Award Lecture . . .

INTERACTIVE DYNAMICS OF CONVECTION AND CRYSTAL GROWTH

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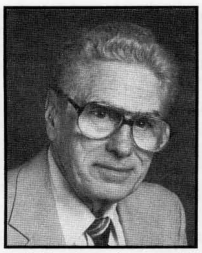
I would have liked this article to be the story of how basic theoretical and experimental breakthroughs have contributed to the spectacular advances in making and using new materials for the various "revolutions" we have witnessed since World War II. But it quickly became clear to me that the scope of such a story is beyond my competence to tell coherently. Therefore, I will attempt to give here a chemical engineer's view of some rather recent work on dendritic growth, which has generated many surprises over the last fifty years—only a part of the story I had hoped to tell.

The macroscopic properties of materials and the uses to which they can be put depend on their internal structure. In turn, the microstructure of materials depends on the details of the processes used to make them. Furthermore, the making of a material and its placement in a system for final use may occur simultaneously as, for example, with thin dielectric or metallic films made by chemical vapor deposition for integrated circuits. In this case, both the microstructure and the macrostructure (uniformity, absence of keyholes, step coverage, etc.) of the materials profoundly influence the performance of the devices which are made from them.

Microstructure and composition determine the properties of advanced materials, including alloy castings, polymer-ceramic composites, and films, and convection may profoundly affect all of them. The transport of energy and the redistribution of solute in the solidification of alloys are influenced by convection in different ways (depending on whether the transformation occurs in the vapor or liquid phase) because the transport properties differ enormously in gases and liquids.

The subject of interactive dynamics has very practical implications because it includes efforts to understand how process conditions, the microstructure, and macroscopic configuration of materials go together, as they are manufactured, to maximize their usefulness in various applications. If our goal is to create materials and structures with the properties we wish them to have, then an understanding of how to control the way they may be made to achieve this goal is of prime importance.

When crystals grow in a melt of pure liquid, from a solution of several components or from the vapor



William N. Gill *was presented with the Thirtieth Annual Chemical Engineering Division Lectureship Award for this lecture, which he gave at the ASEE ChE Division summer school meeting at Montana State University in August of 1992. The award is bestowed annually on a distinguished engineering educator and is designed to recognize and encourage outstanding achievements in important fields of fundamental chemical engineering theory or practice.*

Professor Gill is Russell Sage Professor in Chemical Engineering and the Center for Integrated Electronics at Rensselaer Polytechnic Institute, and has been a faculty member at Syracuse University, Clarkson University, and the State University of New York at Buffalo. He served as chairman of the ChE departments at Clarkson and RPI, Dean of Engineering at Buffalo, Fulbright Senior Research Scholar in England and Australia, and Glenn Murphy Distinguished Professor at Iowa State.

Gill's research has focused on several areas, including turbulent convection, Taylor diffusion, membrane separations, and various aspects of crystal growth. His contributions are summarized in more than 150 articles balanced between theoretical and experimental content. His motivation is the understanding of chemical and physical phenomena that underlie processes of practical importance from both industrial and environmental viewpoints. His work has been cited about 2000 times in a large number of journals and books in many fields, including medicine, biophysics and biochemistry, chemistry, geophysics, applied mathematics, condensed matter physics, virtually all areas of engineering, and materials science.

He has investigated the effects of concentration polarization, the interaction among components in the feed, ways to model hollow fiber and spiral wound systems, and the fundamental processes that underlie transport in membranes. His most recent area of interest has been crystallization, including dendritic growth and rapid solidification, and currently with chemical vapor deposition of copper and interlayer dielectric films for integrated circuits.

Gill has worked with many graduate students and has been thesis advisor to thirty-one PhD recipients. These former students currently have faculty positions at universities in the United States, Israel, Korea, Taiwan, India, Iran, and Europe. He has been editor of Chemical Engineering Communications *since 1979 and serves on several boards.*

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phase by deposition on a substrate, an interface is created, the shape and movement of which depends on the conditions that prevail on both sides of the interfacial region and within that region itself. In other words, the movement and shape of the interface creates an active pattern which evolves in time and which is inherently three-dimensional in systems of practical interest. The quantitative description of this spontaneous nonlinear dynamic process is one of the objectives of the theory of pattern formation, and one particular aspect of it includes the interactive dynamics of convection and crystal growth. Convection is coupled with the growth of the crystals involved, and each affects the other because of the movement of the mobile interface.

The problems associated with studying interactive dynamics are formidable, but progress is being made which will contribute to the development of some of the extraordinary materials of the future. The October 1986 issue of *Scientific American* contains thirteen articles, well worth reading, on various aspects of the development of materials, providing an interesting perspective on the modern aspects of materials engineering and science by people who are not chemical engineers. A physicist's viewpoint on dendrites and the theory of pattern formation is given by J.S. Langer in *Science* (March 1989); this perspective was updated with his more recent article in *Physics Today* (October 1992) in which he appealed for a rational approach to materials research on the national level.

An applied mathematician's viewpoint on interactive dynamics in crystal growth is given in two review articles published recently by Huppert[1] and Davis[2] in the *Journal of Fluid Mechanics.* Brown,[3] a chemical engineer, has reviewed various interesting aspects of single semiconductor crystal growth from the melt, especially for electronic and optoelectronic devices.

The *Handbook of Crystal Growth* (Edited by Hurle; Elsevier, 1993) contains a number of chapters which review subjects related to this one. Also, a conference was held in March of 1992 in Chamonix, France, the papers from which have been published as *Interactive Dynamics of Convection and Solidification* (Kluwer Academic Publishers, edited by Davis, *et al.*).

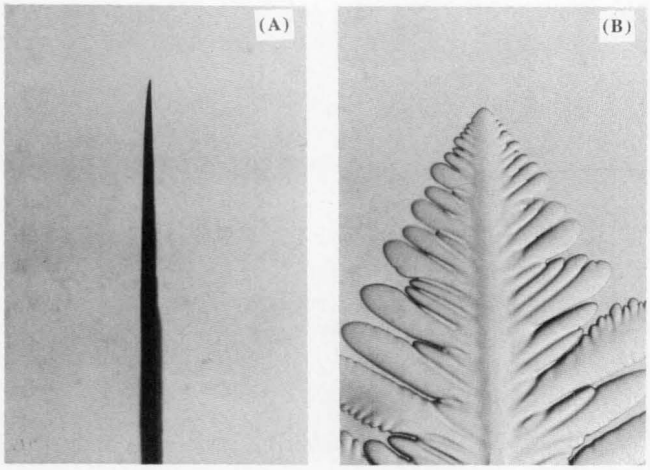

Figure 1. *Photographs of an anisotropic ice dendrite with $\Delta T = 0.6°K$, $R_2/R_1 \sim 28$, magnification 26x:*
(A) edge view with tip radius, R_1;
(B) Basal plane with tip radius, R_2.

When they are still undergraduates, chemical engineers begin learning to deal with heat, mass, and momentum transport coupled with chemical reactions in the bulk phase or on the boundaries of the system. At Rensselaer, for example, individual courses in fluid mechanics, heat transfer, separation processes, and chemical reactor design are required in the third and fourth years of the undergraduate program. The exposure to separation processes, reactor design, and the option to take a fourth-year course in transport phenomena provides a good foundation for graduate-level course work and research in materials processing.

THE ROLE OF TRANSPORT PROCESSES IN CRYSTAL GROWTH

It is important to understand the roles of heat, mass, and momentum transfer in crystal growth processes. One or more of these phenomena may be of crucial importance, depending on the phase change system involved and how the process is carried out. To illustrate this, we will consider dendritic growth which, according to Glicksman and Marsh,[4] is the most prevalent form of solidification.

In the simplest case involving dendrites, the free growth of crystals (unencumbered by walls) from pure undercooled melts produces dendritic structures

in which the leading tip of the main stem of the dendrite propagates in the preferred crystallographic direction, as shown in Figure 1. These reproducible patterns have small dimensions and large curvature. They seem to occur in nature due to the competition among the kinetic resistance to the attachment of molecules at the surface, the effect of the surface energy of the solid-liquid interface, and the rate of removal of heat from the surface into the subcooled melt. The underlying reasons for why dendrites form the patterns they do are not completely clear and currently are being investigated on a worldwide basis. Figure 1 illustrates how different ice dendrites appear when viewed from the basal plane, where one sees a rich sidebranch structure, and from the edge plane, which has no sidebranches at all. Figure 2 shows the typical shape of a water drop in equilibrium with the ice matrix in which it is encased. This drop also demonstrates strong anisotropy.

Surface tension depresses the melting point of an interface which is convex to the melt, and attachment kinetics reduce the driving force for heat transfer, both of which stabilize the solidification front. The rate of heat removal by conduction to the undercooled melt may be influenced significantly by thermal or forced convection, which will have an effect on the properties of the crystal. A successful theory of this seemingly straightforward problem from which one hopes to predict the size, shape, and growth rate of the crystal, has proved to be elusive.

Grain size (and its distribution in a material) is one of the properties that is of importance in crystal growth processes, ranging from the making of steel to the deposition of thin metallic films in integrated circuits. Within each grain of a structural material (such as a titanium-based alloy) is a microstructure formed by dendritic growth. The microstructure consists of a pattern in which the interstices between the main stems and the sidebranches of alloy dendrites are filled with material which is rich in solute. The solution in the interstices solidifies more slowly than that contained in the dendrites. This microstructure determines many of the macroscopic properties of structural materials, and thus the uses for them. Therefore it is of major importance to understand how to control the rate of dendritic growth, the size of the dendrites, and the spacing between their sidebranches.

It has been known for a long time that processing techniques can have a profound influence on the microstructure of materials. Indeed, the basic ideas regarding some of the methods currently used to produce exotic materials, such as directional solidification, modified directional solidification to produce single crystals, and rapid solidification, had their roots in the 1960s. These processing methods have had a major impact on the development of advanced structural materials because they influence the microstructure in predictable ways. Furthermore, the methods used are all related to controlling the transport processes involved in the making of these materials.

Examples of the dramatic improvements in materials due to the procedures used to process them abound. The use of directionally solidified metal structures has resulted in an increase of 150°C to 200°C in inlet temperatures for turbines, which substantially increases their efficiency. One can actually obtain amorphous metal alloys by rapid solidification, and these have properties which may substantially reduce the energy losses which occur in the distribution of power. Thus it is important from both a basic and a practical viewpoint to understand the role of various transport mechanisms in determining rates of production, the length scales that correspond to the rates of production, and how a particu-

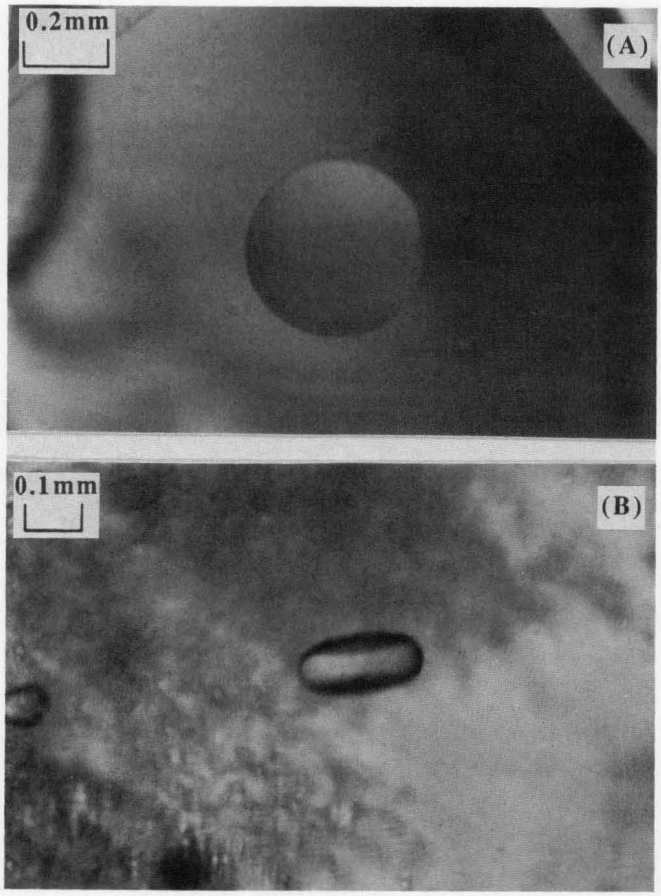

Figure 2. *Equilibrium shape of highly anisotropic water drop in ice matrix: (A) basal plane; (B) edge plane.*

Convection is the organized movement of large groups of molecules on a macroscale, and (as required by Newton's laws of motions) it results from force fields which may be subtle and difficult to manipulate or control.

lar growth path is selected, which determines what the length scales and rates are. Here, I will concentrate on some of the ways natural and forced convection affect the crystal growth process—it is a subject on which some of my students and I have spent a considerable amount of time.

CONVECTION AND CRYSTAL GROWTH

Convection is the organized movement of large groups of molecules on a macroscale, and (as required by Newton's laws of motions) it results from force fields which may be subtle and difficult to manipulate or control. For example, flow may be caused primarily by buoyancy forces which are ubiquitous on earth due to its gravitational field. If one uses the microgravity levels offered by a space station to reduce gravity by several orders of magnitude, surface tension forces may become more important than gravity in determining the nature of the flow and, therefore, in materials processing. Also, convection may be useful in enhancing heat and mass transfer rates and in reducing processing time. Or it may adversely affect product uniformity, as can happen in the chemical vapor deposition (CVD), of thin films in a flow field that is not oriented properly.

Essentially, all of the mathematical work aimed at understanding the role of convection in dendritic growth has dealt with needle crystals in which the sidebranches are neglected and one concentrates on the region near the tip of the dendrite. The first exact solution for the growth of a dendrite from the melt was obtained by Ivantsov almost fifty years ago. His classical analysis applied to isothermal needle crystals growing in a stationary melt with zero velocity, $\overline{U} = 0$. Some of the work my students and I have done has focused on trying to add a realistic description of convection in the melt to generalize Ivantsov's analysis. Thus, Dash and Gill[5] showed that a similarity variable could be used to solve the energy equation given by

$$\frac{\partial T}{\partial t} + \overline{U} \bullet \nabla T = \alpha \nabla^2 T \qquad (1)$$

for an isothermal needle crystal including convection in the melt, if the velocity field $\overline{U}$ in the convective term $\overline{U} \bullet \nabla T$ on the left-hand side of Eq. (1) is given by the Oseen or potential flow models.

Ananth and Gill[6] and Saville and Beaghton[7] for Oseen flow, and Ananth[8], and BenAmar, *et al.*,[9] for potential flow, used variations on the similarity approach to study the growth of parabolic dendrites. Ananth and Gill considered the growth of crystals in the form of an elliptical paraboloid and gave exact solutions to the Oseen flow and energy equations in an effort to understand better the growth of ice crystals which are anisotropic. The body shapes, flow fields, and thermal fields which yield self-consistent solutions for the steady growth of dendrites into a melt which itself is in motion, $\overline{U} \neq 0$, were determined by Ananth and Gill. Among surfaces of revolution only a parabolic crystal grows steadily, and either the Stokes flow, Oseen viscous flow, or potential flow approximations must be made in order to obtain an exact solution for the growth Peclet number

$$P_G = \frac{V_G R}{\alpha}$$

where V_G is the growth velocity, R is the tip radius, and α is the thermal conductivity of the melt. Stokes and potential flows are valid in the limit

$$Re = \frac{UR}{\nu} \to 0 \quad \text{and} \quad Re \to \infty$$

respectively. As discussed by Lagerstrom and Cole[10] and Lagerstrom,[11] the Oseen approximation to the Navier-Stokes equations is uniformly valid for a semi-infinite three-dimensional paraboloid of revolution at very small Reynolds numbers.

The growth of shape-preserving (near the tip) dendrites in subcooled melts has been observed experimentally in detail for succinonitrile (SCN) by Huang and Glicksman[12] and for ice by Fujioka,[13] Tirmizi and Gill,[14] and Koo, *et al.*[15] In these experiments crystals grow from a capillary tube into a melt which is quiescent. Near the crystal-melt interface, however, there is a natural convection flow generated by gravity acting on a density distribution created by the temperature gradients, caused by the spreading of the heat of fusion into the melt. The intensity of this natural convection is indicated by the Grashof number, Gr.

In order to compare their Oseen flow solutions to experimental data on the dendritic growth of SCN, Ananth and Gill interpreted $Re = \sqrt{Gr}$ and showed reasonable agreement with the data of Huang and Glicksman. They found that the Grashof number increases as undercooling, ΔT, which is the driving force for the flow, decreases. This counter-intuitive result occurs because the length scale, R, which appears as R^3 in

$$Gr = \frac{\beta g \Delta T R^3}{\nu^2}$$

increases as ΔT decreases in the experiments on SCN. Koo, *et al.*, demonstrated that the same behavior is observed with ice dendrites if one uses the harmonic mean

$$R_m = \frac{2 R_1 R_2}{R_1 + R_2}$$

of the tip radii, R_1 and R_2 of the edge and basal planes as the length scale. This result is shown in Figure 3, and Gr is seen to change by several orders of magnitude. Ananth and Gill also solved approximately the fully nonlinear thermal convection problem, where Gr arises naturally, and they demonstrated excellent agreement between the mathematical results and the data on SCN. Subsequently, Canright and Davis[16] studied the limiting case of the effect of very weak buoyancy driven flows on the shape of dendrites and showed that their analysis was complementary to that of Ananth and Gill.

THEORIES OF SELECTION IN DENDRITIC GROWTH

The experiments mentioned above all show that the growth velocity, V_G, and the tip radius of the dendrite, R, are determined uniquely when the undercooling, ΔT, is fixed. In contrast, Ivantsov's pure conduction solution indicates that the Peclet number, P_G, is fixed when the dimensionless undercooling given by the Stefan number, St, is specified. Consequently, only the product, $V_G R$, is determined by fixing ΔT, and the individual values of V_G and R cannot be predicted. Thus, the theory is incomplete if one assumes that the dendrites are isothermal and smooth.

If one includes natural convection, or forced convection, V_G increases and R decreases, but the same degeneracy exists in the theory. Therefore considerable effort has been expended over many years to find a selection criterion for V_G and R, and these efforts in one way or another revolve around the introduction of surface tension in the problem. The selection criterion is an additional relationship between V_G and R which enables each of them to be chosen uniquely for given values of ΔT and U_∞.

Until the middle of the 1970s, the maximum-velocity hypothesis was used for this purpose, and it assumed that the operating point was the maximum in the V_G versus R curve. This assumption was used for about thirty years until it was shown to be incorrect by the careful experiments on succinonitrile of Glicksman, *et al.*,[17] in which both V_G and R were measured as a function of ΔT with $U_\infty = 0$.

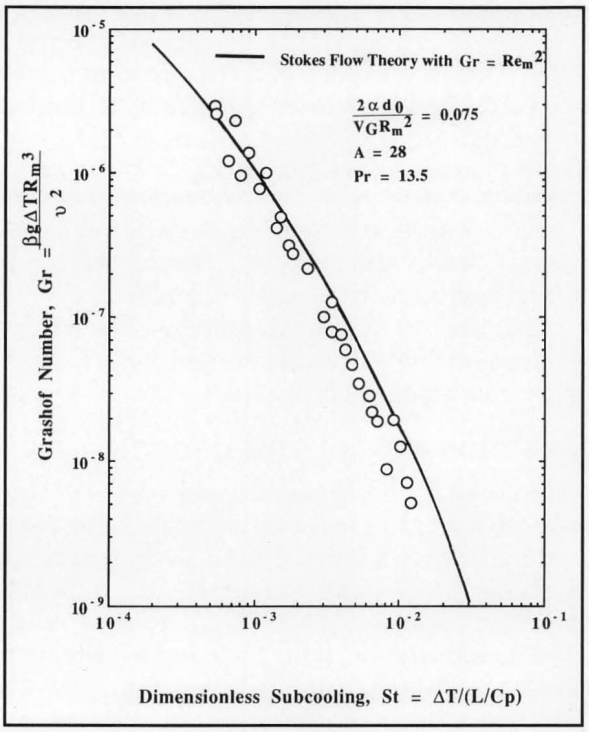

Figure 3. *Use of thermal convection analogy, $Gr = Re^2_m$, with selection parameter, $\sigma^* = 0.075$ to estimate level of natural convection. Comparison of Grashof numbers observed experimentally with those predicted by Stokes flow theory.*

Then Langer and Muller-Krumbhaar[18] proposed the marginal stability theory based on the introduction of surface tension as a perturbation. This led to the relationship

$$\sigma^* = \frac{2 d_o \alpha}{V_G R^2} = \text{constant} \qquad (2)$$

where

d_o = $\gamma T_m C_p / L^2$, the capillary length
α = thermal diffusivity
γ = surface tension
T_m, C_p, L = melting point, heat capacity, heat of fusion

The notion that $V_G R^2$ is independent of ΔT also had been proposed by Oldfield[19] on the basis of computer experiments in which he balanced the heat transfer by conduction against the stabilizing effect of surface tension.

More recently the selection of the growth velocity and length scale in the absence of fluid flow has been addressed in many articles which discuss the microscopic solvability theories. For detailed reviews of this work I refer the reader to the work by Langer[23] and Kessler, *et al.*[20] They showed that no solutions can be found in the presence of finite surface tension unless anisotropic effects (variation in surface tension around the surface of the crystal), no matter

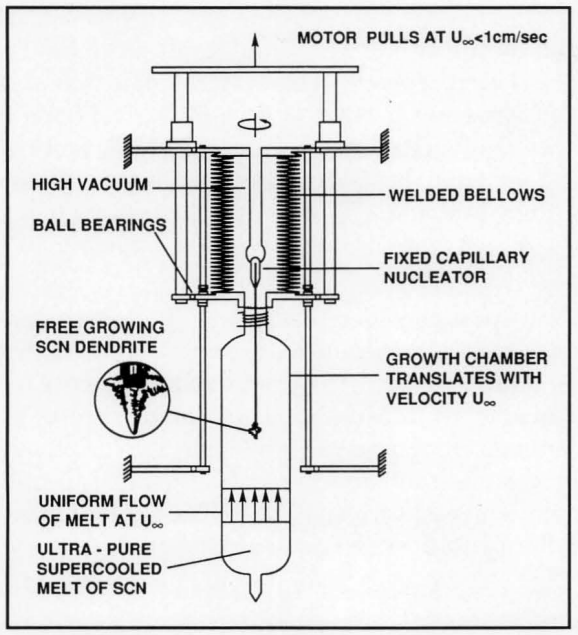

Figure 4. *Forced convection growth cell provides rigid body motion of melt with dendrite fixed in space.*

how small, are taken into account, and that the fastest growing tip is selected because it gives the only dynamically stable solution.

Anisotropy refers to the variation in surface tension around the dendrite in the azimuthal direction. When the anisotropy, ε, is about 0.5%, the microscopic solvability theory yields according to Mushol, *et al.*,[21] a result which is given by

$$\sigma^* = \frac{2\,\alpha d_o}{V_G R^2} \sim 0.01$$

This result is smaller by a factor of two than that obtained from the marginal stability hypothesis of Langer and Muller-Krumbhaar which neglects the anisotropic effects. Both theories are in qualitative agreement with the experimental value of $\sigma^* \sim 0.02$ for the data on succinonitrile of Huang and Glicksman, and Lee, *et al.*,[22] when the subcooling is large and the effect of convection is small.

The agreement between microscopic solvability theory (MST) and the experiments for succinonitrile was encouraging, and it led to a feeling that real progress was being made in our understanding of dendritic growth. But Langer,[23] who is an original contributor to MST, raised warning signals several years ago, indicating that the predictions of MST had not been confirmed experimentally. Unfortunately, the experimental evidence which has been accumulating lately on different materials does not seem to be consistent with MST and suggests that Langer's concerns were well founded.

SOME EXPERIMENTAL TESTS OF SELECTION THEORIES

Two types of experiments yield results which are significantly different from those predicted by MST. I will try to describe both of these experiments, but I will concentrate more on the forced convection experiments carried out by my research group. Because we had worked on the theoretical problem of including forced convection in the melt, my students and I naturally were interested in performing experiments under conditions which corresponded as closely as possible with the assumptions we made to obtain forced convection solutions of Eq. (1).

Forced convection experiments seemed attractive because they introduced the velocity of the melt at a large distance from the crystal, which is a new quantity, U_∞, that can be varied independently of ΔT. As usual, these experiments proved to be more difficult and time consuming than we originally thought they would be.

First, we wanted to study a material whose properties were well known so that we could reproduce previous results for $U_\infty = 0$ before attempting to do something entirely new. *Second*, the material (succinonitrile) which we settled on had to be ultrapure (99.999% or better) and was extremely expensive at the time. Therefore we had to think small in designing an apparatus. *Third*, we realized that our forced convection velocities, U_∞, would have to be very well defined and large enough to render natural convection negligible. *Fourth*, the melt had to be transparent because we wanted to track the crystal-melt interface and measure precisely both the growth rate and the tip radius in real time. *Fifth*, we would have to photograph and make video tapes of the experiments to enhance our understanding of the physical processes involved. *Sixth*, we would have to control the temperature of the growth cell to extremely close limits. The solutions to these and other problems are described in a thesis by Lee[24] which had significant contributions by Ananth (who was working with me as a post doc at the time).

In forced convection experiments on crystal growth, one has to deal with three different velocities: V_G, U_N, and U_∞. It is important to understand clearly the meaning of each because they all exist simultaneously. In our experiments we observed the shape of the interface of the dendrite which we characterize by the tip radius, R, the motion of the interface denoted by the velocity V_G, and the velocity of the melt with respect to a fixed frame of reference, U_∞. We could not observe or measure the velocity due to natural convection, U_N. Therefore we used theory to

estimate it and conducted our experiments so that U_∞/U_N and U_∞/V_G ranged up to about 40 and 300, respectively, as indicated by Gill, *et al.*[25]

The apparatus in which the forced convection experiments were carried out is described in detail by Lee[24] and is shown in Figure 4. The basic idea is to keep stationary the capillary from which the crystal emerges and to have the cell containing about 100 cm^3 of SCN move so that it creates a rigid body motion of the melt relative to the crystal over the range $0 \le U_\infty \le 1$ cm/sec. The cell is operated in a constant-temperature bath which is controlled to ±0.001 K, and after purification the SCN remains hermetically sealed throughout the experiments. The patterns formed by the dendrite-melt interface are observed through a microscope and recorded on video film. The measurements of V_G and R deviated from their mean values by less than 2.5% and 5%, respectively.

The goals of this work were to measure various characteristics of the dendrite, including V_G, R, distance from the tip to the first sidebranch, distance between sidebranches, and other quantities, as functions of the undercooling, ΔT, and the forced convection velocity, U_∞, which can be varied independently. However, here I shall discuss only the steady state measurements of V_G and R and their implications when they are combined to obtain σ^*, which is defined in Eq. (2). In this way we can obtain

$$\sigma^* = \sigma^*(\Delta T, U_\infty) \tag{3}$$

The results obtained by Lee, *et al.*,[22] show that V_G increases and R decreases monotonically with both ΔT and U_∞. All four of the quantities (V_G, R, ΔT, and U_∞) were measured in our experiments. Therefore we were able to determine if V_G increases more or less quickly than R^2 decreases, and this shows how $V_G R^2$ behaves either with ΔT or U_∞. In this way one can establish experimentally the behavior of the selection parameter, σ^*, as a function of (ΔT, U_∞).

By determining the dependence of σ^* on (ΔT, U_∞), we can test one of the basic predictions of microscopic solvability theory. Bouissou and Pelce[26] used microscopic solvability theory to calculate this dependence, and they reported that σ^* **decreases** with **increased** (U_∞/V_G), which is the opposite of what the experiments of Gill, *et al.*,[25] and Lee, *et al.*,[22] show. Figure 5 displays the measured values of σ^* as a function of U_∞/V_G.

In Figure 5 we see that σ^* **increases** by more than 50% as U_∞/V_G **increases**, as shown by Gill, *et al.*[25] Since this behavior is contrary to that predicted by the two-dimensional analysis of Bouissou and Pelce, it appears that microscopic solvability theory does not predict even qualitatively the correct dependence of σ^* on U_∞ for SCN. A more dramatic contrast between theory and experiment is shown by Lee, *et al.*,[22] in their Figure 4 in which the slope of the data is negative and opposite to the prediction of MST.

Let us now consider the growth in quiescent melt of dendrites other than SCN and attempt to determine if MST predicts the proper dependence of σ^* on the anisotropy, ε. Muschol, *et al.*, made careful measurements of ε, and we shall outline briefly only their results and the information needed to understand them qualitatively. The Gibbs-Thompson equation for a cubic crystal can be written

$$T_i = T_m[1-(\gamma+\gamma_{\theta\theta})\kappa/L] \tag{4}$$

where

- γ surface tension
- $\gamma_{\theta\theta}$ second derivative of γ with the angular coordinate, θ
- θ measures position around the surface of the dendrite
- κ total curvature
- L heat of fusion
- T_m melting point of a flat surface
- T_i local interfacial temperature

If the surface tension is represented by

$$\gamma = \gamma_0[1+\varepsilon_m \cos m\theta] \tag{5}$$

combining Eqs. (4) and (5) obtains the approximation

$$T_i = T_m - \frac{2\gamma_0 T_m}{RL}[1-\alpha\cos m\theta] \tag{6}$$

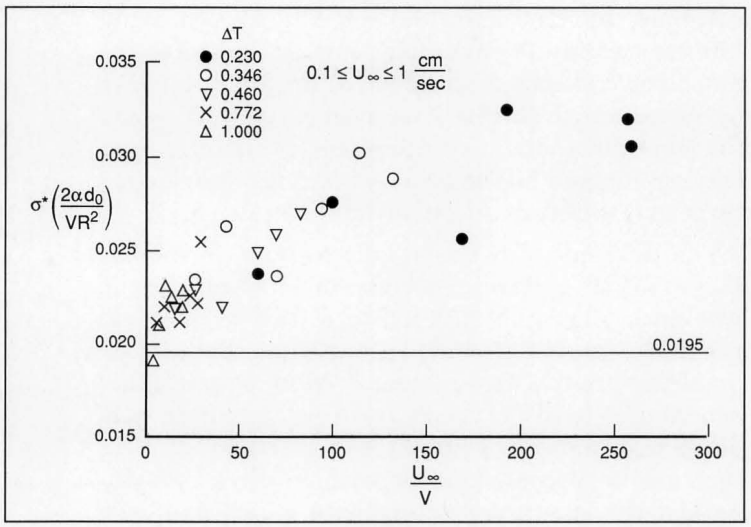

Figure 5. *Selection parameter, $\sigma^* = (2\alpha d_o)/(V_G R^2)$ for SCN dendrites increases by more than 50% as ratio of forced convection velocity, U_∞, to growth velocity, V_G, increases.*

where

R mean value of the tip radius

α anisotropy factor used by Mushol, *et al.*[21]

In Eq. (5), α is related to ε_m by $\alpha = (m^2-1)\varepsilon_m$. As pointed out by Glicksman and Marsh, (who subsequently made similar comparisons to those of Mushol, *et al.*) Eq. (6) shows that the equilibrium melting point at a point on an anisotropic solid-liquid interface depends on both its curvature and the orientation of the interface normal with respect to the principal crystallographic axes.

Mushol, *et al.*, computed theoretical values for the selection parameter, σ^*_{th}, based on various MST analytical models and numerical codes for materials for which α had been measured. The results were disappointing, and they concluded (as did Glicksman and Marsh) that the predictions of MST do not agree well with the available experimental evidence for dendrites grown in quiescent melts. Therefore, it seems that both forced convection experiments ($U_\infty > 0$) with SCN[22,25] and experiments on the relationship of σ^* with ε for other materials including alloys, do not support MST. It would appear that something fundamental is missing from the theory.

CONCLUDING REMARKS

Where does this leave us? Where do we go from here? It is obvious that great progress has been made in developing new materials, and this progress has been aided and abetted by a continually expanding body of experimental and theoretical knowledge about crystal growth. Furthermore, the available experiments have been getting more precise, which is important when testing new ideas and insights. Indeed, one can expect that as new experiments accumulate, they will suggest new theoretical concepts, as has been the case in virtually all fields of science. Also, incredibly fast computers and intriguing mathematical methods have been developed which will facilitate the integration of theory and experiments.

It is impossible to predict just when a fully satisfactory, predictive theory of dendritic growth will be available. I doubt it will take another 350 years, as apparently has been required to prove Fermat's last theorem. When it arrives, new doors no doubt will open up, and it probably will be helpful in controlling the microstructure of even more useful and exotic materials. In the meantime, there is a need for precise experiments on multicomponent materials of practical importance. Based on past experience, I think such experiments will stimulate new developments of technological importance—and deeper theoretical understanding as well.

ACKNOWLEDGMENT

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