ChE **class and home problems**

The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. We request problems that can be used to motivate student learning by presenting a particular principle in a new light, can be assigned as novel home problems, are suited for a collaborative learning environment, or demonstrate a cutting-edge application or principle. Manuscripts should not exceed 14 double-spaced pages and should be accompanied by the originals of any figures or photographs. Please submit them to Dr. Daina Briedis (e-mail: briedis@egr.msu.edu), Department of Chemical Engineering and Materials Science, Michigan State University, East Lansing, MI 48824-1226.

MODELING OF AN INDUSTRIAL ANAEROBIC DIGESTER:

A Case Study for Undergraduate Students

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The traditional approach in undergraduate chemical reaction engineering courses, as practiced in our university, is typically limited to design problems that can be solved by analytical methods. The study of single reaction systems presents the alternative of graphical resolution and easy analysis. Furthermore, single autocatalytic or inhibitory kinetic reactions are generally used to study the optimal combination of ideal reactors or the optimum recycle ratio.[1, 2] However, in most practical applications, a serial-parallel multiple reaction system takes place and the resolution of numerical schemes is required. The increasing use and development of computers is gradually changing the design approach in chemical engineering education. Indeed, the current technological tools facilitate quantitative analysis and, therefore, allow the student to attain a deeper learning approach to chemical engineering fundamentals by solving authentic systems.[3]

An example of a multiple-reaction system is anaerobic digestion, a process that is being increasingly used for wastewater treatment. Anaerobic microorganisms digest the organic materials to produce methane and carbon dioxide as end products under ideal conditions. Biomass growth is inherently an autocatalytic system since the rate is proportional to the biomass concentration. Moreover, it can be inhibited by certain substrate and product concentrations.[4,5]

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The pH is an essential parameter that should be taken into account. The methanogenic bacteria have a pH range of 6.5–7.2[6] whereas the acid-producing bacteria can grow at a wider range of 4.2–8.5.[7] As acetic acid is produced by acid-producing bacteria and consumed by methanogenic bacteria, its net production has to be balanced to prevent the collapse of the bioreactor.

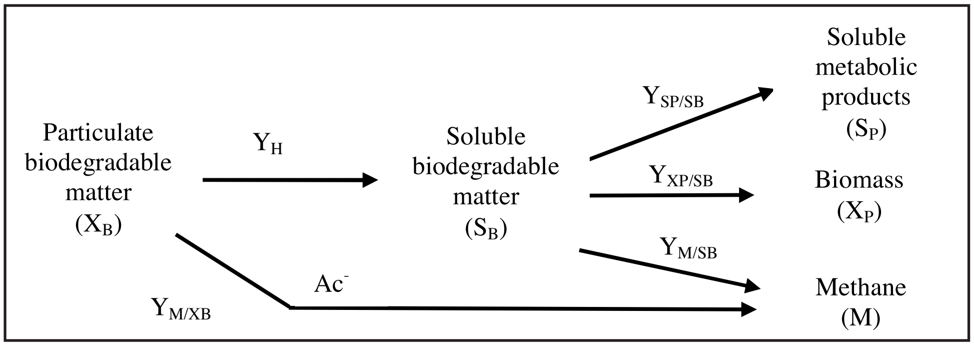


Figure 1. *Proposed kinetic reaction scheme.*

In addition, the nutrients' availability is another significant parameter in anaerobic digestion.[8] In particular, the C:N ratio is very important during anaerobic digestion of starch or other carbohydrate-based feedstocks given their relatively low nitrogen content and thus their poor pH buffer capacity.[9]

An industrial digester is proposed as a case study, aiming at promoting active learning in undergraduate chemical reaction engineering courses. This problem is a good candidate for teamwork in groups of two to four members to stimulate debate and enhance the learning experience.

Students are given two weeks to prepare a short written report describing model building, solution method, interpretation of results, and sensitivity analysis. At the end of the first week, model building and its resolution are supposed to be accomplished so that the encountered difficulties can be shared and discussed in class. The authors assume that the students have prior knowledge of the use of mathematical software. Some students will specifically need help with the convergence of the model. The second week is devoted to performing the sensitivity analysis and identifying key operating parameters. Finally, a roundtable discussion is proposed to present the final project in which each group will present a 10 minute talk about their work. When everyone has completed their individual presentations, the instructor will facilitate discussion and debate to reach general conclusions.

GOALS

The goals of this work are to:

1) *Present and discuss a serial-parallel reaction system based on an authentic biological system.*
2) *Generate a representative computer-based design problem.*
3) *Help undergraduate students attain a deeper realistic learning approach to chemical engineering fundamentals.*

THE ASSIGNMENT

The objective is for the students to model an industrial digester as a combination of ideal reactors. The digester consists of a covered rectangular pool of 130 m of length, 15 m high, and 80 m wide with biomass recycle. The recycle ratio ($R=Q_R/Q_F$) is set to 0.0077. The operating conditions in the industrial anaerobic digester are presented in Table 1.

TABLE 1
Operating Conditions in the Industrial Anaerobic Digester

Variable	Value
Q_F ($m^3.d^{-1}$)	6500
Inlet Total COD ($mgO_2.L^{-1}$)	3800
Recycle ratio ($R=Q_R/Q_F$)	0.0077
L_{CSTR} (m)	20
L_{PFR} (m)	100
$L_{thickener}$ (m)	10
pH	7.0
COD : N : P	200 : 5 : 1

The reactor presents three zones with different mixing patterns. The first zone is a mixing zone. This section is stipulated as the first 20 m length, fixed by the distance to which the feed tubes are introduced into the reactor and the turbulence is generated. In the second zone (20 m < L < 120 m) the flow develops longitudinally, while the gas produced mixes in transverse direction. Moreover, the high length-to-cross-sectional area allows the assumption of negligible axial dispersion. In the third zone, the particulate material is deposited since the reaction ends and no gas is generated. Design equations for this section are known and they are provided to students [Eqs. (18) and (19)].

The kinetic model and parameters are also given to students. This information can be found in sections on Kinetic Model and Kinetic Parameter Estimation.

Students are asked to validate the model assumptions, that is, model predictions should be contrasted with experimental outcomes. The data measured in the industrial digester are provided to students (see section on Model Validation). Once the model is validated, students are asked to implement the model (simulation) using different input assumptions and scenarios.

MODEL DEVELOPMENT

Kinetic model

The extent of chemical reaction is evaluated by measuring Chemical Oxygen Demand (COD), which is a method widely used on wastewater treatment to indirectly measure the concentration of organic matter in water.[11] Industrial potato wastewaters contain particulate and soluble organic load, mainly starch and soluble carbohydrates, which are assimilated by anaerobic microorganisms at different rates.

As described in Durruty, *et al*.[10] the total COD of the wastewater must be divided into a refractory or inert (I) organic fraction and a biodegradable (B) organic fraction. Both fractions—the inert and the biodegradable—must in turn be split into soluble (S) and particulate (X) fractions. Based on this COD distribution, Figure 1 outlines the kinetic model. It is assumed that the particulate biodegradable matter (X_B) is degraded to soluble biodegradable matter (S_B) and to acetate that is not detectable by the COD method.[11] It can be assumed that the acetate produced during hydrolysis is directly converted to methane (M) by acetoclastic methanogenic bacteria.[6] Thus, the kinetic model assumes that the metabolic products (S_P), biomass (X_P), and methane (M) are produced from soluble biodegradable matter (S_B) degradation.[6,10]

Therefore, a specific first order serial-parallel reactions kinetic model can be used to describe the kinetic behavior. Taking into account the reaction scheme shown in Figure 1, the particulate biodegradable organic fraction degradation (r_1) and the net degradation rate of S_B (r_2) are given by:

$$\frac{dX_B}{dt} = (r_1) = -k_1 \cdot X_P \cdot X_B \tag{1}$$

$$\frac{dS_B}{dt} = (r_2) = Y_H \cdot k_1 \cdot X_P \cdot X_B - k_2 \cdot X_P \cdot S_B \tag{2}$$

The biomass production (r_P) can be depicted by:

$$\frac{dX_P}{dt} = (r_P) = Y_{XP/SB} \cdot k_2 \cdot X_P \cdot S_B \tag{3}$$

Even though more complex kinetics, such as Contois or Monod, have been proposed to predict the biomass growth,[12,13] the specific first-order power law has been successfully used in most of anaerobic applications.[10,14,15] Hence, given its simplicity, it appears as the best alternative for practical applications.

Analogously, the soluble product (S_P) and the methane production rate (r_M) can be described as follows,

$$\frac{dS_P}{dt} = Y_{SP/SB} \cdot k_2 \cdot X_P \cdot S_B \tag{4}$$

$$\frac{dM}{dt} = (r_M) = Y'_{M/XB} \cdot k_1 \cdot X_P \cdot X_B + Y_{M/SB} \cdot k_2 \cdot X_P \cdot S_B \tag{5}$$

Kinetic parameter estimation

The industrial wastewater stream is biomass free and its composition (X_I, S_I, X_P, X_B, S_B), shown in Table 2, was estimated with the modified Orhon method developed by Durruty, *et al*.[10] The yield coefficients ($Y_{SP/SB}$, $Y_{XP/SB}$, and Y_H) were previously evaluated by Durruty, *et al*.[10] and their values are also listed in Table 2. The feed has the same composition as that used in the industrial reactor.

Experiments in a laboratory scale batch reactor were carried out to estimate the kinetic parameters and the yield coefficients. The original wastewater assay buffered at pH 7.0 was inoculated and the initial biomass concentration was 103 $mgO_2.L^{-1}$. The COD:N:P was 200:5:1 ensuring the nutrient availability.

The values of k_1, k_2, $Y_{M/XB}$, and $Y_{M/SB}$, listed in Table 2, were obtained by fitting experimental data. Regression analysis was performed using a data analysis tool of Origin 8.0®, OriginLab

TABLE 2
Feed Composition and Model Parameters

		Value	Reference
Wastewater composition	X_I ($mgO_2.L^{-1}$)	328	Lab (this work)
	S_I ($mgO_2.L^{-1}$)	146	Lab (this work)
	X_P ($mgO_2.L^{-1}$)	0	Lab (this work)
	X_B ($mgO_2.L^{-1}$)	1352	Lab (this work)
	S_B ($mgO_2.L^{-1}$)	1899	Lab (this work)
Yield coefficients and kinetic parameters	Y_H	0.616 ± 0.023	[10]
	$Y_{SP/SB}$	0.034 ± 0.007	[10]
	$Y_{XP/SB}$	0.133 ± 0.009	[10]
	$Y_{M/XB}$	0.377± 0.014	Lab (this work)
	$Y_{M/SB}$	0.143± 0.017	Lab (this work)
	k_1 ($L.mgO_2^{-1}.d^{-1}$)	9.611×10^{-4}	Lab (this work)
	k_2 ($L.mgO_2^{-1}.d^{-1}$)	2.757×10^{-4}	Lab (this work)

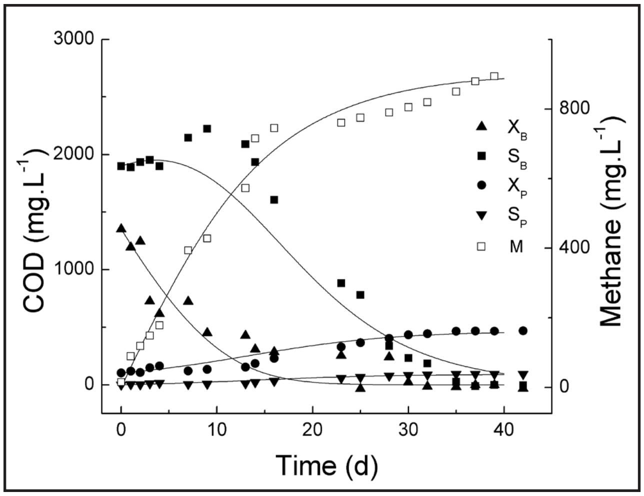


Figure 2. *Solid and soluble biodegradable organic fractions and methane measured during laboratory assay. The kinetic fitting also is shown as solid lines.*

Corporation, based on the least square method. All the variables were successfully predicted, as shown in Figure 2. As previously reported by Durruty, *et al.*,[10] this kinetic model under-predicts the maximum S_B concentration. However, the difference between experimental and predicted values is within the measurement error.

The kinetic model and parameter values listed in Table 2 are provided to students.

Digester model development

The industrial digester presents three zones with different mixing patterns as shown in the schematic in Figure 3. The first zone is a mixing zone that can be modeled as a continuous stirred-tank reactor (CSTR). Thus, L_{CSTR} =20 m. In the second zone (20 m < L < 120 m) the flow can be modeled as an ideal plug flow reactor (PFR). The third zone can be modeled as a thickener whose design equations and their parameters are obtained experimentally. The information about the third zone is provided to students. Figure 3a represents this scheme including the recycle flow. Students should write mass balances for the inlet node and for the first and for the second zones, as explained below.

According to Figure 3a, mass balances for each component in the inlet node are given by:

$$Q_I = Q_F + Q_R = Q_F \cdot (1+R) \tag{6}$$

$$X_{B,I} = \frac{Q_F \cdot X_{B,F} + Q_R \cdot X_{B,PU}}{Q_F \cdot (1+R)} = \frac{X_{B,F} + Q_R \cdot X_{B,PU}}{(1+R)} \tag{7}$$

$$S_{B,I} = \frac{Q_F \cdot S_{B,F} + Q_R \cdot S_{B,PU}}{Q_F \cdot (1+R)} = \frac{S_{B,F} + R \cdot S_{B,PU}}{(1+R)} \tag{8}$$

$$X_{P,I} = \frac{Q_F \cdot X_{P,F} + Q_R \cdot X_{P,PU}}{Q_F \cdot (1+R)} = \frac{R \cdot X_{P,PU}}{(1+R)} \tag{9}$$

The resolution of Eqs. (6) to (9) allows the evaluation of the composition in the stream "I".

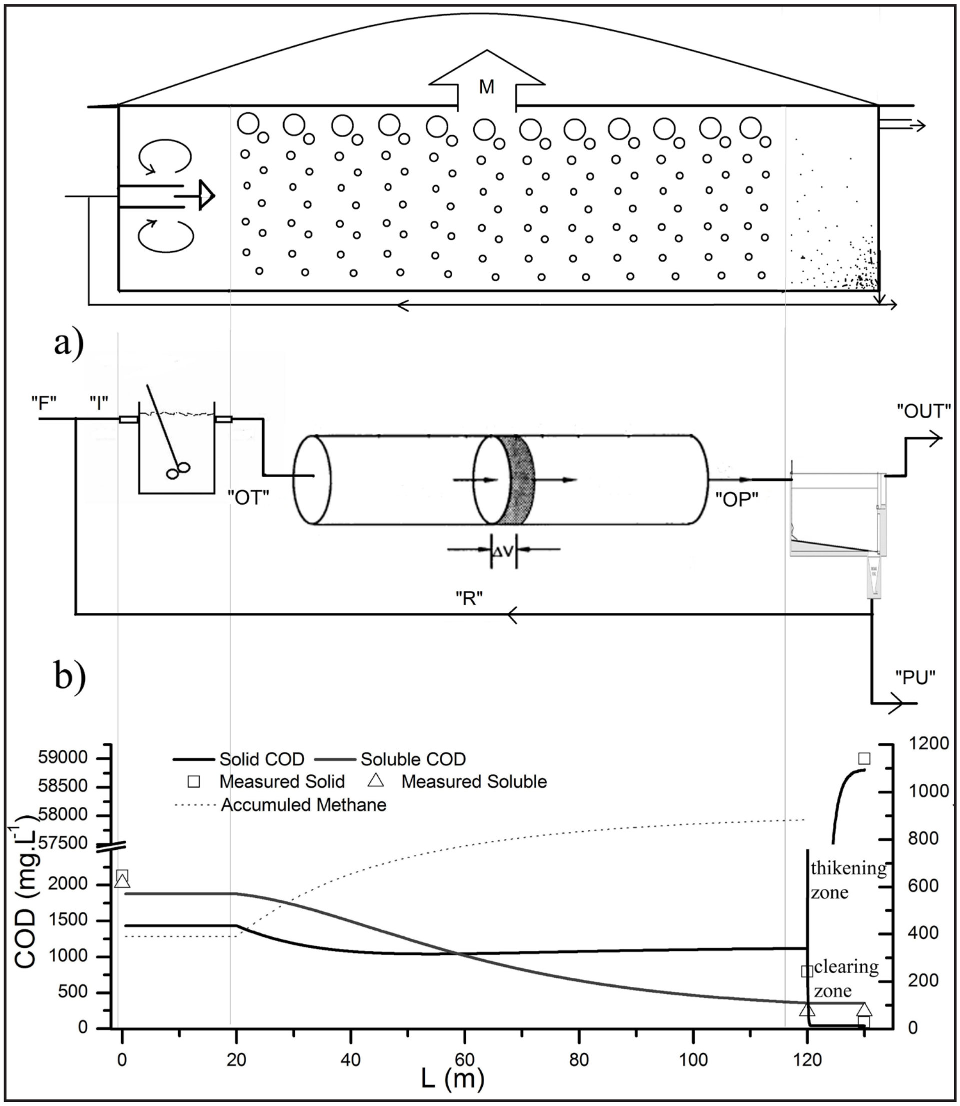


Figure 3. *Diagram of industrial digester. a) model scheme proposed for the industrial real anaerobic digester. b) measured and predicted solid and soluble COD and methane production profiles. Operating conditions detailed in Table 2.*

For L<20m, the design equations for X_B, S_B, X_P, and M can be obtained from the corresponding mass balances of CSTR as follows, where the subscript OT represents the position corresponding to the output of the first reactor.

$$HRT_{CSTR} = \frac{X_{B,I} - X_{B,OT}}{(-r_1)} = \frac{X_{B,I} - X_{B,OT}}{k_1 \cdot X_{B,OT} \cdot X_{P,OT}} \tag{10}$$

$$HRT_{CSTR} = \frac{S_{B,I} - S_{B,OT}}{(-r_2)} = \frac{S_{B,I} - S_{B,OT}}{-Y_H \cdot k_1 \cdot X_{B,OT} \cdot X_{P,OT} + k_2 \cdot S_{B,OT} \cdot X_{P,OT}} \tag{11}$$

$$HRT_{CSTR} = \frac{X_{P,OT} - X_{P,I}}{(r_p)} = \frac{X_{P,OT} - X_{P,I}}{Y_{XP/SB} \cdot k_2 \cdot S_{B,OT} \cdot X_{P,OT}} \tag{12}$$

$$HRT_{CSTR} = \frac{M_{OT}}{(r_M)} = \frac{M_{OT}}{Y_{M/XB} \cdot k_1 \cdot X_{B,OT} \cdot X_{P,OT} + Y_{M/SB} \cdot k_2 \cdot S_{B,OT} \cdot X_{P,OT}} \tag{13}$$

where the hydraulic retention time (HRT) is $\frac{A_T \cdot L_{CSTR}}{Q_I}$. These balances should be solved simultaneously.

The output of the CSTR reactor enters the PFR reactor. Therefore, students can evaluate the composition profiles along the PFR reactor from the following mass balances:

$$-\frac{dX_B}{dHRT} = -\frac{Q_I}{A_T}\frac{dX_B}{dL} = (-r_1) = k_1 \cdot X_B \cdot X_P \tag{14}$$

$$-\frac{dS_B}{dHRT} = -\frac{Q_I}{A_T}\frac{dS_B}{dL} = (-r_2) = -Y_H \cdot k_1 \cdot X_B \cdot X_P + k_2 \cdot S_B \cdot X_P \tag{15}$$

$$\frac{dX_P}{dHRT} = \frac{Q_I}{A_T}\frac{dX_P}{dL} = (r_p) = Y_{XP/SB} \cdot k_2 \cdot S_B \cdot X_P \tag{16}$$

$$\frac{dM}{dHRT} = \frac{Q_I}{A_T}\frac{dM}{dL} = (r_M) = Y_{M/XB} \cdot k_1 \cdot X_B \cdot X_P + Y_{M/SB} \cdot k_2 \cdot S_B \cdot X_P \tag{17}$$

Eqs. (14) to (17) are simultaneously solved using a fourth order Runge-Kutta routine with the following boundary condition:

For $L = L_{CSTR}$; $X_B = X_{B,OT}$; $S_B = S_{B,OT}$; $X_P = X_{P,OT}$; $M = M_{OT}$.

Thus, for $L = L_{OP}$ or $HRT = HRT_{OP}$ concentrations $X_{B,OP}$, $S_{B,OP}$, $X_{P,OP}$, and M_{OP} were obtained, where the subscript OP represents the position corresponding to the outlet of modeled PFR.

To model the clarifier, an upper area of clarification and a thickening zone at the bottom were defined. The expressions in Eqs. (18) and (19) represent the behavior in the clarification and in the thickening zones, respectively.

$$\delta_{clarifying} = \frac{COD_T - COD_S}{(COD_T - COD_S)_{t=0}} = 0.041 + 0.958 \cdot e^{-2.279(\sigma)} \tag{18}$$

$$\delta_{thickening} = \frac{COD_T - COD_S}{(COD_T - COD_S)_{t=0}} = 60.679 + 60.199 \cdot e^{-0.195(\sigma)} \tag{19}$$

where COD_T and COD_S are the total and the soluble chemical oxygen demand, respectively, and σ is the residence time expressed in days. Parameters in these equations have been obtained from experimental data obtained in the lab and provided to students.

The model equations [Eqs. (6) to (19)] can be solved following the algorithm depicted in Figure 4 (page 76) using Mathcad 15.0©, Parametric Technology Corporation. The mass balances on the CSTR are solved with a given-find block whereas the mass balances on the PFR are solved with the rkfixed function.

Model validation

Direct validation is a necessary condition, but by no means a sufficient condition to evaluate the predictive power of the model. With this aim, a cross-validation vs. fresh data is mandatory.[16] Cross-validation can be used to assess the accuracy of the parameters and to evaluate digester model performance.

An independent set of data taken from the industrial digester is provided to students. For an industrial reactor of total length L = 120 m and the operating conditions listed in Table 2, it is known that the measured solid and soluble Chemical Oxygen Demand (COD) at the end of the second zone (OP) are 795 mgO_2/L and 239 mgO_2/L, respectively; X_B=0.035 mgO_2/L and S_B=136 mgO_2/L. Then, the biodegradable COD conversion (x) observed is 0.96.

Figure 3b shows the predicted solid and soluble organic loads profiles. The solid and soluble COD correspond to the sum of X_P, X_B, and X_I, and the sum of S_P, S_B, and S_I, respectively. This figure shows that the outcomes are satisfactorily predicted along the digester. In this way, both the reactor and the kinetic model are simultaneously cross validated against fresh industrial data. The importance of the knowledge of an adequate kinetic model should be highlighted.

DISCUSSION

For a single reaction system, the hydraulic retention time (HRT) needed to achieve a given conversion can be graphically estimated by plotting the inverse of reaction rate vs. the integration variable.[1] This graphical analysis is frequently taught in chemical engineering undergraduate courses. However, in a serial-parallel multiple reaction system a given reactor configuration that benefits some stages will adversely affect others and the graphical analysis of individual steps is useless.

The first step to study this complex system is to develop and validate a model able to represent the performance of the process. Once that has been achieved, a parametric study is a useful alternative tool to assess the effect that certain parameters' changes can have on the process. This analysis also allows not only the verification of the model in idealized test cases but also the exploration of different scenarios.

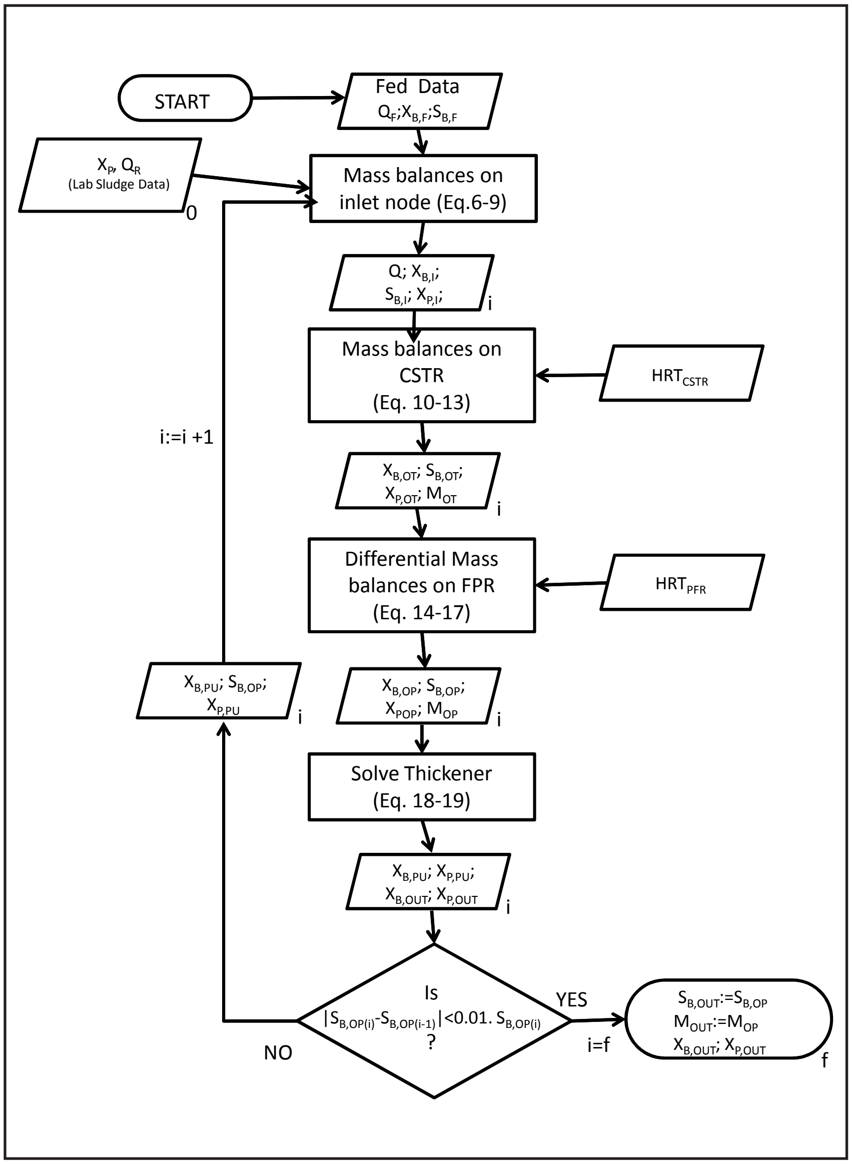


Figure 4. *Algorithm structure.*

In this context, the following questions are proposed with the aim of encouraging discussion and debate among the undergraduate students:

Is the actual configuration of the digester the optimal one? What if this process is carried out in a single PFR or a single CSTR? Is it possible to maximize production in the industrial digester? Is it possible to diminish reactor volume?

To address this, the recycle ratio (R) and the agitated zone length (L_{CSTR}) are selected as key parameters, considering the self-catalytic nature of the system. Students are asked to evaluate methane and biomass production and the total volume necessary to attain biodegradable COD conversion (x) =0.96. Some teams are asked to investigate the effect of varying the value of R for a given L_{CSTR} whereas the rest of the groups analyze the effect of varying the value of L_{CSTR} for a given R. Final results should be presented as plots showing the profiles of methane, biomass production, and the total volume necessary as a function of LCSTR. Partial conclusions will be exposed by each group at the roundtable class aiming at summing up results and building general conclusions.

Figures 5 (a-c) show the methane and biomass production and the total volume necessary to attain x=0.96 as a function of L_{CSTR} with R as parameter. It is expected to build this figure taking into account each group's contribution. Limit cases are identified as follows: L_{CSTR}=0 and L_{CSTR}= 120 m represent the single PFR and CSTR, respectively.

In the extreme case, when R=0, a single PFR does not work since no biomass enters the reactor. Thus, with no recycle, L_{CSTR} positively affects the overall performance. Indeed, the single CSTR presents the best performance. However, the possibility of washout exists during the operation of a no-recycle CSTR.[17]

The minimum HRT needed to ensure the biomass growth can be calculated following analytical or numerical approaches. The analytical procedure was widely studied for simple substrate and Monod kinetics.[17, 18] However, for more complex systems, like multi-substrate or serial-parallel multiple-reactions systems, numeric resolution is the only viable way to find the minimum HRT to avoid washout.

X_B, S_B, and X_P profiles vs. HRT for a no-recycle CSTR with sterile feed detailed in Table 1 have been obtained by solving numerically Eqs. (10) to (13). Figure 6 (page 78) shows the outcomes. The production of biomass (X_P/HRT) presents a maximum with respect to the HRT or Q_F.

This figure clearly shows that the HRT that maximizes the biomass production is around 23 days, and the HRT that maximizes the methane production is around 19 days. The washout will occur when HRT is lower than 14 days. In this case study, HRT=14 days corresponds to L_{CSTR} = 75 m. Figure 5 shows that the reactor collapses for L<75 m when recycle is not used. The effect of cell recycle is to extend the range at which washout of cells will not occur.[17]

When R>0, Figure 5 shows that for a given recycle ratio, the reactor volume necessary to achieve x=0.96 increases with higher L_{CSTR}, suggesting that the use of a CSTR may not be necessary to improve the performance. A large mixed volume at the beginning of the reactor leads to a low particulate biodegradable concentration that cannot be compensated by the poor biomass growth (smaller and slower) leading to a low particulate degradation rate. In contrast, the soluble biodegradable COD consumption is enhanced due to the increase in both the soluble biodegradable concentration and the biomass. Consequently, the use of a single recycled PFR seems to be the better choice to improve the reactor performance.

As R increases, the overall performance is significantly improved since methane and biomass production increase. At the same time, the reactor volume necessary to attain x=0.96 is reduced. In fact, an optimal recycle ratio that minimizes the total reactor volume is not found within the range of parameters studied. This contrasts with the results obtained in the conventional reaction systems studied.[1] Nevertheless, an optimal R that minimizes the operational cost implicitly exists due to the pumping cost of recycling high amounts of slurry sludge with high density and viscosity.

As a whole, when biomass is recycled, the use of a CSTR before the PFR is not required for design purposes in the self-catalytic reaction system studied here. At the same time, the recycle of the wet digestate stream results in a higher biomass concentration in the inlet and therefore, higher degradation rates. Therefore, it is desirable to operate the digester at the highest possible recycle ratio.

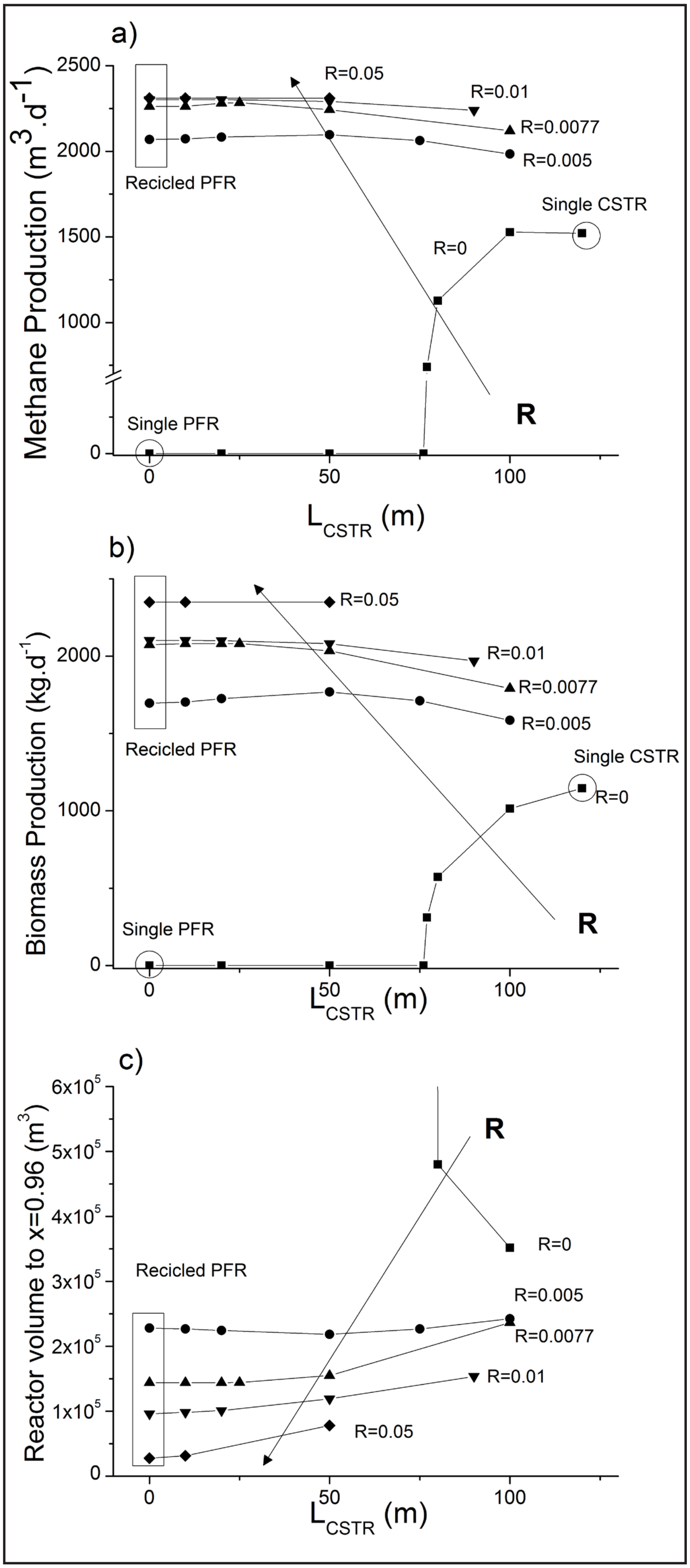
STUDENT FEEDBACK

Students responded well to this realistic case study. The availability of the data taken from the treatment plant motivated the groups' work and discussion. This helped them understand the relevancy of building a correct model and its implementation to predict outcomes in an industrial scale reactor using different values of operational variables.

CONCLUDING REMARKS

A case study for chemical reaction engineering students is proposed. Students are encouraged to build, solve, validate, and implement the model by exploring and discussing different input assumptions and scenarios.

The case study presented in this paper is particularly interesting given that it represents an actual industrial process. This activity can be proposed in undergraduate chemical reaction engineering



Figures 5. *a) methane, b) biomass production, and c) the Reactor volume necessary to attain x=0.96 vs. L_{CSTR} using R as parameter.*

courses as an overarching exercise to raise students' interest and motivation to incorporate the academic knowledge to real applications.

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NOMENCLATURE

COD	Chemical Oxygen Demand ($mgO_2.L^{-1}$)
HRT	Hydraulic retention time (d)
k	Specific first order kinetic constant ($L.\ d^{-1}.mgO_2^{-1}$)
M	Methane expressed as chemical oxygen demand per volume of reactor ($mgO_2.L^{-1}$)
Q	Volumetric flow (m^3/d)
R	Recycle ratio (dimensionless)
A	Reactor cross sectional area (m^2)
L	Reactor length (m)
V	Reactor volume (m^3)
x	Conversion of total biodegradable COD
X_I	Inert particulate COD fraction ($mgO_2.L^{-1}$)
S_I	Inert soluble COD fraction ($mgO_2.L^{-1}$)
X_P	Biomass COD fraction ($mgO_2.L^{-1}$)
X_B	Particulate biodegradable COD fraction ($mgO_2.L^{-1}$)
S_B	Soluble biodegradable COD fraction ($mgO_2.L^{-1}$)
S_p	Product soluble COD fraction ($mgO_2.L^{-1}$)
Y	yield coefficient (dimensionless)

Symbols

$\delta_{clarifying}$	Change of the solid chemical oxygen demand in the clarifying zone
$\delta_{thickening}$	Change of the solid chemical oxygen demand in the thickening zone
σ	Residence time in the settler zone (d)

Subscripts

F	Feed stream
I	Inlet stream
CSTR	Continuous stirred tank reactor
PFR	Plug flow reactor
S	Soluble
T	Total
1	Referred to reaction 1, biodegradable particulate degradation.
2	Referred to reaction 2, biodegradable net soluble degradation.
OT	outlet CSTR stream
OP	outlet PFR stream
OUT	outlet stream
PU	purge stream
R	recycle stream.

REFERENCES

1. Levenspiel, O., *Chemical Reaction Engineering*, 3rd Ed., New York: John Wiley and Son (1999)
2. Missen, R.W., C.A. Mims, and B.A. Saville, *Introduction to Chemical Reaction Engineering and Kinetics*, New York: John Wiley & Sons (1999)
3. Harb, J.N., *et al.*, "Use of Computational Tools in Engineering Education," *Chem. Eng. Ed.*, **31**(3), 180 (1997)
4. Mösche, M., and H.-J. Jördening, "Comparison of Different Models of Substrate and Product Inhibition in Anaerobic Digestion," *Water Research*, **33**(11), 2545 (1999)
5. Kus, F., and U. Wiesmann, "Degradation Kinetics of Acetate and Propionate by Immobilized Anaerobic Mixed Cultures," *Water Research*, **29**(6),1437 (1995)
6. Gerardi, M.H., *The Microbiology of Anaerobic Digesters*, New Jersey: John Wiley and Sons (2003)
7. Hwang, M.H., *et al.*, "Anaerobic Bio-Hydrogen Production From Ethanol Fermentation: the Role of pH," *J. Biotechnology*, **111**(3), 297 (2004)
8. Demirel, B., and P. Scherer, "Trace Element Requirements of Agricultural Biogas Digesters During Biological Conversion of Renewable Biomass to Methane," *Biomass and Bioenergy*, **35**(3), 992 (2011)
9. Demirel, B., and P. Scherer, "Production of Methane From Sugar Beet Silage Without Manure Addition by a Single-Stage Anaerobic Digestion Process," *Biomass and Bioenergy*, **32**, 203 (2008)
10. Durruty, I., N.E. Zaritzky, and J.F. González, "Kinetic Studies on the Anaerobic Degradation of Soluble and Particulate Matter in Potato Wastewater," *Biosystems Eng.*, **111**(2). 195 (2012)
11. APHA, *Standard Methods for the Examination of Water and Wastewater*, 20th Ed. 1998, Washington DC, USA: American Public Health Association/American water works Association/Water environment federation
12. Bhunia, P., and M.M. Ghangrekar, "Analysis, Evaluation, and Optimization of Kinetic Parameters for Performance Appraisal and Design of UASB Reactors," *Bioresource Tech.*, **99**(7), 2132 (2008)
13. Beltran, J., T. Gonzalez, and J. Garcia, "Kinetics of the Biodegradation of Green Table Olive Wastewaters by Aerobic and Anaerobic Treatments," *J. Hazardous Materials*, **154**(1-3), 839 (2008)
14. Neves, L., *et al.*, "Influence of Composition on the Biomethanation Potential of Restaurant Waste at Mesophilic Temperatures," *Waste Management*, **28**, 965 (2008)
15. Siles, J.A., *et al.*, "Kinetic Modeling of the Anaerobic Digestion of Wastewater Derived from the Pressing of Orange Rind Produced in Orange Juice Manufacturing," *Chem. Eng. J.* (Lausanne), **140**(1-3), 145 (2008)
16. Donoso-Bravo, A., *et al.*, "Model Selection, Identification, and Validation in Anaerobic Digestion: A Review," *Water Research*, **45**(17), 5347 (2011)
17. Blanch, H.W., and D.S. Clark, *Biochemical Engineering*, New York: Marcel Dekker Inc. (1996)
18. Bailey, J., and D. Ollis, *Biochemical Engineering Fundamentals*, McGraw-Hill, New York (1986) ❐

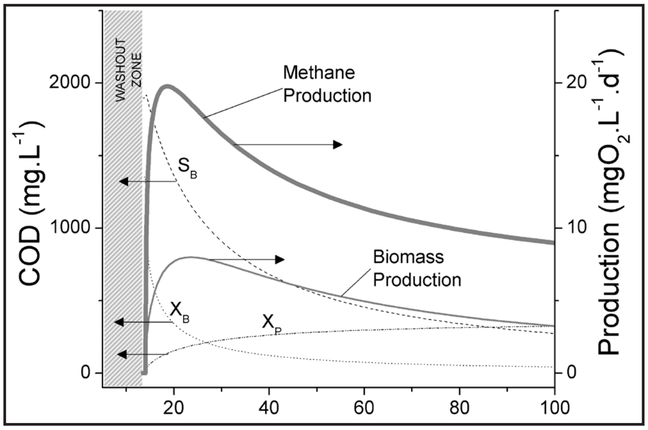

Figure 6. *Numerical solution for CSTR for different HRT. X_B, S_B, and X_P (Biomass) must be read on left axis and production of methane and biomass must be read on the right axis.*