ChE laboratory

THE OSCILLATING SINK

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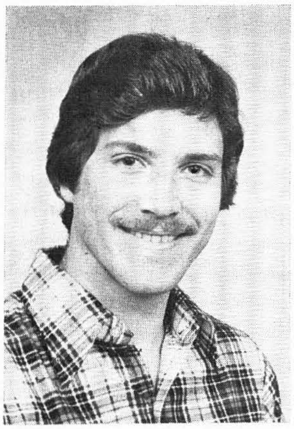
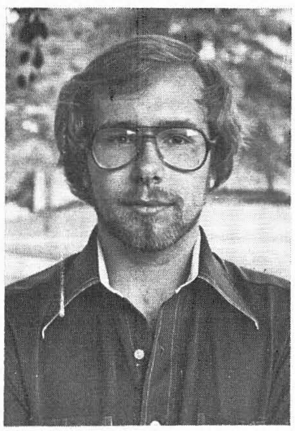
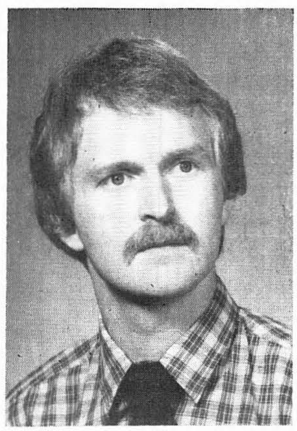
IN THE SUMMER OF 1978, while cleaning glassware in Room 3 of Fenske Laboratory, one of us (THL) observed that the level of water in the laboratory sink would periodically drop to a near empty level and then slowly climb back to a moderate height—all in the presence of what appeared to be a steady inflow of water from the faucet. These curious events, although memorable, were not accorded any particular significance until the following winter. At that time, THL attended a graduate Process Dynamics Course in which the subjects of Autonomous Oscillations and Bifurcations were presented in some detail. In this course, it was emphasized that systems with steady inputs (autonomous systems) could produce oscillatory outputs under appropriate conditions, and various analytical and computer exercises, mainly involving continuous stirred tank reactors, were worked out while a series of papers from van Heerden [1] and Bilous and Amundson [2] through Uppal, Ray and Poore [3] and Schmitz, Graziani and Hudson [4] were studied. This experience provided a framework for interpretation of the strange events observed in Room 3. In fact, THL may have been the only student in that class who regarded autonomous oscillations as other than an academic hoax, for although experimental demonstrations had been alluded to in lectures, (perhaps) only he had actually observed them.

To further satisfy his curiosity, as well as the term paper requirement for his Process Dynamics course, THL conducted primitive experiments on the sink in Room 3. He measured level as a function of time with a meter stick and stop watch at various faucet flow rates which were measured with a graduated cylinder. He observed steady levels at low flow rates, oscillatory levels at intermediate flow rates and a return to steady levels at high flow rates. The oscillatory states were not truly periodic as the amplitudes and periods appeared to be randomly distributed, but over a fairly narrow range. THL was inclined to describe the oscillations as "chaotic" in light of the paper by Schmitz, Graziani and Hudson [4]. The source of the strange oscillations was not obvious, but the U-shaped trap in the drain line at the bottom of the sink was highly suspect.

To gain better insight into the oscillatory phenomena, a transparent model of a sink and drain was constructed, and a more exhaustive and







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accurate set of data was obtained by another of us (DJJ). We were amply rewarded for this effort as additional curious phenomena attendant to the oscillations were soon uncovered.

The apparatus, procedures and many of the results of these experiments are described in the following sections. Our main purpose in communicating this experience is to suggest a simple system, adaptable to a classroom or laboratory environment, which demonstrates some basic dynamic phenomena (instability, bifurcation, and oscillation).

APPARATUS AND PROCEDURES

A schematic of the experimental apparatus is shown in Fig. 1. A small, translucent, plastic holding tank was fitted with a drain which consisted of an elbow and two short pieces of metal tubing (~ .375″ i.d.) on the inside of the tank,

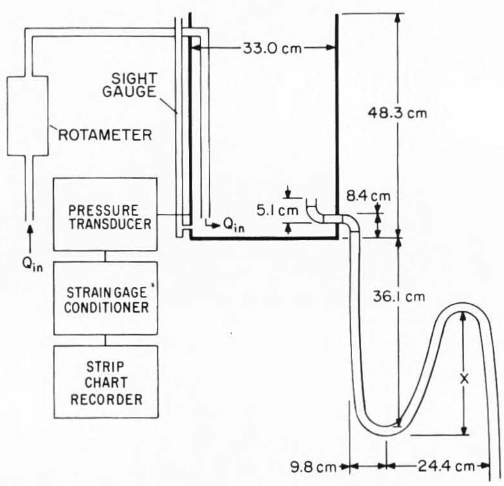

FIGURE 1. Experimental Apparatus.

with a length of flexible, transparent (Tygon) tubing (~ .375″ i.d.) on the outside. A bend of variable height (X) was maintained in the exit line with several clamps, and the line drained directly into a bench top sink. For most of the experiments, the drain inlet was oriented perpendicular to the tank bottom and open upward. An electronic pressure transducer was connected to the side of the tank below the level of the drain by means of a short piece of plastic tubing, and the instantaneous pressure was continuously re-

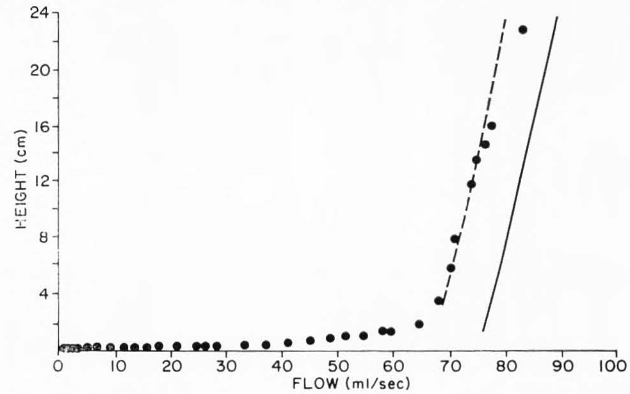

FIGURE 2. Height Versus Flow Rate (X = 0.0 cm.), Smooth Curve is from Mechanical Energy Balance with Friction Losses for Smooth Tubing and Bends. Dashed Curve Includes Drain Losses.

corded and assumed to be indicative of the instantaneous fluid level. Metered water flow was admitted to the tank through plastic tubing (~ .375″ i.d.) with the inlet maintained below the drain level in order to minimize splashing and associated noise.

Steady inlet flows were varied over a range from 0.5 m*l*/sec to 90 m*l*/sec resulting in Reynolds numbers in the outlet line up to 9,280 under steady level conditions. The bend height (X) was varied between 0.0 cm and 90.0 cm to provide conditions of: no bend, bend below the drain, and bend above the drain.

RESULTS AND DISCUSSION

A baseline case with no bend in the exit line (X = 0.0) was characterized by the data shown in Fig. 2. The ordinate is the height of the tank level above the drain and the abscissa is the steady inlet flow rate. The tank level was steady for all flow rates, but a rather marked steady state transition (bifurcation) occurred at ~ 65 m*l*/sec and a more subtle one at ~ 3 m*l*/sec. At flow rates between ~ 3 m*l*/sec and ~ 65 m*l*/sec, the exit line was in a two-phase (air/water) flow condition, while the flow became single-phase above 65 m*l*/sec. In the two-phase flow regime, there was a swirling vortex above the drain which sucked air into the exit line. As the flow rate was

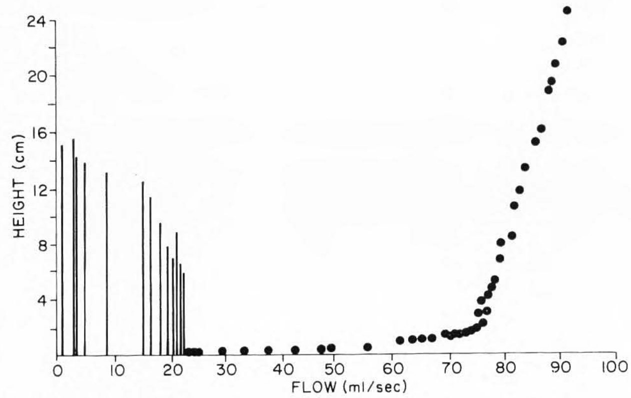

FIGURE 3. Height Versus Flow Rate (X = 41.4 cm.).

increased in this regime, the vortex contracted leading to diminished entrainment of air. As the flow rate was reduced below ~3 ml/sec, two-phase flow subsided and a slug of fluid remained stationary in the bottom of the U in the exit line with fluid trickling into and out of the slug.

The smooth curve in the high flow rate regime is based on the Mechanical Energy Balance incorporating friction factors for the smooth tubing and bends. When a loss coefficient of 2.0 is assigned to the drain, the dashed curve, which fits the single-phase flow data quite well, is obtained. Clearly, it is only in the high flow rate regime that the simple tank draining model of elementary process control [5] applies.

Fig. 3 shows the results of experiments with a 41.4 cm bend in the exit line. The vertical lines in the 4-25 ml/sec range represent oscillatory states. The oscillations were not truly periodic (perhaps chaotic?—see Fig. 4), and the heights of the vertical lines represent only average amplitudes. There are four bifurcations(changes in qualitative behavior) apparent in Fig. 3:

(1) Trickling steady states (with a stationary slug at the bottom of the U) transform to large amplitude oscillatory states at ~3 ml/sec;
(2) Large amplitude oscillatory states transform to small amplitude oscillatory states at ~12 ml/sec;
(3) Small oscillatory states become two-phase flow steady states at ~25 ml/sec; and
(4) Two-phase flow steady states become single phase flow steady states at ~65 ml/sec.

The observed physical mechanism of large amplitude oscillations is summarized in Fig. 5. Consider that an empty tank (and exit line) is being filled up, and the tank level has just reached the mouth of the drain (A). Fluid spills over the drain and fills up the U in the exit line as the level gradually rises (B). Eventually, fluid completely covers the drain trapping a column of air in the U (C). As the tank level rises, the air column is forced further down the tube until it passes around the bottom of the U. At this point the buoyant force of the rising air is sufficient to push fluid over the top of the bend (D). Once fluid passes over the top of the bend, a siphon is created and the tank quickly empties until the level has descended to the drain (E). Now air is sucked into the drain and a brief period of slug flow follows (F) until condition (C) is reestablished. At this point the cycle (C-F) repeats itself. The small amplitude oscillation mechanism is quite similar. The major difference is that the trapped air slug is smaller and the emptying is triggered by fluid passing over the top of the bend rather than air passing under the bottom of the U as in (D).

The transition from oscillatory to steady flow at 25 ml/sec can be understood if we realize that the siphon flow which drains the tank has a fixed rate determined by the configuration of the exit line. If the inflow rate exceeds the siphon flow rate, emptying is prohibited. The transition at 3 ml/sec

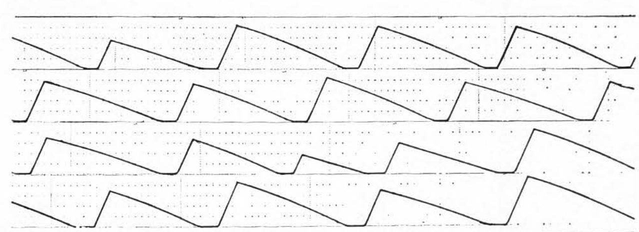

FIGURE 4. Height vs. Time Recording of Typical Oscillations (X = 21.6 cm., Flow Rate = 16.5 ml./sec.).

is not so easily understood. In fact, with the bend height set at X = 21.6 cm, oscillatory flows were observed down to our lowest inflow rate (~0.5 ml/sec) without a steady state transition (see Fig. 6). The results of Figs. 3 and 6 suggested the possibility of bi-stability (two stable flow regimes) in the low flow rate range, but limited hysteresis experiments did not reveal a range of coexisting steady and oscillatory states.

The effect of bend height on flow regimes was further investigated at X = 5.8, 30.1, 51.1, and 90.0 cm. Oscillations were observed for all bends, with their amplitudes strongly proportional to bend height. Steady states always existed at high flow rates, but with sufficiently high bend height, the two-phase flow regime disappeared. There was no obvious pattern in the appearance of low flow rate steady states. They appeared for X =

0.0, 5.8, 41.4 and 51.1 cm, but not for X = 21.6, 30.1 and 90.0 cm.

In a final desperate attempt to eliminate oscillations with a finite bend, we inverted the drain to an open downward configuration thinking that this would eliminate the sucking of air which was considered vital to the oscillation mechanism. To our surprise, the siphon effect was strong enough to suck air around the inverted drain, and we observed oscillations following the mechanism of Fig. 5. We conclude that the only way to completely eliminate the oscillatory regime is to completely eliminate the bend.

CONCLUDING REMARKS

If you are interested in practical engineering applications for our sink, you might consider it as a primitive level controller when operated in the two-phase flow regime, where level is nearly independent of inflow rate, or, as a periodic feed pump to a batch process when operated in the oscillatory flow regime.

On the other hand, if you are looking for theoretical significance, you should realize that it is an unusual hydrodynamical system in which intermediate time-dependent states are surrounded by steady states at both lower and higher values of a parameter. For example, in pipe flow there is a transition from steady (laminar) flow to time-dependent (turbulent) flow as the Reynolds number is increased, but a return to steady flow at higher Reynolds numbers is not observed. The same may be said of the classical Taylor (circular couette) and Rayleigh-Bénard (heated layer) flows [6]. As the Taylor or Rayleigh number is increased, a bifurcation from steady to time dependent motion occurs, but further parameter increase does not result in steady motion. Intermediate time dependent states are readily observed in chemical reaction systems. For example, a constant volume continuous stirred tank reactor (CSTR) always produces stable steady states at sufficiently high and low values of the flow rate [7, 8], but time-dependent states may occur at intermediate flow rates [4].

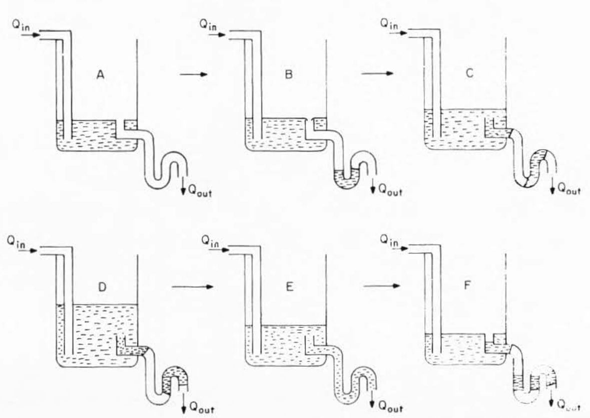

FIGURE 5. Schematic of the Oscillation Mechanism.

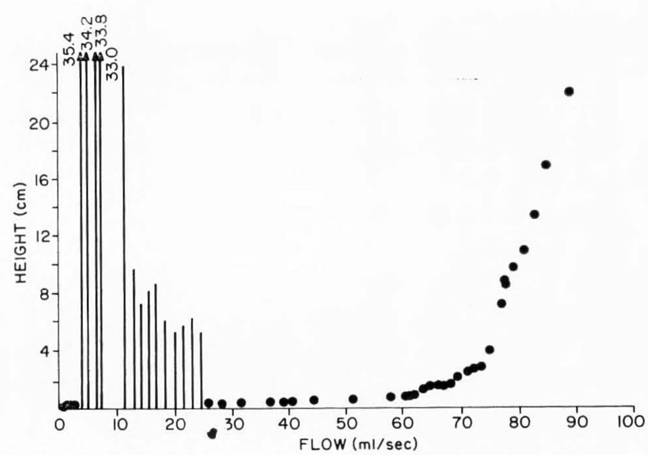

FIGURE 6. Height Versus Flow Rate (X = 21.6 cm.).

Finally, if your interests are pedagogical, "the oscillating sink" provides a simple demonstration of autonomous oscillations and a variety of bifurcation phenomena. As a parting note, the demonstration value of the oscillating sink can be improved by the use of a smaller diameter tank than we have shown in Fig. 1. This will result in shorter oscillation periods (we observed periods of several minutes) and a more dynamic demonstration. The tank diameter should not affect the oscillation mechanism. □

REFERENCES

1. van Heerden, C., Ind. Eng. Chem., *45*, 1242 (1953).
2. Bilous, O., Amundson, N. R., A.I.Ch.E. J., *1*, 513 (1955).
3. Uppal, A., Ray, W. H., Poore, A. B., Chem. Eng. Sci. *29*, 967 (1974).
4. Schmitz, R. A., Graziani, K. R., Hudson, J. L., J. Chem. Phys., *67*, 3040 (1977).
5. Coughanowr, D. R., Koppel, L. B., "Process Systems Analysis and Control," McGraw-Hill, New York (1965).
6. Fenstermacher, P. R., Swinney, H. L., in "Bifurcation Theory and Applications in Scientific Disciplines," Eds., Gurel, O. K., Rössler, O. E., Ann. N.Y. Acad. Sci., *316* (1979).
7. Plau, F., Tarbell, J. M., Chem. Eng. Commun., *4*, 677 (1980).
8. Gavalas, G. R., "Nonlinear Differential Equations of Chemically Reacting Systems," Springer-Verlag, New York (1968).