BERNOULLI'S EQUATION WITH FRICTION

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This demonstration shows examples of flows in which inertial effects are greater than frictional effects and the converse. These are explained in terms of Bernoulli's equation with friction.

APPARATUS

The apparatus consists of:

1. A one-gallon solvent can, into which a ¼″ IPS coupling is soldered near the bottom.
2. A one-foot length of ¼″ IPS pipe with five ¼″ OD holes drilled in it on 2″ centers. These holes are all aligned on one line in the pipe surface, parallel to the pipe axis. Both ends of the pipe are threaded.
3. Same as No. 2, except that into each of the five holes a ½″ long piece of ¼″ OD copper tubing has been slipped and soft-soldered in place.
4. A piece of ¼″ rod, 12″ long.
5. A ¼″ IPS pipe cap.
6. A short length of rubber tubing.
7. One gallon of water thickened with 1.0 wt% Methocel 4000, estimated viscosity 100 to 150 cp.

DESCRIPTION OF DEMONSTRATION

1. The instructor sets the can on the lab bench over a sink, with part No. 3 inserted in the coupling and the end of No. 3 covered with the cap (No. 5). He shows it to the class, describes it to them, and tells them he is about to fill the can with water. He asks them to predict which jet from the pieces of tubing will rise the highest. After taking a poll of the class, he fills the can (using the rubber hose). The water jet squirts highest from the tube farthest from the can and successively lower from the tubes nearer to the can.
2. The instructor then removes the cap from the end of the pipe, inserts the rod, replaces the cap, and tells the students that he will again fill the can with water. He again asks which jet will go highest and records the results of his poll. He then fills the can with water. This time the water squirts highest from the tube nearest the can.
3. The instructor removes the rod from the tube and tells the students that he is about to fill the can with the methocel solution. Again he takes an opinion poll on which jet will go highest. He then pours the solution into the can. The jet nearest the can squirts highest, but none of the jets squirts as high as in demonstration No. 2.
4. The instructor then removes part No. 3, rinses the can, and replaces part No. 3 with the pipe which has only holes (part No. 2). He tells the class that he is going to fill the can with water and again quizzes them as to which jet will go highest. After recording the predictions, he fills the can with water. This time there is a hump-shaped distribution of heights, with the highest in the middle. The streams are very unequal in width, with the stream nearest the can the thinnest and the far stream the thickest. The streams are not vertical, as before, but leave the holes at an angle away from the can, with the stream nearest the can about 10° from the vertical and the stream farthest from the can about 30° from the vertical. It is also observed that each of the streams uses only part of the hole to exit through, and that part is the part farthest from the can. The near hole issues a stream which fills about one fifth of the hole area; this fraction grows to about three fourths at the far hole.
5. The instructor explains the foregoing experimental results.

THEORY

For the first three demonstrations with the fluid rising through the small vertical tubes, the flow (and, hence, the height to which each jet rises) depends only on the static pressure at the base of the vertical tubes. The sharp right-angle turn which the fluid makes entering these tubes prevents any of the horizontal momentum of the fluid from being converted into vertical momentum.

To find the pressures at the base of these tubes, we apply Bernoulli's equation:

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = -\frac{dW_{pump}}{dm} - F \qquad (1)$$

Here states 1 and 2 are chosen as shown in Figure 1.

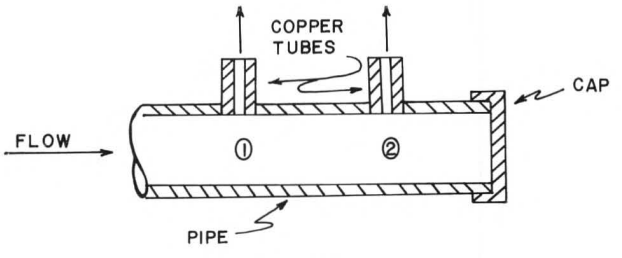
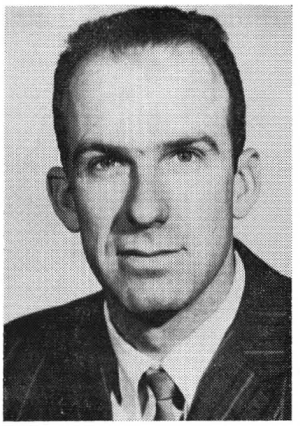


Figure 1.



Noel de Nevers earned his BSChE at Stanford and his PhD at the University of Michigan, with a year out in between to be a Fulbright exchange student at the Technical Institute in Karlsruhe, Germany. He spent five years with what is now Chervon Research and Chevron Oilfield Research, before joining the faculty of the University of Utah. He spent Academic 1971-72 on leave, working for the Office of Air Programs of the Environmental Protection Agency. He is the author of a textbook on Fluid Mechanics, and editor of a book of readings and discussions on Technology and Society.

From 1 to 2 there is no change of elevation or any pump work done on the system, and V_2 is zero; so

$$\frac{P_2 - P_1}{\rho} - \frac{V_1^2}{2} = -F \qquad (2)$$

Substituting for F from the Fanning friction factor equation (1) and solving,

$$\frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{V_1^2}{2}\left(1 - 4f\frac{L}{D}\right) \qquad (3)$$

Equation 3 indicates that if 4f L/D is less than 1, the static pressure at 2 will be greater than at 1; and, if 4f L/D is greater than 1, the static pressure at 2 will be less than at 1. Thus, by adjusting the value of 4f L/D, one can cause the water to squirt highest from either the farthest or nearest tubes.

This analysis is not restricted to the two end holes. It can be applied to holes n and (n+1), yielding

$$\frac{P_{n+1}}{\rho} = \frac{P_n}{\rho} + \frac{V_n^2}{2}\left(1 - 4f\frac{L}{D}\right) - \frac{V_{n+1}^2}{2} \qquad (4)$$

Here V_n is always greater than V_{n+1}; so, if $f = 0$, P_{n+1} is always greater than P_n. Thus, for any two holes, the relative jet heights are determined by the value of (1 - 4f L/D) between them.

In experiments run as described above, with the can held full of water so that the flow was at steady state, it was found that, with no rod in the pipe, the flow rate out of the farthest tube was 1000 cc/min; the Reynolds number in the pipe just upstream of it was about 10^4. If we accept the value of ϵ for galvanized pipe from Perry[1] as 0.006 inches, then we can read f=0.012, which makes 4fL/D equal 0.27. We would expect the fluid to squirt highest from the farthest tube, as it does.

In a similar experiment, with the 0.25″ OD rod in place, it was found that the flow rate from the near tube was about 550 cc/min. By using the hydraulic diameter, as shown in Perry[1], we can calculate the Reynolds number as 2×10^4. Using the same absolute roughness as before yields f=0.018 4f L/D = 1.25. This indicates that the fluid would squirt highest from the nearest tube, as it does.

Raising the water viscosity has the same effect as inserting the rod. The flow is laminar, and the friction factor is proportional to the viscosity. For the thickened water, the friction factor is large enough that 4f L/D is greater than one; and the water squirts highest from the hole near the can.

The whole subject of flow in this type of manifold has been reviewed by Acrivos *et al.*[2]

The fourth demonstration is much more complicated to explain than the first three because there is no simple, one-dimensional representation of this flow. If we had placed a pitot-type tube in each of the holes, with its exit vertical, we would expect that from each hole the jet would have risen the same amount if friction were negligible. This result would have been the same, even if the various pitot-type tubes were of different diameters. If, however, we had tilted the exits of these pitot-type tubes at differing angles to the vertical, the heights of rise of the jets would not have been equal because, at their highest points, the jets would have had varying kinetic energies in the horizontal direction.

In the fourth demonstration, the fluid forms its own pitot tubes. The wall thickness of the pipe is only about one third of the hole diameter, so the fluid can turn through the hole without losing all its horizontal momentum. The interesting thing here is that, the faster the fluid is flowing, the greater is the angle through which the fluid must turn to exit; so the fastest-moving fluid exits nearest the vertical. One may look at this another way by saying that the fluid builds up a turning vane for itself, made up of stagnant fluid trapped in part of the orifice.

This conversion of horizontal to vertical momentum is not perfect, so the jet nearest the can

does not rise as high as it would if the conversion were perfect. As we proceed out the pipe, we see two conflicting effects producing the hump-shaped distribution:

1. The increasing efficiency of the "turning vanes" as the velocity falls leads to higher rise; and
2. As the angle through which the fluid must turn to exit decreases, the fluid exits more nearly horizontally and thus does not rise as high.

A VARIATION

For graduate courses, one may insert a 5/16" thin-wall tubing about 10" long, open at both ends, into the pipe in place of the rod. In this case the flow through the tube will be great enough that the two end jets will both rise higher than the ones in the middle. This presents an analysis problem suitable for graduate students.

SOME PRACTICAL CONSIDERATIONS

1. The pipe connections should be finger tight. A small leak doesn't hurt.
2. In placing the copper tubes in the holes in part No. 3, little care is needed to align them. After they have been soft-soldered in place, they can be easily bent into line by holding the pipe in a vise, inserting a steel rod in each tube, and tapping it gently into line with a hammer.
3. To make the pipe without the tubes (part No. 2), use extreme care to get the holes in one axial line.
4. While running the experiments, tilt the plane of the orifices a few degrees to one side (by rotating the pipe) so that the jets fall free of the pipe and do not interfere with each other.
5. Check the apparatus carefully for burrs. They can have a pronounced influence.
6. In running the first part of the experiment, it is well to show that the result is independent of which end of the pipe is inserted in the can. As long as all the connections are finger tight, the instructor can reverse the position of the pipe in the can quite quickly.
7. Allow adequate time to make up methocel solution. Sugar solutions could be substituted, but they are messy. Any viscous solution will do, but methocel is probably the cheapest and easiest.

ACKNOWLEDGMENT

This device was used by Dr. J. Q. Cope, former vice president of the Chevron Chemical Corporation, to teach young engineers several important lessons. His procedure was:

1. Describe the apparatus to the new engineer, without mentioning the insertable rod.
2. Ask him from which end of the pipe the jet will squirt highest.
3. When he answers, goad him into betting that he is right.
4. Go get the device, inserting the rod if necessary.
5. Demonstrate—and take his money.

The educational qualities of this procedure are obvious.

NOMENCLATURE

D	diameter	ft
f	Fanning friction factor.	
F	lost work per pound due to friction	ft-lbf/lbm
g	acceleration of gravity	ft/sec^2
L	length	ft
m	mass	lbm
P	pressure	lbf/in^2
V	velocity	ft/sec
W	pump work	ft-lbf
z	elevation	ft
ϵ	absolute roughness	ft
ρ	density	lbm/ft^3

REFERENCES

1. Perry, J. H., et al. **Chemical Engineers' Handbook** (4th Ed.; McGraw-Hill, 1963), p. 5-19 et sec.
2. Acrivos, A., B. D. Babcock, and R. L. Pigford, 'Flow Distributions in Manifolds," Chem. Eng. Science. 10 (1959), pp. 112-124.

BOOK REVIEW: (Continued from p. 110)

Atom balances, purge and recycle are not discussed. Although reactors form a central theme; conversion, yield and extent of reaction are not used or defined. The enumeration of the number of equations (mass balances, mass fraction constraints) does not appear, for this text does not deal with multi-unit processes in multicomponent systems.

The mathematical content is such that a student should have finished calculus and differential equations. Parenthetically, the student is asked to deal with a large amount of notation to appreciate the content. Computer methods are not stressed, although there is material on numerical methods. Any computer applications would require a supplementary text. Linear algebra, which some people refer to as "the language of stoichiometry," is not used.

In short, one must compliment the authors on their attempt to inject vitality and meaning into the first text in chemical engineering. However, their approach does not fit well in a so-called "standard" core curriculum. Perhaps our curricula will adapt: I for one concur with their philosophy. For the present, the book will find use in peripheral ways, such as refresher courses.

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