ChE classroom

SIMULATION EXERCISES FOR AN UNDERGRADUATE DIGITAL PROCESS CONTROL COURSE

DEBORAH E. REEVES, F. JOSEPH SCHORK
Georgia Institute of Technology
Atlanta, GA 30332

In seeking to develop a set of homework exercises for a senior level course in digital process control, the authors have devised an alternative to the classical approach of short homework problems assigned at the end of each lecture segment. Instead, we have tried to provide longer-term exercises which complement the lecture material while allowing for more creativity and independence on the part of the student. The concept is to define a control problem, have the students analyze its dynamics, and then have them digitally simulate both the open-loop process and its closed-loop dynamics under various control schemes. Digital simulation has some advantages as a learning tool. It forces the students to understand the process in order to write the code to simulate it, and it requires an understanding of how each control scheme is actually implemented in quasi-real time. In addition, since digital simulation is, by its nature, digital, the concept of discrete control is emphasized.

Deborah E. Reeves received her BS from Clemson University in 1986 and her MS from Georgia Tech in 1988. She is presently a PhD student in chemical engineering at Georgia Tech. As a National Science Foundation Fellow she is concentrating her research in the field of process control. **(L)**

F. Joseph Schork received his BS (1973) and MS (1974) from the University of Louisville. He was employed as a Research and Development Engineer with DuPont for three years before pursuing a PhD in chemical engineering at the University of Wisconsin, which he received in 1981. He was the 1981 recipient of the American Chemical Society's Arthur K. Doolittle Award. He joined the faculty at Georgia Tech in 1982. **(R)**

These exercises were designed as the primary homework set for a two quarter-hour senior-level course in digital process control. The students have already taken a three quarter-hour course in classical control theory, and a one quarter-hour laboratory in system dynamics and analog and digital control. The exercises are meant to supplement lectures from Deshpande and Ash [1] or Stephanopoulos [2]. Extensive use is made of Program CC (a control design package for the personal computer available from Systems Technology, Inc., of Hawthorne, California [3]), but only as an analysis tool to aid in implementation via digital simulation. Simulations are run on a VAX 11/780 and may make use of numerical integration routines from the IMSL Library [4]. Students work in groups of two or three members of their own choosing. Although it is not explicitly stated in the problem statements below, students are expected to plot and discuss all results. Due to space limitations, only the Problem Statements are included here. Full solutions, documentation of the simulation program, and the Program CC results may be obtained from the authors.

We feel that these exercises give the students experience in implementation, allow them to compare various algorithms on a single process, and stimulate initiative and creativity.

PROBLEM 1
System Model

Problem Statement

Consider an example from Ray [5] consisting of a system of two continuous stirred tank reactors in series as shown in Figure 1. The irreversible reaction A → B is carried out isothermally in the two-stage reactor system. The composition of the product streams, c_1 and c_2, must be controlled. However, there is a substantial analysis delay. The manipulated variables are the feed compositions to the two reactors, c_{1f} and c_{2f}, and the process disturbance is the concentration of an additional feed stream, c_d. The flowrates to the system are constant, and only the compositions vary. An additional delay arises due to the transportation lag in the recycle stream.

A. For the reactor system above, write the material balances for c_1 and c_2 around the reactor train. Note that

$$F_{p2} = F_1 + F_d - F_{p1} + F_2$$

B. Cast the equations from (A) in deviation variables. Use the definitions below. (Subscript s denotes steady state value.)

$$\theta_1 = \frac{V_1}{F_1 + R + F_d} \qquad \theta_2 = \frac{V_2}{F_{p2} + R}$$

$$\lambda_R = \frac{R}{F_1 + R + F_d} \qquad \mu = \frac{F_{p2} - F_2 + R}{F_{p2} + R}$$

$$\lambda_d = \frac{F_d}{F_1 + R + F_d} \qquad Da_1 = k_1\theta \qquad Da_2 = k_2\theta_2$$

$$U_1 = c_{1f} - c_{1f_s} \qquad U_2 = c_{2f} - c_{2f_s}$$

$$x_1 = c_1 - c_{1_s} \qquad x_2 = c_2 - c_{2_s} \qquad d = c_d - c_{d_s}$$

C. Show that the results of (B) can be expressed in

The concept is to define a control problem, have the students analyze its dynamics, and then have them digitally simulate both the open-loop process and its closed-loop dynamics under various control schemes.

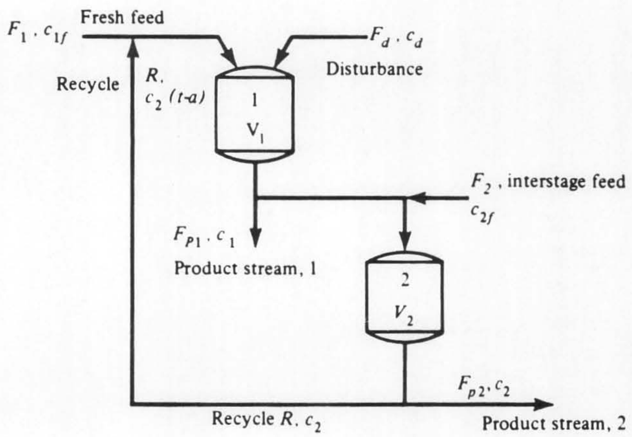

FIGURE 1. Two-CSTR System with Delayed Recycle (Reprinted with permission from* Advanced Process Control*, W. H. Ray (1981), McGraw-Hill Co., page 220, figure 4.23.)

matrix form as

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}_0\mathbf{x}(t) + \mathbf{A}_1\mathbf{x}(t-\alpha) + \mathbf{BU}(t) + \mathbf{L}d$$

where

$$\mathbf{A}_0 = \begin{bmatrix} -\dfrac{1+Da_1}{\theta_1} & 0 \\ \dfrac{\mu}{\theta_2} & -\dfrac{1+Da_2}{\theta_2} \end{bmatrix} \qquad \mathbf{A}_1 = \begin{bmatrix} 0 & \dfrac{\lambda_R}{\theta_1} \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \dfrac{1-\lambda_R-\lambda_d}{\theta_1} & 0 \\ 0 & \dfrac{1-\mu}{\theta_2} \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} \dfrac{\lambda_d}{\theta_1} \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

D. Assume there are pure delays of τ_1 and τ_2 on the measurements of x_1 and x_2 respectively. Thus

$$Y_{m_1}(t) = x_1(t - \tau_1) \qquad Y_{m_1}(s) = e^{-\tau_1 s} x_1(s)$$

$$Y_{m_2}(t) = x_2(t - \tau_2) \qquad Y_{m_2}(s) = e^{-\tau_2 s} x_2(s)$$

or

$$\mathbf{Y}_m(s) = \mathbf{H}(s)\mathbf{x}(s)$$

$$\mathbf{H} = \begin{bmatrix} e^{-\tau_1 s} & 0 \\ 0 & e^{-\tau_2 s} \end{bmatrix} \qquad \mathbf{Y}_m = \begin{bmatrix} Y_{m_1} \\ Y_{m_2} \end{bmatrix}$$

E. Take the Laplace transform of the result from (B) to obtain

$$\mathbf{Y}_m(s) = \mathbf{H}(s)\ \mathbf{G}(s)\ \mathbf{U}(s) + \mathbf{H}(s)\ \mathbf{G}_d(s)\ d(s)$$

i.e., find

$$\mathbf{G}(s) \quad \text{and} \quad \mathbf{G_d}(s)$$

F. Into the result of (E), insert the operating parameters given in (G). Let $\lambda_R = 0$.

G. This reactor system will form the basis for the following problems. If we wish to deal with an SISO system, we will set $\lambda_R = 0$ and focus on the first reactor. If we wish to deal with a MIMO system, we will set $\lambda_R = 0.5$ and deal with the coupled reactors. Base case parameters will be as follows:

$$x_1(0) = x_2(0) = U_1(0) = U_2(0) = 0$$

(System is initially at steady state.)

$$\theta_1 = 1; \qquad \theta_2 = 1$$
$$Da_1 = 1; \qquad Da_2 = 1$$
$$\lambda_R = 0 \quad \text{or} \quad 0.5$$
$$\lambda_d = 0.1 \quad \text{(where appropriate)}$$
$$\mu = 0.5; \qquad \alpha = 0.5$$
$$T = 0.1 \text{ min} \quad \text{or} \quad 0.01 \text{ min}$$
$$\tau_1 = \tau_2 = 0.5$$

PROBLEM 2
Open-Loop Simulation

Problem Statement

A. Digitally simulate the system of two reactors in series as analyzed in Problem 1. Simulate the open-loop response of both reactors to a 0.1 step change in d at time = 1 minute. Do this for both values of λ_R (0 and 0.5). Use a sampling period of 0.1.

B. Use Program CC to produce the open-loop simulation above for no recycle. Use the s-domain transfer functions developed in Problem 1.

C. Use Program CC to produce the open-loop simulation above for no recycle. Do this simulation in the z-domain with sampling period = 0.1 min.

PROBLEM 3
PID Control

Problem Statement

A. Using the FORTRAN simulator developed in Problem 2, digitally simulate the closed-loop response of Reactor 1 to a 0.1 step change in the disturbance (d). Use the velocity form of the PID algorithm. Assume zero recycle. Do the simulation for sampling times (T) of 0.1 and 0.01. Use the Ziegler Nichols method to tune the controller, either by constructing a Bode plot (with Program CC), or by finding the ultimate gain and period on-line by the loop tuning method. Include in your program a calculation of the integral squared error associated with the above disturbance.

B. Use Program CC (in the z domain) to repeat the simulation in (A) for the case of T = 0.01 only.

PROBLEM 4
Dahlin Algorithm

Problem Statement

A. Design a Dahlin control algorithm for the system which has been studied in Problems 2 and 3. The design should be based on a first order plus deadtime response to a step change in set point. This should be an SISO controller which manipulates the feed concentration to Reactor 1 in order to control the concentration in Reactor 1 (as before). Use T = 0.01. There is no recycle.

B. Using the FORTRAN simulator you developed previously, simulate the response of the Dahlin algorithm you derived in (A) to a 0.1 step change in d. Plot the concentrations in both reactors, even though only the concentration in Reactor 1 is being controlled.

PROBLEM 5

Analytical Predictor

Problem Statement

A. Design an Analytical Predictor time delay compensator control algorithm for the system which has been studied in Problems 2-4. This should be a SISO controller which manipulates the feed concentration to Reactor 1 in order to control the concentration in Reactor 1 (as before). Use T = 0.01. There is no recycle.

B. Using the FORTRAN simulator you developed previously, simulate the response of the Analytical Predictor algorithm you derived in (A) to a 0.1 step change in d.

PROBLEM 6

Noninteracting Control

Problem Statement

A. Consider the reactor system in Problem 1. Let λ_R equal 0.5. Calculate the Relative Gain Array. Discuss the loop pairings.

B. Simulate the system with both reactors under PI control. These should be two SISO loops. Sampling time should be 0.01. Include the delay in the recycle loop. Tune each controller separately. The loop not being tuned should be open. Simulate the response of the system (both loops closed) to a step change of 0.1 in the set point of Loop 1 (Reactor 1). Repeat for a step change of 0.1 in the set point of Loop 2. Simulate the response of both loops to a step change of 0.1 in the disturbance.

C. Design and implement a steady-state decoupler for this system. Repeat the simulations above. (Use PI controllers with only slight integral action.) Do the loops interact more or less than in (B)? Why?

SUMMARY

Integral-squared error results for the various problems are summarized in Table 1. In summary, the PID results indicate that a smaller sampling period produces a better response. This is consistent with digital control theory. Deadtime compensation inherent in the Dahlin and Analytical Predictor algorithms improves the controlled behavior of the system. The predictive capacity of the Analytical Predictor also appears to upgrade the response slightly. It is imperative to note however that the quality of the response generated by each method is highly dependent upon the tuning parameters employed. Thus the above observations should not be taken as conclusive.

In the multivariable case (Problem 6), no clear advantages result upon decoupling; however several undesirable response characteristics are produced by the steady state decoupler. This is probably due to the fact that interactions are inherently small for this system. Hence, a steady state decoupler would not be recommended, but a dynamic decoupler might be a reasonable option for future expansion of the simulation.

TABLE 1

Integral-Squared-Error values of Y_{m_1} and Y_{m_2} for a time interval of 10 minutes. The sampling period, T, is 0.01 minutes unless stated otherwise.

	ISE ($x10^5$)	
Disturbance Step Change	Y_{m_1}	Y_{m_2}
$\lambda = 0$		
Open Loop Response (T = 0.1)	19.5	1.12
PID Control (T = 0.1)	0.568	0.0240
PID Control	0.541	0.0221
Dahlin Algorithm	0.611	0.0257
Discrete Analytical Predictor	0.555	0.0226
$\lambda = 0.5$		
Open Loop Response (T = 0.1)	21.7	1.25
SISO PI Control	1.11	0.0225
Steady State Decoupler	1.12	0.0888
Y_{m_1} Set Point Step Change		
$\lambda = 0.5$		
SISO PI Control	736.0	22.7
Steady State Decoupler	725.0	15.9
Y_{m_2} Set Point Step Change		
$\lambda = 0.5$		
SISO PI Control	22.8	735
Steady State Decoupler	49.2	722

ACKNOWLEDGMENTS

This material is based upon work supported under a National Science Foundation Graduate Fellowship.

REFERENCES

1. Deshpande, P. B., and R. H. Ash, *Computer Process Control*, ISA, Research Triangle Park, NC (1981).
2. Stephanopoulos, G., *Chemical Process Control*, Prentice-Hall, Englewood Cliffs, NJ (1984).
3. Thompson, P. M., *User's Guide Program CC*, Systems Technology, Inc., Hawthorne, CA (1985).
4. *IMSL Library*, IMSL, Houston, TX (1983).
5. Ray, W. H., *Advanced Process Control*, McGraw-Hill, NY (1981). □