A Modular Course in

MULTIVARIABLE CONTROL AND ESTIMATION

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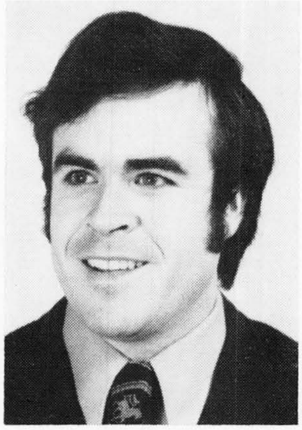
IN THE 1970 Graduate Education Issue of *Chemical Engineering Education,* Lowell Koppel lamented that advanced control techniques had not been considered to be practical or effective in spite of the significant number of engineers with graduate level training in process control. Today, however, it appears that there is a real opportunity for advanced control techniques to have a significant impact on the practice of process control in the chemical industry. Concomitantly, graduate education in control theory can contribute to the emergence of the new control methods.

Let us examine the current situation in more detail. First, the dedicated process computer has been made a reality via the development of inexpensive process control software and hardware. Second, some of the ideas which have received theoretical attention in the control literature have now been subjected to experimental verification. For example, the increase in effectiveness of multivariable control, where the controller is fed information from all outputs, over single loop control (single measurement feedback) has been clearly demonstrated by several investigators[1, 2]. Third, increased energy costs have caused supervisory personnel to re-examine the economic trade-off between energy consumption and product specifications, both for steady state and dynamic operation. Fourth, the use of the computer for data acquisition and supervisory control as well as in single loop DDC has been accepted in the process industries—a development which clears the way for further advances in sophistication.

Given the current industrial situation, how does one attempt to structure the graduate curriculum in control so that it will present the important concepts but also eventually have some impact on control practice? There are a number of relevant facts to consider here:

- Today there are fewer graduate students specializing in process control, most of them M. S. candidates with relatively short holdup times. This situation together with faculty logistics usually permit the offering of only one graduate control course.
- A chemical engineering graduate course in control should not and need not duplicate other engineering control courses. It should emphasize theory and application indigenous to the chemical process industry.
- Since a classical control course based on frequency domain analysis is traditionally taught undergraduates, the graduate course should interface with that background. In order to communicate with a practicing control engineer, the graduate must be able to speak in terms of transfer functions and PID controllers. Unfortunately these subjects have not been addressed in most advanced control theory books based on time domain analysis.



Tom Edgar is an assistant professor at The University of Texas at Austin. He came to Texas from Princeton University (Ph.D. 1971), where he specialized in control theory and collision and trajectory analysis, the latter two topics mainly applied to intercollegiate competition in rugby and volleyball. His B.S. degree is from the University of Kansas, and he has worked as a process engineer for Continental Oil Company. At Texas he is engaged in teaching and research in the fields of multivariable control, optimization, process modeling, and energy systems.

- Experimental computer control facilities, if available, should be integrated into the control course. This is the surest way to lend credibility to advanced control concepts. The 1970 survey of universities by the CACHE Real-Time Task Force has shown that nearly fifty chemical engineering departments had acquired or were planning to acquire computers for use in their laboratories.

COURSE PEDAGOGY

GIVEN THE ABOVE considerations, the graduate offering in modern control theory at the University of Texas has evolved into a course on multivariable control and estimation. The course emphasizes the development of control strategies based on state variable models but not necessarily limited to the use of optimization theory. The concepts of transfer functions, both continuous and discrete, are introduced, and the design of feedback control laws for single input-single output systems is shown to be a subproblem of the multiple input-multiple output design problem. The majority of the course material is based on linear(ized) systems, for which many useful mathematical results have been developed.

Coverage of basic mathematical concepts, especially those of static optimization and matrix techniques, is minimized in the interests of time. Variations in the mathematical background of the students can be rather wide, but it has been found that most students will accept the scale-up of a two dimensional example to a matrix expression. By later studying a higher order example, they do obtain an appreciation for the power of matrix notation.

An important ingredient of the course is the providing of experience via computer simulation and real-time computer control experimentation. The experiments require knowledge of computer programming (Fortran, Basic); however, the student does not need to learn details on instrumentation or computer hardware although that option is available.

As part of a large project on computer-based education at the University of Texas, modularizing of certain portions of the course has been attempted to strengthen the learning process, with good success. A module consists of explanatory material (both theory and application) on a specific topic in which the student behavioral objectives or goals are clearly defined by the instructor. The student then proceeds to independently learn the concepts via conjunctive use of textbooks and material written by the instructor. Study questions are used to reinforce the understanding of the module. By formulating the module as a project with many options and alternatives and requiring the student or group of students to write a report on their results, the students' creativity in thought and expression is stimulated. This procedure along with several examinations indicate whether the student has attained the desired behavioral objectives. If the students do not learn the specific concepts, then the module should be altered so that they do. Modules also provide additional experience in independent study; the capacity for self-study is a valuable trait for continued professional development.

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Student evaluation of the module approach has shown that it definitely enhances the quality of the course. The mathematics of modern control theory are rather difficult to master, and supplementary information as well as study questions on the various subjects prove to be helpful. The modules can sometimes stand in place of a lecture; less than twenty percent of my lectures have been displaced by this medium. In those cases the lecture time is used for informal discussion of the concept or experiment under study. The transferability of a module to another school is another important consideration, and great care has been exercised to design the modules so that they could be implemented elsewhere.

COURSE CONTENT

A GENERAL OUTLINE of the course is given in Table I. A heavy emphasis is placed on linear system theory, both for control and estimation, since these topics have a much higher probability of near-term application in the chemical industry.

Table I
COURSE OUTLINE

I. Review of Static Optimization
II. State Representation of Dynamic Systems
  A. State Equations
  B. Eigenvalues, Modal Analysis, Modal Control
  C. Controllability, Observability
III. Dynamic Optimization—Continuous Time
  A. The Variational Approach
  B. The Linear Quadratic Problem (LQP)
  C. Constrained Control, Minimum Time Control
  D. Nonlinear System Control
IV. Dynamic Optimization—Discrete Time
  A. State Equations
  B. Discrete Dynamic Programming—LQP
V. State and Parameter Estimation
  A. Observer Theory
  B. Kalman Filtering
  C. Nonlinear System Estimation

Modules have been written on the following subjects:

IIB: modal analysis and control

IIIB: optimal multivariable control of a distillation column

IIIC: minimum time control of linear systems (phase plane analysis)

V: sequential parameter estimation in a stirred tank

The parameter estimation module has been used with real-time computer data acquisition and computation, while the other modules have used simulation (digital and analog) for demonstrating the concepts. Equipment limitations have previously prevented the application of actual experimentation to the first two modules, but this problem has recently been resolved.

The textbook used is *Modern Control Engineering* by Maxwell Noton; the text more or less covers the topics listed in Table I. The book is interdisciplinary in its presentation, although not as extensive in scope as those books used for additional study in the course[3-8]. After a short review of static optimization using the book, the study of linear continuous system dynamics is undertaken. Such subjects as eigenvalues/eigenvectors and their relationships to transient response, canonical forms, state variable notation, multivariable Laplace transforms, the transition matrix, and the modal equations[9] are presented here.

The course emphasizes the development of control strategies based on state variable models but not necessarily limited to use of optimization theory . . . an important ingredient is providing experience via computer simulation and real-time computer control experimentation.

At this point the student is prepared for the first application of multivariable control. Proportional control of the states is assumed to be the most practical strategy for process regulation. It can be easily shown that the addition of feedback control in effect shifts the eigenvalues of the open-loop model. The proposed controller should realize a quick-responding closed-loop system where the eigenvalues have large negative real parts. Thus the so-called pole placement or modal control technique offers one multivariable control approach. One can adjust the elements of the feedback matrix, K, to obtain the desired closed loop behavior. This can be done intuitively, by optimization techniques, or by other methods[9], [10], [11]. The students are cautioned, however, that the system eigenvectors can cause unpredictable behavior. These factors are studied in the first module.

TYPICAL PROBLEM

A PILOT SCALE distillation column system in the laboratory can be introduced at this juncture as a typical multivariable control problem. Since most multivariable systems are derived from physical principles (black box multivariable modeling techniques are not yet well-developed), this approach is used for the column model development. The Huckaba model[12] for a column with n trays and reboiler and condenser yields a set of n + 2 nonlinear ordinary differential equations. The derivation is explained in detail in a student handout. This model has been experimentally verified and thus assumes some credibility. By linearizing the equations, a state space model of the form,

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Cd}$$

is derived, where $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{d}$ are the state, control, and disturbance vectors. This system can be used as the focus of various linear multivariable control strategies, such as the modal control technique mentioned above.

The second major approach for design of multivariable controllers utilizes the the minimum principle applied to the linear state equation with quadatic objective function, the well-known linear-quadratic problem (LQP). The basic optimal control structure for the LQP is linear feedback; if the disturbance, d, is non-zero, the LQP solution consists of proportional feedback plus feedforward control. Thus the notion of feedforward control to anticipate the effect of the disturbance, a concept which is now well-established in control practice (via transfer function analysis), arises in optimal multivariable control. By proper choice of the objective function, an optimal PID controller can be computed. For simple systems this correlates closely with the PID controller tuned using classical control theory.[13] Optimal control theory clearly demonstrates the effect of the integral model; it only makes the controller more sluggish, but its advantages include compensation for model errors and the smoothing of the control action.

One of the more interesting applications is the control of a fluid catalytic cracker system . . . the distributed parameter version of the LQP is briefly treated in class by discretization of the spatial variable—"if you don't like it, lump it."

The computation of multivariable control via the LQP is rather straightforward, and there are "canned" computer programs available for controller design. Such a program, VASP[14] (Variable Dimension Automatic Synthesis Program), links available Fortran subroutines (e. g., integration of Riccati equation, formation of transition matrix, etc.) and requires a minimum of programming effort, thus permitting the student to concentrate on the interpretation of his results. In the second module the student applies the LQP computation to the distillation column model. The articles on optimal feedforward/feedback control by Hu and Ramirez[15] and Newell et al.[1] serve as good supplementary papers.

THEORY VERSATILITY

EXTENSIONS AND APPLICATIONS of the LQP are also discussed. The recent survey article by Edgar et al.[16] has reviewed the versatility of LQP theory and its applications; one of the more interesting applications is the control of a fluid catalytic cracker system.[17] The distributed parameter version of the LQP is briefly treated in class by discretization of the spatial variable ("if you don't like it, lump it"). The discrete version of the LQP is solved using discrete dynamic programming, which permits the discussion of Bellman's principle of optimality. The discrete LQP is discussed in conjunction with digital control, and the conversion from continuous time to discrete time and the definition of discrete state variables are covered here.

In the third module the subject of continuous time dynamic optimization is continued with discussions of the linear minimum time problem and various algorithms for solving it. Phase plane analysis is an important tool for understanding control synthesis, and real-time simulation of the phase plane on an analog computer readily shows how difficult it is to perform minimum time control. While minimum time control is open loop control, it does exhibit a multivariable feedback nature in that a switching function based on the adjoint variables is defined via the minimum principle.

The final section of the course is state and parameter estimation. This area is relatively difficult for the student because of the need to use probability theory. For no noise in the system, the Luenberger observer is used; for noisy systems, the Kalman filtering algorithm must be introduced. In order to show how a simple sequential linear least squares algorithm is developed (vs. a non-sequential algorithm), the fourth module utilizes an experiment where the computer sequentially estimates a single parameter in a linear discrete-time equation. This equation is derived from an energy balance describing heat transfer in a stirred tank. The theory follows the presentation of Young.[18] This experiment demonstrates many of the convergence features of sequential estimators while including real-life features such as process and measurement noise as well as modeling errors. It is simple enough (one unknown parameter, first order o. d. e.) that the student can interpret the experimental and computational results. The discrete-time filter is then extended to continuous-time systems; the analogy to the LQP is pointed out. The experimental testing of state estimation by Hamilton et al.[19] at the University of Alberta is a good applications paper for this section.

Due to a lack of time, the course does not cover topics such as Lyapunov functions (particularly as applied to suboptimal control and model reference adaptive control), non-interacting control, or multivariable frequency response design.

(Continued on page 199.)

MULTIVARIABLE CONTROL AND ESTIMATION: Edgar

Continued from page 171.

This latter approach is an interesting extension of classical single loop design.[20]

SUMMARY

The course stresses those elements of modern control theory which appear to have the most promise of eventual applications and economic justification. The usefulness of the proposed techniques is tested via simulation and experimentation. A pilot plant distillation column has been chosen as a prototype system for testing multivariable strategies; focusing on a real system seems to enhance the students' interest. There is no question that use of a computer control laboratory strengthens the overall course, and hopefully the experience will motivate the students to use multivariable control and estimation to solve the difficult problems of process operation.

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