ChE class and home problems

The object of this column is to enhance our readers' collection of interesting and novel problems in Chemical Engineering. Problems of the type than can be used to motivate the student by presenting a particular principle in class or in a new light or that can be assigned as a novel home problem are requested as well as those that are more traditional in nature that elucidate difficult concepts. Please submit them to Professor H. Scott Fogler, ChE Department, University of Michigan, Ann Arbor, MI 48109.

SOLUTION: MIRROR FOG PROBLEM

R. L. KABEL
Pennsylvania State University
University Park, PA 16802

Editor's Note: Professor Kabel presented the "Mirror Fog Problem" in the Fall 1979 issue of CEE. *We extended an invitation for student solutions to this problem at the time of publication and would like to congratulate Mauricio Fuentes of Ecole Polytechnique, Montreal, Canada, who submitted the winning entry and by so doing has won a year's subscription to* CEE. *Professor Kabel graded the responses and, in his words, Mr. Fuentes' entry was both "correct and excellently done." The following is Professor Kabel's solution to the problem.*

Derivation of equations:

Use a microscopic model because momentum and energy equations are not required due to isothermality and no bulk flow.

Mass balance equation:

$$\frac{\partial C_A}{\partial t} + v_x\frac{\partial C_A}{\partial x} + v_y\frac{\partial C_A}{\partial y} + v_z\frac{\partial C_A}{\partial z}$$

$$= D_{AB}\left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right] + R_A$$

Since $v_x = v_y = v_z = 0$, $C_A \neq f(x,z)$ and there is no generation in the vapor space this equation becomes.

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

which shows that the concentration at any point changes with time because of diffusion in the y-direction.

Initial condition: At $t = 0$, $C_A = C_{A,sat}$ at all y

Boundary conditions: At $y = 0$, $C_A = C_{A,room}$ at all time
At $y = Y$, $C_A = C_{A,sat}$ at all time

where Y is the location of front edge of the remaining fog on the mirror. Note however that Y varies with time going from 0 when $t = 0$ to 0.3 m when $t = t_f$.

If an analytical solution of the equation is to be sought this second boundary condition should be respecified. If the solution is to be numerical, then one merely needs to keep track of Y(t). The end of the calculation is $t = t_f$ when $Y = Y_{max} = 0.3$ m. Y(t) can be obtained by equating the total amount evaporated to the integrated mass flux into the room neglecting the slight accumulation of water vapor in the enlarging vapor space.

Let M_{Ao} be the initial total mass of water condensed, then

$$\text{Amount evaporated} = M_{Ao}\frac{Y}{0.3} = M_{Ao}\frac{Y}{Y_{max}}$$

$$\text{Amount transferred to room} = \int_0^t D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0} S\,dt$$

where S = ZX

$$Y = \frac{Y_{max}}{M_{Ao}}\int_0^t ZX\,D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0} dt$$

$$Y = \frac{XZY_{max}D_{AB}}{M_{Ao}}\int_0^t \left.\frac{\partial C_A}{\partial y}\right|_{y=0} dt$$

The above is an adequate answer to the exam question. A very simple analytical solution can be obtained as follows. We can say that the flux at $y = Y$ is equal to the amount of moisture evaporated there per unit time. Then the amount of moisture can be related to the rate at which the boundary moves. Thus, if R = thickness of liquid film,

$$D_{AB}\left.\frac{dC_A}{dy}\right|_{y=Y} = \frac{\text{g } H_2O \text{ evap.}}{\text{area} \cdot \text{time}} =$$

$$\frac{\rho_{H_2O}\,ZR}{ZX}\;\frac{dY}{dt}\;\frac{\text{g } H_2O \text{ evap/time}}{\text{area of transfer}}$$

$$D_{AB}\left.\frac{dC_A}{dy}\right|_{y=Y} = \frac{\rho_{H_2O}\,R}{X}\;\frac{dY}{dt}$$

If we assume that a steady state concentration profile is established rapidly and maintained (shown by dynamic analysis to be an excellent assumption) we get

$$\left.\frac{dC_A}{dy}\right|_{y=Y} = \frac{C_{A,sat} - C_{A,room}}{Y}$$

A further simplification is obtained by taking $C_{A,room} = 0$. Let $C_{A,sat} = C_{AS}$.

$$D_{AB}\frac{C_{AS}}{Y} = \frac{\rho_{H_2O} R}{X}\frac{dY}{dt}$$

$$\int_0^{t_f}\frac{D_{AB}C_{SA}X}{\rho_{H_2O}R}dt = \int_0^{t_f} B\,dt = \int_0^{Y_{max}} Y\,dY$$

$$B\,t_f = 0.5Y^2\Big|_0^{Y_{max}} = 0.5\,Y^2_{max}$$

and

$$t_f = 0.5\,Y^2_{max}/B$$

for the mirror fog problem

$$\begin{aligned}
\rho_{H_2O} &= 10^6\ g\cdot m^{-3}\\
R &= 1.3\times10^{-5}\ m\ \text{(obtained from experiment)}\\
D_{AB} &= 2.47\times10^{-5}\ m^2\cdot s^{-1}\\
C_{AS} &= 17.3\ g\cdot m^{-3}\\
Y_{max} &= 0.3\ m\\
X &= 4\times10^{-3}\ m\\
t_f &= 3.5\times10^5\ s = 96\ m = 4\ \text{days}
\end{aligned}$$

This result appears high by about a factor of 4. There are several explanations and we have calculated for different assumptions. Probably the experimental circumstances (e.g. leakage around edges, etc.) do not meet the idealizations of the model. □

IN THE "HEAT" OF THE NIGHT

R. J. GORDON
University of Florida
Gainesville, FL 32611

YOU ARE SPENDING the evening in a small town on your way home for the holidays. At about 11:00 p.m. the local sheriff calls you and asks for your help. He knows from the desk clerk that you are a chemical engineer, and naturally assumes you have some knowledge of forensic chemistry.

It seems that the body of John Lurie, a local car dealer, had been found somewhat earlier in a wooded area just outside of town. The local coroner had gone fishing and there was no one else to estimate the time of death. John Lurie had been known to deal in "hot" cars and was thought to be going to the police to confess and name his four accomplices, Gus Nusselt, Bill Gurney, Ed Reynolds, and Bob Prandtl. Nusselt had been known to be out of town until 11:00 A.M. that morning, Gurney had a solid alibi from 1:00 p.m. on, Reynolds was with his girlfriend until about 6:00 A.M., when he left to go fishing, and Prandtl was in jail the night before for drunkenness, and was not released until about 8:00 A.M.

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When you finally get to the body it is about 12:00 p.m. (midnight). You measure a rectal temperature of 80°F, and an air temperature of 70°F. The air temperature has been about 70°F all day.

Luckily, you brought your Perry's along. Recognizing that the human body is mostly water, 1) calculate the latest possible time the murder could have occurred and 2) state the possible suspect.

NOTE: For practical purposes, John Lurie can be assumed to be shaped like a rectangular slab. He is 10 inches thick from his back to his breastbone. Body temperature is 98.6°F. Rectal temperature is equivalent to core or centerline temperature.

For comparison, a pathology formula sometimes used to estimate the time of death is*

$$\text{No. of hrs. since death} = \frac{98.6 - \text{rectal temperature}}{1.5}$$

ONE-DIMENSIONAL SOLUTION

We will use the Gurney-Lurie charts, Perry's 4th Ed., p. 10-6, 10-7. To calculate the latest time the murder could have occurred, assume maximum rate of cooling, or in other words that the surface of the body is at 70°F (same as saying $h = \infty$ or $m = 0$). We also neglect radiative losses since the body was found in a "heavily wooded" area.

Then, if we assume infinite width and depth,

$$Y = \frac{T_s - T}{T_s - T_o} = \frac{70-80}{70-98.6} = 0.35$$

From graph, for $n = 0$

$$X = \frac{\alpha t}{(x_1^2)} \cong 0.54$$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.36}{62.4\times1.0} = 0.0058\ ft^2/hr$$

$$x_1 = \frac{5\ \text{inches}}{12} = 0.42\ ft.$$

$$t \cong \frac{0.54\times(0.42)^2}{0.0058} = 16.4\ \text{hours or}\ 16\ \text{hours, 24 minutes}$$

1) Murder had to occur before 7:40 A.M.

*"Medical Jurisprudence and Toxicology," Glaister and Rentoul, 12th Ed., Livingston Ltd., Edinburgh, 1966, p. 110.

2) Possible Suspects: Gurney, Reynolds

From the pathology formula:

$$\frac{98.6 - 80}{1.5} = 12.4 \text{ hours}$$

(or murder occurred at 12 noon)

This formula, however, takes no account of changes in room temperature, or body thickness, and in fact is known to underpredict the time of death except for the first few hours. From our superior knowledge of heat transfer, we have eliminated Prandtl and Nusselt as suspects. □

ACKNOWLEDGMENT:

Helpful comments were provided by Professor J. H. Hand, University of Michigan.

Editor's Note: Professor Gordon's purpose in his solution to the foregoing problem, "In The Heat of the Night," was to illustrate the use of the Gurney-Lurie charts assuming a simple one-dimensional model. Professor Fogler, CEE Problem Section Editor, asked his student, Alan Basio, to comment on this simplified solution. Mr. Basio's reply follows.

TWO-DIMENSIONAL HEAT TRANSPORT

ALAN BASIO
University of Michigan
Ann Arbor, MI 48109

It was previously assumed that Lurie, the dead man, is an infinite slab. From this assumption, the time is 16.4 hours since Lurie was killed.

I used Newman's Rule and assumed Lurie is an infinitely long slab with a finite width and depth. Newman's Rule in this situation is the following:

$$Y = Y_x Y_y = \frac{T_s - T}{T_s - T_o} = 0.350 \qquad (1)$$

Let Lurie be 10″ deep, as previously specified, and 1.3 feet wide. Use the same values as before for Y and α. There are now two values of X to be found on the Gurney-Lurie Charts:

$X_x = \alpha t/(5/12)^2$ and $X_y = \alpha t/(1.35/2)^2$. The time must be the same in both X_x and Y_y, and the product $Y_xY_y = 0.350$.

Criteria for solution: (1) $Y_xY_y = 0.35$

$$(2) \quad \frac{X_x(x)}{\alpha} = \frac{X_y(y)}{\alpha} = t$$

Results: By trial and error the times are found to be within 2.7% of each other.

$$Y_x = 0.420 \qquad Y_y = 0.833$$
$$X_x = 0.45 \qquad X_y = 0.18$$
$$Y_xY_y = (0.42)\ (0.833) = 0.350$$

$$t = \frac{(0.45)\ (5/12)^2}{0.0058} = 13.47 \text{ hrs.}$$

$$t = \frac{0.18\ (0.65)^2}{0.0058} = 13.10 \text{ hrs.}$$

$$\frac{13.47 - 13.10}{13.47} \times 100 = 2.7\% \text{ difference}$$

If the width of Lurie is 1.3 ft., he died 13.3 hrs. ago, not 16.4 hrs.

The width of Lurie is important. If Lurie is 2.6 ft. wide, for example, he dies 16.3 hours earlier. In other words, the infinite slab assumption improves when Lurie is assumed over 2.0 feet wide, approaching an answer of t = 16.4 hrs. □

BOOK REVIEW: Reactor Design

Continued from page 24.

The book is an excellent work. The author has covered a very large area of relatively difficult material in a highly readable fashion and has provided enough detail so that the reader is able to come to grips with the realities of chemical reactor design. It is accurate and relatively complete. There is a considerable amount of specialized knowledge, based upon over 1000 references, augmented by the author's own considerable experience. In many areas, it stands at the edge of chemical reactor design knowledge that is in the public domain. As such it will continue to be a valuable reference work for many years to come.

Its only major shortcoming is insufficient illustrations and a lack of exercises or problems for the student. The fourteen case studies of Volume II serve to illustrate design principles but only cover a fraction of the material in Volume I. In order to serve as a text for a graduate course in chemical reactor design, it would have to be supplemented by problems developed to reinforce specific points and others which would require the student to integrate these ideas into a chemical reactor design. The latter would be an undertaking of the order of a term paper.

These two volumes are a major contribution to the chemical engineering literature. They belong in the library of every chemical engineer who is concerned with research, development, design, or, in many cases, operation of chemical reactors or conversion processes. □