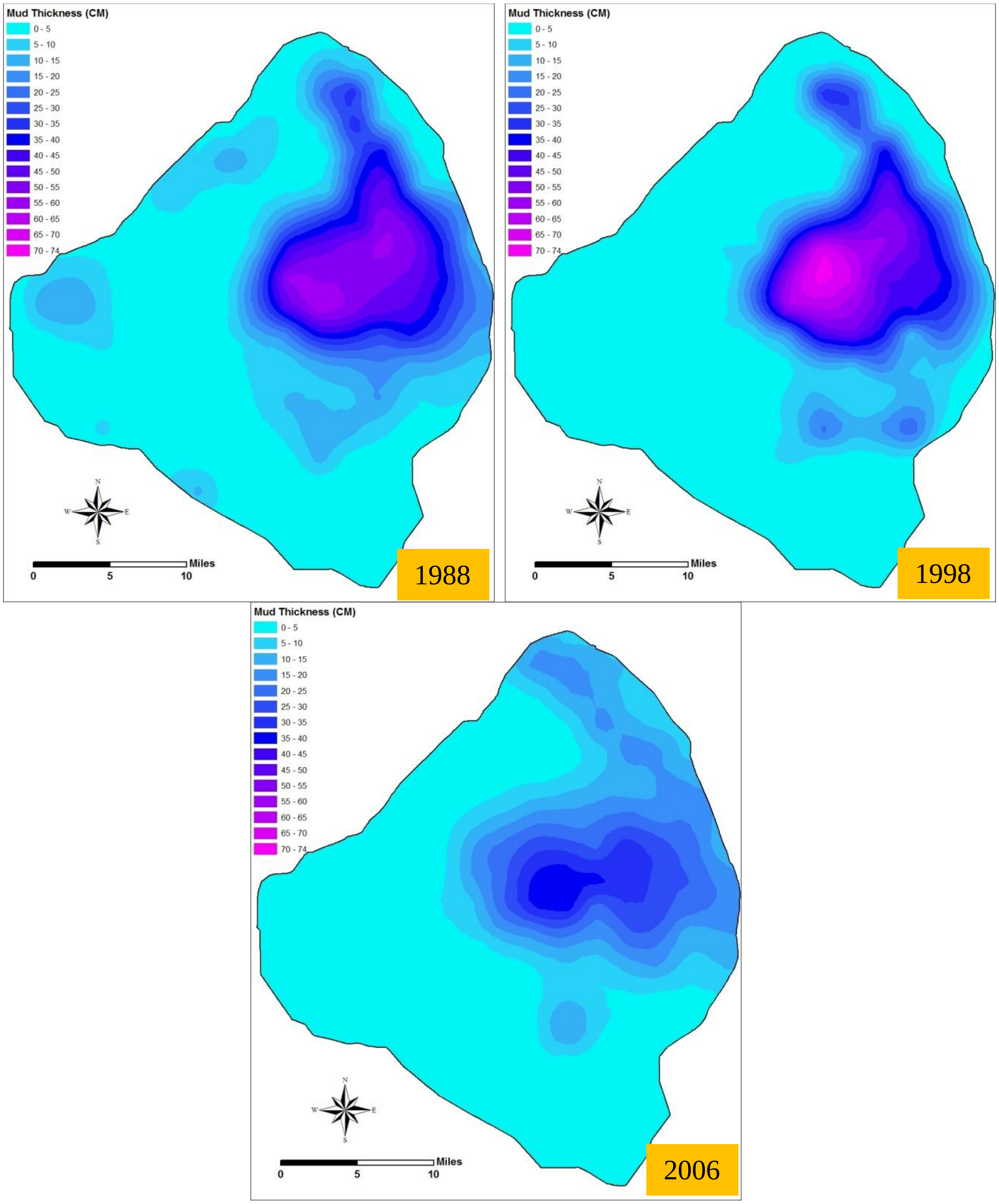
Validated kriging models estimated the mud thickness distributions for 1988, 1998 and 2006, respectively (Figures 7 and 8).

Figure 7: Fitted variogram models using the complete mud thickness data for 1988, 1998 and 2006, respectively



Source: Author

Figure 8: Mud thickness maps for 1988, 1998 and 2006 data, respectively



Source: Author

5.2 Mud Surface Area and Volume Calculations

Area/Volume and **Cut/Fill** tools from ArcGIS 3D Analyst and Spatial Analyst were used to calculate 2D area, surface area, and volume of each data model and the changes between models. The 2D area of a rectangular patch of surface model is its length times its width. The surface area is measured along the slope of the surface. Unless the surface is flat, the surface area will always be greater than the 2D area. The difference between the values for the 2D area and surface area indicates the roughness or slope of the surface - the larger the difference between the values, the rougher the surface. The volume is the space between the surface and a reference plane set at a particular height. ArcGIS ModelBuilder models were developed to integrate several tools to streamline the calculations of the mud area, mud volumes and their changes over-time. For the mud sediments, the area differences are less than 3 square meters for all three data sets (Table 3), which suggest that the mud surfaces are nearly flat. Mud area decreased 13.78% during 1988-1998 and increased 0.74% during 1998-2006. The mud volumes reduced 10.26% and 26.62% during 1988-1998 and 1998-2006, respectively (Table 4).

Table 3: Mud surface area and volume

Year	2D M^2	3D M^2	Acres	M^3
1988	1,652,057,022	1,652,057,024	408, 232.18	2.19E+08
1998	1,424,408,250	1,424,408,252	351, 978.94	1.97E+08
2006	1,434,925,672	1,434,925,672	354, 577.85	1.44E+08

Source: Author

Table 4: Changes of mud surface area and volume

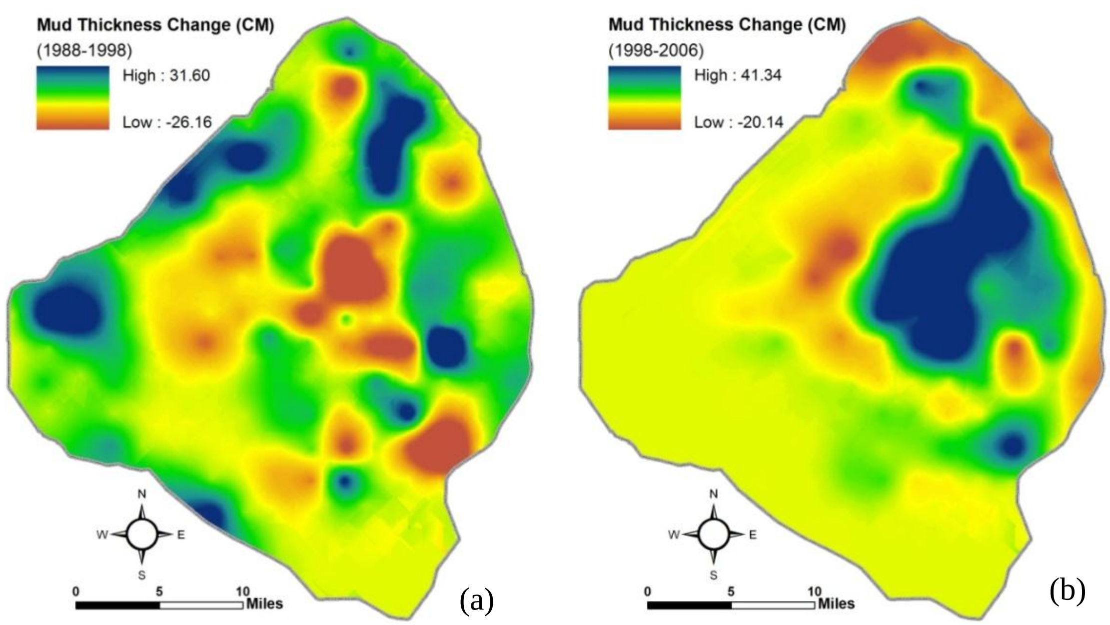
Period	Area change (acres)	Area change (%)	Volume change (M^3)	Volume change (%)
1988-1998	-56,300	-13.78	-2.25E+07	-10.26
1998-2006	2,600	0.74	-5.24E+07	-26.62
1988-2006	-53,700	-15.13	-7.49E+07	-51.85

Source: Author

The spatial variations of mud thickness are displayed in Figure 9. From 1988 to 1998, mud sediments were continually moving toward the center of the Lake. Mud thicknesses increased up to 26 cm in the central lake area (Figure 8 and 9). Mud thickness reduction mainly occurred near the shore during this period (Figure 9a). From 1998 to 2006, the area of mud sediments declined slightly (0.74%) (Figure 9b, and Table 3-4). Mud thickness is a maximum of 51 cm as compared to mud thicknesses of up to 74 cm previously (Figure 8). Mud depths declined by up to 41 cm in the central lake area and increased by up to 20 cm in surrounding areas, with small amounts of mud being deposited throughout the rest of the lake.

Changes in mud depth between years are informative regarding the potential effects of hurricane induced mixing. Between 1988 and 1998 (Figure 9a), with no major hurricanes occurred, the change vector is spatially heterogeneous, with local areas in primarily deeper water showing marked increases juxtaposed with areas in shallower regions showing equally marked decreases in mud depths. Notably, the areas in the center of the lake where mud is currently deepest accumulated large quantities, while outlying areas where mud is largely absent today (western lake) lost appreciable mud depth. Near the inflow of the Kissimmee River (northern lake), mud depths also declined. Overall, the weak spatial structure of the change pattern for 1988-to-1998 is suggestive of the major current gyres that exist in the lake (Jin and Ji, 2004). In contrast, mud depth changes between 1998 and 2006 occurred over much larger scales (Figure 9b). The middle of the lake appears to have lost appreciable mud, while areas around the central zone appear to have accumulated mud. This is suggestive of re-depositional processes, perhaps in response to high wind mixing events during Hurricanes Frances and Jeanne in 2004 and Wilma in 2005.

Figure 9: Spatial changes of mud thickness over-time
(Note: The changes were calculated by subtracting the earlier data set with the later data set. Therefore the negative values indicate an increase of mud thickness, positive values indicate a reduction in mud thickness)



Source: Author

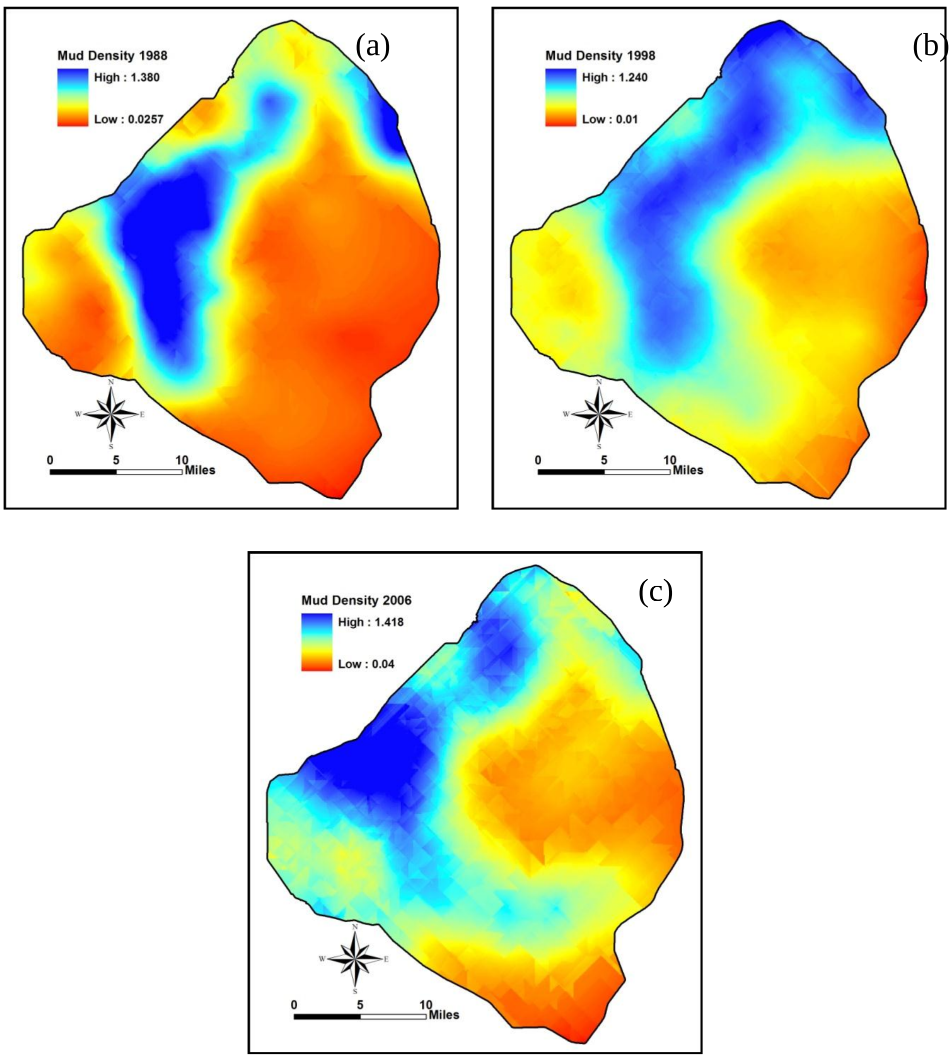
6. Mud Weight Calculation and Loads Change

Mud weight was calculated by multiplying mud volume with mud density for each location. Because mud density changes over distance in the lake, it’s necessary to map the density changes first before calculating the total mud weights.

6.1 Bulk density change

The 2006 data set had a total of 142 samples with valid density values. The maximum value of 5.43 g/cm^3 is an outlier (Site N3) and is removed for further analysis. The 1998 data set had 149 valid density values and the 1988 data set had 134 valid values. The mean density values increased from 0.376 to 0.529 to 0.58 g/cm^3, from 1988 to 1998 and from 1998 to 2006, respectively. Density values for all data sets had a skewed distribution (Table. 1). Both the 1988 and 2006 density data had first order trend changes in both E/W and N/S directions and the 1998 data had 2nd order trend changes in both E/W and N/S directions. All three data sets had weak spatial auto-correlation. Therefore, a spherical variogram was fitted for each data set. Spatial changes of the mud density were mapped using Ordinary Kriging (Figure 10). The central mud area and southern peat areas had the lowest density (red colors in Figure 10) for all three data sets. The near-shore zones had the highest density values (blue colors in Figure 10).

Figure 10: Spatial variations of mud density (g/cm^3)



Source: Author

6.2 Mud weight calculations and loads change

The total mud weight was calculated by summing all mud cell weights. The cell mud weight was equal to the cell volume multiplied by the cell density. The cell edge is 500 ft, the mud thickness in centimeters, and the density in g/cm^3. The mud cell weight (in kilogram) equation is:

Mud cell weight = 232257.60* [Mud Thickness] * [Mud Density]

where 232257.60 is a constant for unit conversion to produce mud weight in kilograms
The calculation process was implemented using ArcGIS ModelBuilder and Map Algebra. Based on 1988 data, the total weight of mud in the Lake was about 57.2 million metric tons. This increased to 73.9 million metric tons based on the 1998 data, and decreased to 58.6 million metric tons based on the 2006 data (Table 5). Mud weight increased over 29% from 1988 to 1998, decreased over 20% from 1998 to 2006, with a net increase of less than 3% (well within the measurement error).

Table 5: Mud weights and their changes over-time

Data	Cell #	Weight (Kg)	Weight (metric Ton)	Change %		
				(88-98)	(98-06)	(88-06)
1988	68815	5.19E+10	5.72E+07	Increase + 29.32		
1998	60545	6.71E+10	7.40E+07		Decrease -20.73	
2006	62465	5.32E+10	5.86E+07			Increase + 2.52%

Source: Author

The solids budgets were also calculated on a calendar year basis using the methods described for phosphorus budgets in Lake Okeechobee (James et al. 1995, Havens and James 2005). It was assumed that no solids were deposited through atmospheric deposition. The annual net loads were determined, and the cumulative net loads for the same periods as the change detection in mud sediments were determined. The change in the mass of mud sediments is between one and two orders of magnitude greater than the cumulative net loads for the same period (Table 6). While the changes in mud mass fluctuate between positive (net deposition) and negative (net removal) depending on the year comparisons, the net solids load is consistently negative. These net negative loads indicate that more solids are being discharged from the lake than are being loaded to the lake from tributaries. It is likely that the lake is a source of solids primarily in the form of organic material originating from plant growth in the lake. This hypothesis is consistent with the high percentage of volatile suspended solids (approximately 46% ±25 data not shown) found in the lake's mud sediments.

Table 6: Change in mass of mud sediments and the cumulative net loads

Year Comparisons	Change in mass (mud sediments)	Cumulative Solids net load for period
1998-1988	1.68E+07	-2.89E+05
2006-1998	-1.54E+07	-5.87E+05
2006-1988	1.40E+06	-7.72E+05

Source: Author

Conclusions

Ordinary kriging models were calibrated and validated to track the spatial-temporal changes of the sediment zones and mud thickness of Lake Okeechobee from 1988 to 2006. The potential impacts of the hurricanes which occurred from 1998-2005 are also discussed briefly. From 1988 to 1998, mud sediments were continually moved toward the center of the Lake, and mud thicknesses increased up to 26 cm in this region and decreased in the near shore zone. The mud area and volume increased up to 13.78% and 10.26%, respectively during this time period (Table 3 and Table 4). From 1998 to 2006, mud depths declined by up to 41 cm in the central lake area and increased by up to 20 cm in the surrounding areas, with small amounts of mud being deposited throughout the rest of the lake. The area of mud sediments increased slightly but the mud volume was reduced by about 27%.The reduction of mud sediments is likely due to re-suspension and redistributed by wind-induced waves and currents produced by Hurricanes during 2004 and 2005. Mud weight increased over 29% from 1988 to 1998, and declined over 20% from 1998 to 2006. Overall, the mud weight increased about 2.5% from 1988 to 2006. The major part of the sediments accumulated during 1988 to 1998 may have been released from the lake through water control structures during the three major hurricanes of 2004-2005.

Acknowledgements

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