SELF-INSTRUCTION IN THERMODYNAMICS

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A LTHOUGH SUBSTANTIAL attention has been focused on self-paced instruction in engineering, the majority of courses continue in the traditional form of professorial lectures. A middle path can be followed in which a portion of the material is presented in the form of "written lectures" akin to, but different from SPI. In the self-instruction (not self-paced) technique, standard lectures can be intermixed with written lectures. Students come to scheduled classes and read rather than listen. A monitor is available to answer questions. With the two lectures described in this article, there were few questions. The students appeared to understand the material. None were able to complete the entire assignment in the 1.5 hour (30 minutes) second period.

The self-instruction process represents a moderate step away from classical procedures. The advantage is that students continue to attend class and do not have to break abruptly from accustomed practice. At the same time, part of the responsibility for learning as in SPI is transferred to their shoulders. If half of the lectures in a course were given with the self-instruction system, one instructor and



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The author decided to test the self-instruction technique with fundamental and presumably difficult topics, (1) first law of thermodynamic and (2) entropy. No statistical control evaluation was carried out. Informally it was learned that good students developed excellent comprehension of the subjects; and poor students performed about as expected.

The following material is exactly as presented to students except for portions which were omitted for editorial purposes.

FIRST LAW OF THERMODYNAMICS

THIS "LECTURE" IS PART of an experiment to see if it is possible to present effectively a portion of the material in EGR 234 in written rather than oral form. The topics discussed in this "lecture" are equivalent to those which would be given by the lecturer. The material represents the single most important principle in the course of thermodynamics. It is a building block for all science and engineering. It is essential that every student has a thorough mastery and understanding of the subject.

It is the object to present the underlying basis for the first law of thermodynamics also known as the law of conservation of energy. The concepts which are necessary to an understanding are as follows:

- Work and heat
- Properties and non-properties, point and path functions
- Tests for exact differentials in the form

a.
$$dz = Pdx + Qdy$$
 is exact if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial y}$
b. $\int_{C} dz = 0$

Note: Line integrals formed a part of prior lectures.

BACKGROUND MATERIAL

The concept of work and quasiequilibrium processes is important to the first law. (At this point a reading assignment is given.) The authors (of the book assigned) imply that a quasiequilibrium process is inherently frictionless. That is not true. A quasiequilibrium process can be carried out *with* as well as without friction. A body can be moved very slowly up a frictional plane. A piston moving in a heat engine could work against an internal friction generating device. Therefore, the term quasiequilibrium as used by the authors should always be accompanied by the adjective "frictionless". In general, thermodynamicists use the term "reversible process" as equivalent to the works "frictionless, quasiequilibrium process". I use the expression "frictional quasiequilibrium process" to describe a large group of operations which are not reversible.

WORK AND HEAT

THE CONCEPTS OF WORK and heat are essential to the development of the first law. Heat is a transitory phenomenon which represents the passage of "something" between bodies having different temperatures. Work is similarly another transitory phenomenon which results from mechanical, electrical, magnetic, or gravitational effects. After establishing the concept of "conservation" of the effects to be studied, work and heat will be viewed as energy in transitory passage from one body to another.

You should be clearly aware that d'W is a path function. Written in the form for a frictionless, quasiequilibrium process involving only work of expansion, we have

$$d'W = pdv \tag{1}$$

W is given by the area under a curve connecting A to B. (Two paths connecting the same endpoints are presented in a figure). As the areas under the curves are different, W is a path function and not a property of the system.

As we do not as yet have variables which give Q in the form YdX, it is not so easy to show that Q is a path function. However, if we resorted to experiment we should discover that heat transferred depends upon the path.

HISTORY OF THE FIRST LAW

A HISTORICAL APPROACH to the first law is helpful in an analysis of the fundamental under-pinnings. Before work could be quantitized and defined, the science of mechanics had to be developed. That required an understanding of motion or the inter-relationships of mass, velocity, and acceleration. Accurate clocks were essential to measurement, and no serious progress was possible prior to the development of accurate measurement of time. Sun dials were of no value, and water clocks offer a dubious means to satisfactory experimentation. The pendulum clock was one of

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the first instruments to permit reasonably precise measurement.

A study of falling bodies was one of the important first steps in modern mechanics with the recognition of the difference between velocity and acceleration. Out of the Gallileo experiments at the Leaning Tower of Pisa came the basis for the first form of the law of conservation of energy in the form:

$$v \equiv \sqrt{2gz} \tag{2}$$

where v is the velocity and z the distance of fall from rest. Equation (2) can be manipulated into a more general form

$$\frac{v^2}{2g} + z = \frac{v_1^2}{2g} + z_1$$
 (3)

Later Bernoulli modified Equation (3) for a flowing fluid and arrived at

$$\frac{p}{w} + \frac{v^2}{2g} + z = \frac{p_1}{w_1} + \frac{v_1^2}{2g} + z_1$$
(4)

where p is the pressure and w the specific weight. Although different language was employed, Equation (4) represented the first law of the conservation of mechanical energy.

Simultaneously with development of mechanics, experimental studies in heat were taking place. Just as the clock was basic to mechanics, the thermometer played a similar role in the science of heat. Fahrenheit produced the first reliable mercury thermometer in the early part of the eighteenth century just as the steam engine and industrial revolution began to emerge. The thermometer permitted studies to be made in a variety of areas including irreversible mixing of hot and cold bodies. From such experiments came equations of the form

$$m_1C_1(T_1-T) = m_2C_2(T-T_2)$$
 (5)

where T is the final temperature and T_1 and T_2 are the initial temperatures. Equation (5) represents a form of conservation of heat energy.

The idea of "heat energy" is foreign to modern thermodynamicists. As pointed out earlier, heat is a transitory phenomenon and therefore not something which is indestructible and conserved. Nevertheless, the caloric theory of heat evolved upon the basis that heat was an imponderable (no mass) fluid which could neither be created or destroyed.

Thus, at the beginning of the nineteenth century, there were two important theoretical principles which might be referred to as:

- Conservation of mechanical energy
- Conservation of heat energy

During the first half of the nineteenth century, a basis was laid for unifying the two principles which at that time appeared to be irreconcilable to many scientists.

The principle of conservation of energy was developed on slim evidence. A highly specilized case was extrapolated into a great law. The second law of thermodynamics was established before the first law in Carnot's remarkable writings, the numbering of the laws having nothing to do with their chronological development.

A number of people contributed to the development of the first law and to the demise of the caloric theory. The problem of friction entered heavily into the debate over the energy laws. It was known that mechanical actions such as canon boring produced large amounts of heat. It was reasoned that an "imponderable" fluid was not a reasonable hypothesis. Basically it was found that the frictional work as measured by the number of turns of a frictional device was proportional to the increased length of a mercury column in a small bore tube (as shown in a figure). The turns were considered to be proportional to work and the length of the mercury column (temperature) proportional to heat absorbed by the cooling water. Stated mathematically

$$d'W \cong J d'Q$$
 (6)

where the arrow indicates the work was converted to heat but the process (by the second law) could not be reversed. It was a one way frictional process in which mechanical energy disappeared and the cooling water became hotter.

It is interesting to note Equation (6) depends upon the straightness of the line. If the data had yielded a curve, say a parabola of the form $W = KT^2$ instead of W = KT, no one would have presumed there was a direct, linear relation between Q and W. If water with dye had been used as a thermometer, the results would have been drastic, near 4°C, where water has its maximum density. (A figure demonstrates the minimum length of the water column and the lack of linearity. The curve is double-valued). We can philosophize over what might have happended if the length of the mercury column had not been approximately proportional to what we call "temperature" today. Even with mercury, the expansion is not quite linear with temperature. There is no question but that the line shown would not be straight for any carefully done experiment. Nevertheless, a true spark of genius led scientists to assume Equation (6) was valid under all circumstances and to adjust the specific heat of various substances to force Equation₈(6) to remain valid. That has turned out to be a truly monumental "fudge factor" methodology.

On performing the reverse process of adding heat to a gas or vapor as in a steam engine, it was discovered that the work was not all changed into

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heat so that we may write

$$d'Q - d'W = dz$$
 (7)

where dz simply represents the difference between the heat added d'Q and the work of expansion d'W, both of which are inexact differentials. It would be quite natural to inquire about the properties of z. Is it an exact or inexact differential? If it is exact, it is a property of the system. If it is inexact, it is kin to Q and W in that it, is a path function.

If the process involved only reversible work of expansion, d'W could be written as

$$d'W = pdV \tag{8}$$

and (7) would become

$$dz = d'Q - pdV$$
 (9)

We do not know whether to write dz or d'z.

Before reading further, attempt to answer the following questions. Spend no more than five minutes on the questions, and then turn the page and read the commentary.

1) What is your opinion about the probability of dz being (Continued on page 140.)

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exact or inexact? A logical approach would revolve around the fact that both d'W and d'Q are inexact. Work with two inexact differentials like the following:

$$d'u = ydx + dy$$
$$d'v = dx + x^2dy$$

Substract one from the other to obtain some idea of the possibility of the difference leading to an exact or inexact differential.

2) Test Equation (9) for exactness (if you can) by using the relation $\partial p/\partial y = \partial Q/\partial x$ for an exact differential of the form dz = pdx + Qdy.

Work on these two questions before looking at the answers that follow.

ANSWERS

1) It is demonstrated that no linear combination of certain inexact differentials can be made exact and the following comment added:

It would be judicious (but not necessarily correct) to

think that dz in Equation (9) was inexact.

2) Equation (9) is restricted to a reversible, quasiequilibrium process. Therefore, any conclusions drawn from Equation (9) will apply only to those restrictions.

$$\mathbf{d}\mathbf{x} = \mathbf{d}'\mathbf{Q} - \mathbf{p}\mathbf{d}\mathbf{V} \tag{10}$$

We might say the coefficients could be derived from

$$dz = Md'Q + NdV$$
(11)

and apply the rule

$$\left(\frac{\partial \mathbf{M}}{\partial \mathbf{V}}\right)_{\mathbf{Q}} = \left(\frac{\partial \mathbf{N}}{\partial \mathbf{Q}}\right)_{\mathbf{V}}$$
(12)

Such a mathematical process would imply that Q is a state variable, i.e., Q=f(p,V). However, Q is a path function; and the operations used with properties like p,v,T cannot be used with Q and Q. For example, writing

$$dz = d'Q - d'W$$
(13)

it could be argued dz is exact because

$$\frac{\partial}{\partial \mathbf{w}}$$
 (1) = 0 and $\frac{\partial}{\partial \mathbf{Q}}$ (-1) = 0 (14)

Thus Equation (14) does not have the same status as

$$dz = dx - dy \tag{15}$$

where it is obvious z = x - y.

We conclude it is impossible to apply the $\partial p/\partial y=\partial Q/\partial x$ test.

INTERNAL-ENERGY

THERE IS NO PURELY mathematical method to demonstrate the exactness or inexactness of dz. It is necessary to resort to experiment, because the first law rests upon the basis of factual evidence. Place Equation (9) in integrated form

$$\int_{\mathbf{C}} d\mathbf{z} = \int_{\mathbf{C}} d' \mathbf{Q} - \int_{\mathbf{C}} d' \mathbf{W}$$
(16)

in which the circuit integral starts and stops at the same point. From the "half" law of thermodynamics^{*}, we assume there are two independent variables, say p, T, or p, V over which the circuit integral must be performed. In working with d'Q, there will be portions of the cycle when Q and W are positive or negative. Let us call the sum of all the heat transferred to the system Q_1 and all transferred from the system Q_2 where both quantities are considered as positive numbers. Then

$$\int_{\mathbf{C}} \mathbf{d}' \mathbf{Q} = \mathbf{Q}_1 - \mathbf{Q}_2 = \mathbf{Q}_{\text{net}} \tag{17}$$

where Q_{net} represents the net heat transferred to the system. With a similar situation for W, Equation (16) becomes

$$\int_{\mathbf{C}} \mathrm{d}\mathbf{z} = \mathbf{Q}_{\mathrm{net}} - \mathbf{W}_{\mathrm{net}}$$
 (18)

Overwhelming experimental evidence points to-

ward $Q_{net} = W_{net}$ and $\int_{C} dz = 0$. This is the essential point of The Law of Conservation of Energy: The Heart of the Subject.

The quantity z now possesses special interest as it *is a property of the system*. It deserves a name. Thermodynamicists generally call it internal energy. It represents the storage of heat or work as they are transferred to the system. It is frequently given the symbol U, I, or E. In accord with the textbook, we shall choose U and rewrite Equation (18) in the form

$$dU = d'Q - d'W$$
 (19)

The integrated form is

$$\Delta \mathbf{U} = \mathbf{U}_2 - \mathbf{U}_1 = \mathbf{Q} - \mathbf{W}$$
 (20)

For a frictionless, quasiequilibrium process in which there is only work of expansion

$$dU = d'Q - dpV$$
 (21)

Various examples involving the First Law follow.

ENTROPY

IN THERMODYNAMICS, it is frequently useful to define certain quantities which appeal to the senses like pressure and temperature or may be conceptualized as voltage, current, and resistance. In this exercise, a powerful but abstract

*In a previous lecture, students were introduced to the zeroth law and the "half law". The latter which lies between the zeroth and first laws states that Boyle (or Mar-

riot) and Charles showed that $\int c dv=0$ when p and T were changed and, therefore, that v=f(p,T).

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concept which arises out of application of an integrating factor to d'Q will be developed.

Our task is to seek a generalized displacement which yields heat transferred in a reversible process. Concepts necessary to this "lecture" are:

- First Law of Thermodynamics
- Integrating factors which turn inexact into exact differentials

GENERALIZED DISPLACEMENT FOR HEAT TRANSFERRED

In general, work is given by

$$d'W = \pm XdY$$
 (1)

where X is a generalized force and Y is a generalized displacement.

X and Y are both properties, and they possess exact differentials. W, is not a property, and its differential is inexact. We know from the study of differentials an inexact differential multiplied by an integrating factor yields an exact differential. From Equation 1 we can see (1/X) serves as an integrating factor for d'W because dY is exact. Various kinds of generalized quantities are shown in Table 1 along with a description of the type of work involved.

TABLE 1

		GEN- ERAL- IZED	
		DIS-	
	GENERALIZED	PLACE-	EQUA-
PROCESS	FORCE	MENT	TION
Fluid expansion	Pressure P	volume V	PdV
Elastic solid	Force F	length L	FdL
Electric	Voltage E	charge Q	—EdQ
Surface charge	Surface tension s	area A	—sdA
Heat transfer	Temperature T	unknown Y*	$\pm TdY$
Mass transfer	unknown X*	mass m	$\pm Xdm$
Potential energy	weight (—mg)	distance z	—mg dz

In Table 1, it is apparent there are two missing quantities marked by * which have not been previously encountered in non-thermodynamics courses. If we accept Equation (1) as valid for expressing work done in general terms, there must be a generalized displacement Y for transfer of heat. We assume temperature is the generalized force which causes heat to flow. That is not the only assumption which could be made. From elementary kinetic theory, we know that temperature is proportional to the kinetic energy of the molecules. We might assume the generalized force was the square of the velocity or the velocity of the molecules. Such assumptions would lead to a somewhat different if equivalent conclusion. To simplify the problem, we shall assume T is *the* generalized force for heat transfer.

When a crystal dissolves in a liquid, there is necessarily a transfer of energy. In fact, any time there is a transfer of mass from one phase to another, there will be an energy transfer. Thus if a differential mass, dm, passed from one phase to a second, there would be an energy transfer of the type Xdm.

Previous knowledge from non-thermodynamic sources does not provide us with definitions of the generalized quantities needed for heat and mass transfer. We shall seek an answer for heat transfer at this time but will postpone seeking a solution for mass transfer until we study multicomponent systems and define new thermodynamic functions such as Gibbs free energy and the work function.

There is no general formula for finding new thermodynamic functions. There is some controversy among thermodynamicists about rigorous methods for defining our Y in the equation

$$d'Q = \pm TdY$$
 (2)

The method which will be followed is not rigorous. It will apply only to a specialized case of an ideal gas in a frictionless, quasiequilibrium, non-flow process. Thus the method applies to a highly-restricted, non-existent situation. Nevertheless, the method will produce the same answer which can be obtained from more rigorous (and controversial) methods.

We shall give the name of entropy to our generalized displacement Y before we find it. Our method will involve integrating factors. The process we are about to consider was commented on by Clausius in an appendix of his book on the mechanical theory of heat. He recognized the lack of rigor at that time (close to 1850).

Concepts that are needed to understand this development include (1) integrating factors and (2) internal energy of an ideal gas.

QUESTIONS Take the function

$$= \mathbf{x}\mathbf{y} + \mathbf{x}^2 \tag{3}$$

and find its differential. Then divide the differential by x to obtain a new differential du. Show du is inexact by

z

^{*}The generalized forces are generally, but not always, intensive quantities. Pressure, surface tension, and electric potential are examples of intensive variables. The generalized displacements are extensive variables in that they are dependent on length, area, volume, mass, or charge. Gravitational force mg is an example of a force which is extensive rather than intensive.

means of the $\partial P/\partial y = \partial Q/\partial x$ test. Then assume there is an integrating factor $\lambda(x)$ which is a function of x alone and find λ . Attempt to find another integrating factor which is a function of y alone.

Answer

The differential

$$du = (z + y/x)dx + dy$$
 (4)

is shown to be exact. Assuming λ is an integrating factor, it is demonstrated that $\lambda(x) = x$ and that no integrating factor which is a function of y exists.

INTERNAL ENERGY AND ENTROPY

A N IDEAL GAS CONSISTS of point masses having no gravitational, electrical, or magnetic interaction. Therefore an ideal gas can only have translational kinetic energy. From kinetic theory, it can be shown that

$$PV = \frac{2}{3} N \left(\frac{1}{2} Mu^2 \right)$$
 (5)

where u is the square root of the average velocity of the molecules. Thus the molar kinetic energy is directly related to the ideal gas temperature. In fact

$$RT = (2/3) \text{ (molar K.E.)}.$$
 (6)

In deriving (5) all but translational kinetic energy has been neglected. The molecules are assumed to be incapable of having rotational, vibrational, gravitational, magnetic or electrical energy. Therefore, the internal energy of an ideal gas depends only on its temperature. We can write

$$\left(\frac{\partial \mathbf{u}}{\partial \mathbf{v}}\right)_{\mathrm{T}} = 0; \left(\frac{\partial \mathbf{u}}{\partial \mathbf{p}}\right)_{\mathrm{T}} = 0 \tag{7}$$

which states that

$$a = u(T) \tag{8}$$

The first law for a non-flow, frictionless, quasiequilibrium process can be written as

$$d'Q = dU + pdV$$
 (9)

The generalized specific heat is defined by

$$d'Q = NCdT$$
(10)

where C is the molar specific heat. For a constant volume process, combining (9) and (10) yields

$$d'Q = NC_v dT = dU$$
(11)

Integration gives

$$\mathbf{Q} = \Delta \mathbf{U} = \mathbf{N} \mathbf{C}_{\mathbf{v}} \Delta \mathbf{T} = \mathbf{N} \mathbf{C}_{\mathbf{v}} \mathbf{T} + \mathbf{U}_{\mathbf{o}}$$
(12)

At this point, a number of examples involving specific heats and ideal gases are presented.

SEARCHING FOR ENTROPY

WE ARE LOOKING for the generalized displacement in the equation

$$d'Q = \pm TdY$$
 (13)

Basically we want to find a function which is an integrating factor of d'Q. We do not know if T itself will turn out to be the function we are looking for. However, we shall experiment and see what results.

You are to do the following:

- Write the first law for a general non-flow process and solve for d'Q.
- Assume that work is restricted to a frictionless, quasiequilibrium expansion process.
- Assume the gas is ideal and replace dU by NC_vdT.
- Eliminate P by means of PV = NRT.
- Place the differential of Q in the form

$$\mathbf{d}'\mathbf{Q} = \mathbf{A}\mathbf{d}\mathbf{T} + \mathbf{B}\,\mathbf{d}\mathbf{v} \tag{14}$$

• Find an integrating factor which is a function of T.

Do this work before continuing. The answer is on the next sheet.

NOTE: Basically the student develops the concept of entropy in this example by himself. By providing him the appropriate catalyst, a situation is produced in which entropy naturally falls out of the process of finding an integrating factor for d'Q. To the equation

$$d'Q = NC_v dT + P dV = NC_v dT + \frac{NRT}{v} dV \quad (15)$$

an integrating factor $\lambda(T)$ is applied. Then applying the rule for exactness

$$\frac{\partial}{\partial \mathbf{v}} \left(\lambda \mathbf{N} \mathbf{C}_{\mathbf{v}} \right)_{\mathrm{T}} = \mathbf{0} = \frac{\partial}{\partial \mathbf{T}} \left(\frac{\lambda \mathbf{N} \mathbf{R} \mathbf{T}}{\mathbf{v}} \right)_{\mathbf{v}}$$
(16)

The product λT must be constant, and $\lambda=$ 1/T. Therefore,

$$\frac{d'Q}{T} = dY = NC_{v} \frac{dT}{T} + NR \frac{dV}{V}$$
(17)

It can be seen that dY is exact. We shall call it entropy and use the letter S.

SIGNIFICANCE OF ENTROPY

I T HAS BEEN SHOWN that 1/T is an integrating factor for d'Q. Therefore, the quantity dS = d'Q/T must be an exact differential. It fulfills our search for a dY to satisfy the relation d'Q = \pm XdY. The question of sign has been settled; a plus is used. Further as Q is proportional to mass (lbs. or lb-moles), it can be written as Q = Nq. Then

$$dS = \frac{d'Q}{T} = \frac{Nd'q}{T} = Nds$$
 (18)

and we see entropy is an extensive variable which is proportional to mass. (In the accompanying figure (Three graphs show x(generalized force) vs Y (generalized displacement), P vs V, T vs S)) we see that the heat transferred is given by the area under the T-S curve in accord with

$$Q = \int T dS$$
 (19)

There is a direct correlation between the area

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under the X-Y, P-V, and T-S curves. Each area corresponds to some kind of work.

There are an infinite number of integrating factors for d'Q. Each one would lead to an exact differential with a new X and Y. As an example, v^{k-1} where $k = c_p/c_v$ is an integrating factor for the case of an ideal gas. Thus

$$v^{k-1} d' Q = dY \tag{20}$$

or

$$d'Q = v^{1-k} dY$$
 (21)

There is no special value attached to this XY combination. Further, 1/T has general application to all processes, although the present development has been restricted to ideal gases. There may be some useful integrating factors other than 1/T, but we shall not search for them.

WHAT DO WE HAVE?

WE HAVE FOUND a function, entropy, which is a property of the system. We shall be able to show that entropy plays a very useful role in thermodynamics. But that comes later. We now ask, what do we have? Usually when a physical quantity is defined, there is some material concept involved which appeals to our senses. In the case of the other generalized displacements in Table 1, it is simple to visualize distance, area, and volume. The concept of electric charge is not difficult to imagine. In the case of magnetic quantities, we have to use a little more imagination. The generalized forces can be understood in terms of measuring instruments. Everyone thinks of a mercury column in glass as signifying tempera-

ChD book reviews

INTRODUCTION TO PROCESS ECONOMICS

F. A. Holland, F. A. Watson and J. K. Wilkinson, John Wiley & Sons, 290 pages.

Reviewed by James H. Black, University of Alabama, University, Alabama

This book is intended as an undergraduate text for the process engineering disciplines, such as chemical, metallurgical, and mineral engineering. It would also give an excellent introduction to process economics for the practicing engineer or serve him as an excellent reference book. It is, as the title states, a book covering process economics; but some wider aspects, such as some of the management sciences, are also presented. If we went deeply into measurement in a sophisticated manner, we would find the simple concepts must be examined with much more care. We would discover temperature is difficult to define precisely. In the end, we would be asking how can we measure the variables and how can we assure reproducibility and comparative accuracy.

Entropy suffers in comparison with other physical quantities, because we have no *entropometer* from which we can read values of entropy. However, if we can show how to measure entropy, that should enable us to form an intellectual if not a physical notion of what it is.

Returning to the equation for an ideal gas, we have on a unit mass basis

$$ds = d'q/T = C_v \frac{dT}{T} + R \frac{dv}{V}$$
 (22)

This equation can be integrated to give

S

or

$$= C_v \ln T + R \ln v + s_o \qquad (23)$$

$$\Delta \mathbf{s} = \mathbf{C}_{\mathbf{v}} \ln \mathbf{T} / \mathbf{T}_{\mathbf{o}} + \mathbf{R} \ln \mathbf{v} / \mathbf{v}_{\mathbf{o}}$$
(24)

These equations tell us how to measure s or \triangle s for an ideal gas. However, we must make measurements of T and v and then *calculate* s. We could develop an instrument which would translate (24) into something which could be visibly seen. Such an instrument would not be very useful.

We must be satisfied with a mathematical and intellectural rather than a familiar physical concept of entropy. \Box

The organization of the book is excellent. It is divided, logically, into two parts: the elements of profitability assessment and the elements of decision making. Thus, the reader first gets a complete treatment of time value of money calculations, followed by chapters on profitability estimates, uncertainties in profitability estimates, capital cost estimates, and manufacturing cost estimates. The second part of the book covers such decision making tools and techniques as statistical analysis, curve fitting and trend analysis, linear programming, financial and cost accounting, price and cost trends, value engineering, marketing, and some material on risk and insurance.

This is a good book, well worth the cost. It would be of particular interest and value to those who found use in the recent series of articles on engineering economics, by the same authors, in *Chemical Engineering* magazine.