PROCESS CONTROL EXPERIMENT: THE TOILET TANK

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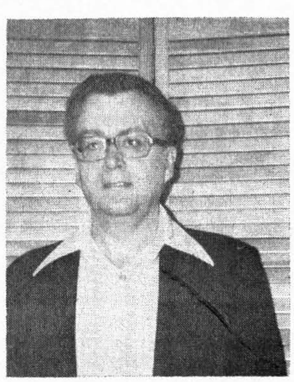
THE LEVEL CONTROL mechanism in the home toilet tank is a nonlinear, proportional control system that illustrates various process control concepts. It can also serve as an introduction to data acquisition, process analysis, and model development. This simple experiment can be developed as an example problem, a classroom demonstration, or a laboratory exercise.

DESCRIPTION

A TYPICAL TOILET TANK is shown in Figure 1. The level control system regulates the tank water level C at the desired steady state C_s by manipulating the inlet water rate M to compensate for the disturbance (the flushing rate U). The control logic is given in the flow diagram of Figure 2A.

An actual toilet tank can be used for the laboratory exercise if a flowmeter is installed in the feed line and a measuring scale is fastened to the inside of the tank wall. For classroom demonstrations, the level mechanism can be installed in a clear plastic tank (0.5 by 0.15 by 0.4 meters high). A quick-opening valve can provide the necessary flush for the plastic tank.



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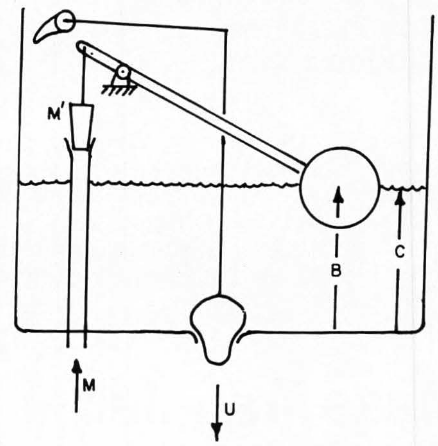

FIGURE 1. A typical tank.

Typical exercises involve the development of analytical models, determination of model parameters, and measurement of the closed-loop response to a flushing disturbance. Two suitable process models are presented in the following sections.

NONLINEAR MODEL

THE EQUATIONS FOR EACH of the elements of Figure 2A can be obtained as follows:

Tank: A mass balance on the tank gives

$$A \frac{dC}{dt} = M - U \tag{1}$$

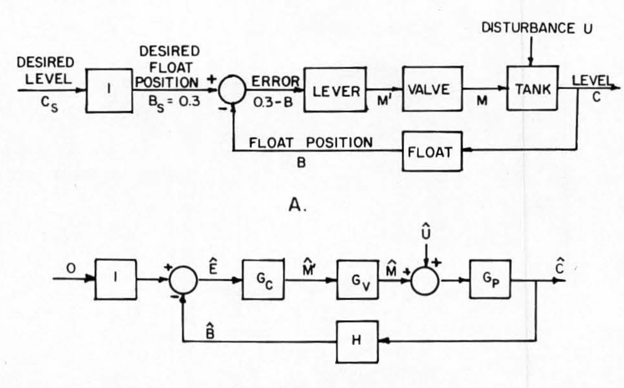

FIGURE 2. Information flow diagrams: A. Nonlinear logic, B. Linear block diagram.

where A is the tank cross-sectional area. A typical value for A is suggested by the plastic tank dimensions given above so that

Tank: A mass balance on the tank gives

$$A = (0.5)(0.15) = 0.075<m>^2 \qquad (2)$$

Float: The float center height B is assumed to correspond to the water level so that the equation for the measuring element is simply

$$B = C \qquad (3)$$

Alternately, it could be assumed that the float is characterized by a first or second order model. While this would lead to an interesting higher-order process model, it might be difficult for students to estimate the parameters in a higher-order model.

Valve: It is assumed that the valve flow rate M can be related to the valve position M′ by the equation

$$M = \alpha (M')^{0.5} \qquad (4)$$

Many toilet valves have an adjustable ring so that the valve coefficient can be changed. A typical value for the coefficient α is $0.0021<m>^{2.5}/s$. The experimental determination of a suitable valve equation and coefficient can provide an interesting short study.

Lever: A reasonable controller equation relating the valve position M′ to the error $(C_s — B)$ can be written as

$$M' = K(C_s — B) \qquad (5)$$

where the controller gain K is given by the lever ratio. A typical value for the desired steady-state level C_s is assumed to be 0.3 meters. If the lever is assumed to be 0.40 meters long (to the float center) and the pivot is 0.04 meters from the valve end, then the controller gain is given as

$$K = 0.04/0.40 = 0.1 \qquad (6)$$

These can be combined to give the nonlinear model as

$$A\frac{dC}{dt} = \alpha[K(C_s — C)]^{0.5} — U \qquad (7)$$

LINEARIZED MODEL

As NOTED ABOVE, the steady state is selected as the filled tank (a nonflow condition). For any variable X, the perturbation from the steady state is defined as $\hat{X} = X — X_s$. In terms of such perturbation variables, the equations for three of the elements are

$$\text{Tank:} \quad A\frac{d\hat{C}}{dt} = \hat{M} — \hat{U} \qquad (8)$$

$$\text{Float:} \quad \hat{E} = —\hat{B} = —\hat{C} \qquad (9)$$

$$\text{Lever:} \quad \hat{M}' = K\hat{E} \qquad (10)$$

The nonlinear valve equation can be linearized around the steady state by the truncated Taylor series approach to give

$$\hat{M} \cong \left(\frac{dM}{dM'}\right)_s \hat{M}' = \beta\hat{M}' \qquad (11)$$

where β should be the slope of the valve curve at the origin. As can be seen in Figure 3, this would

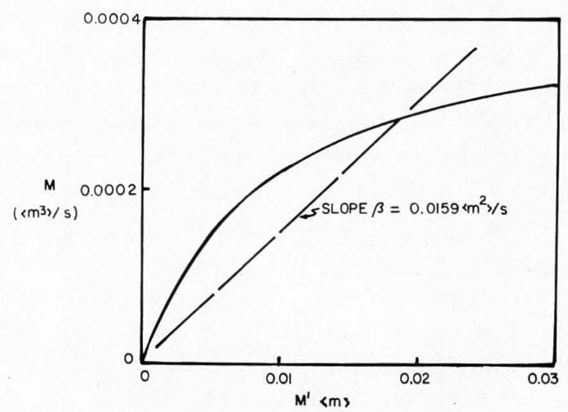

FIGURE 3. Linearization of valve response.

give a value of β that would be much too high away from the origin. A line with slope $\beta = 0.0159 <m^2/s>$ is arbitrarily selected to approximate the value behavior over the region of interest. This illustrates how experimental data can be used to improve on the classical steady-state linearization.

The linear model corresponds to the block diagram of Figure 2B, where $G_c = K$, $G_v = \beta$, $H = 1$, $G_p = 1/AD$, and $D = d/dt$. These can be combined to give the linear model as

$$(TD + 1)\hat{C} = —\alpha\hat{U} \qquad (12)$$

where $T = A/\beta K$ and $\gamma = 1/\beta K$.

DISTURBANCE INPUT

IT IS ASSUMED THAT the flush, given by $\int U dt$, empties the tank. Then $\int U dt = (0.5)(0.15)(0.3) = 0.0225 <m>^3$. Various approximations can be suggested for the disturbance U. A suitable function might be the displaced cosinusoid with a period of 10 seconds

$$U = \begin{cases} -0.00225 \cos 0.2\pi t + 0.00225 & 0 \leq t \leq 10 \text{ sec} \\ 0 & t > 10 \text{ sec} \end{cases} \quad (13$$

Since the flush occurs quickly compared to the filling time, it might be reasonable to approximate the disturbance as an impulse of 0.0225<m^3> occurring at t = 0.

MODEL RESPONSES

THE RESPONSE FOR VARIOUS model and disturbance forms can be compared with experimental responses to show the significance of the approximations made. Three model responses will be given here:

a. **Nonlinear Model With Cosinusoid Disturbance.** The response for this case was obtained numerically for the parameter values assumed above. This is given by the curves NL-C in Figure 4.

b. **Nonlinear Model With Impulse Disturbance.** If the flush is an impulse at t = 0, then the solution for t>0 can be obtained by assuming that U = 0 and C = 0 at the instant t = 0^+. Equation 7 can then be solved by the separation of variables technique to give

$$(C_s - C)^{0.5}/2A)t + C_1 \qquad (14)$$

For the assumed parameter values, the integration constant $C_1 = (0.3)^{0.5}$. The response curves for this case are given as NL—I in Figure 4.

c. **Linear Model With Impulse Disturbance.** If it is assumed that $\hat{U}(0^+) = 0$ and $\hat{C}(0^+) = -0.3$, then the solution to Equation 12 is

$$\hat{C} = -0.3\, e^{-t/T} \qquad (15)$$

The responses for this case are shown in Figure 4 as the curves LI.

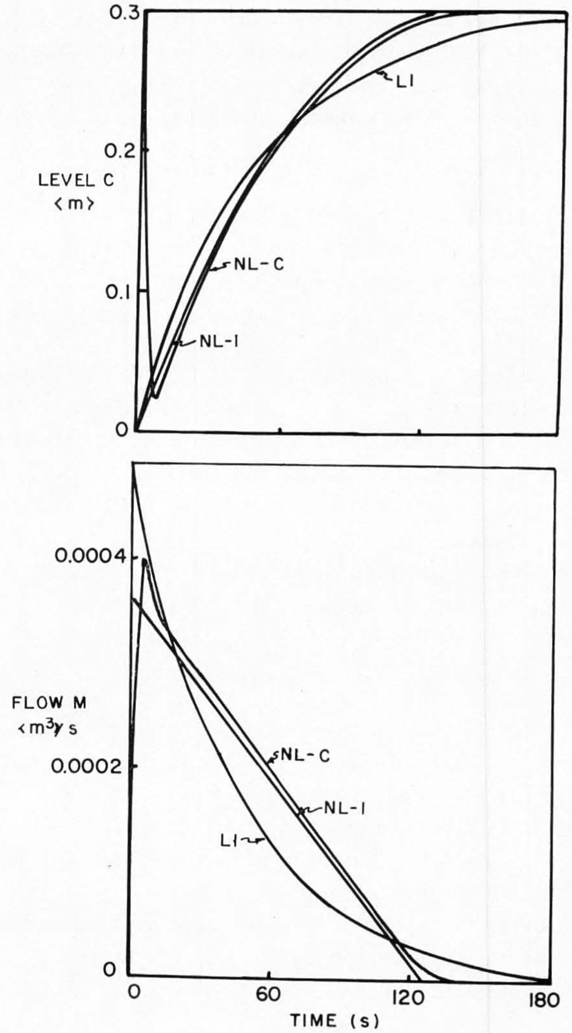

FIGURE 4. Level and flow responses (LI = linear model with impulse disturbance, NL—I = nonlinear model with impulse disturbance, NL-C = nonlinear model with cosinusoid disturbance).

OTHER FEATURES

THE TOILET TANK CAN be used to illustrate other process control concepts. Some of these are:

a. **Offset.** The leaking toilet provides one of the simplest demonstrations of offset. If the outflow at steady-state is not zero, then the inflow at steady state is not zero. Since the flow into the tank is a fixed function of C, then the steady-state level must decrease if the toilet tank is leaking.

b. **Measurement Error.** The water-logged float implies that the float center-line does not correspond to the level. At steady state, the level must increase in order to close the feed valve.

c. **Setpoint Changes.** The float end of the level can be bent to represent setpoint changes.

d. **Controller Gain.** Most toilet tank controls have some provision for changing the loop gain. In some cases, this is accomplished by changing the effective lever ratio. In others, an adjustable ring is used to change the valve gain.

CONCLUSION

The toilet tank, as either a classroom problem or experiment, provides a simple introduction to process control. □