The object of this column is to enhance our readers' collection of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested as well as those that are more traditional in nature, which elucidate difficult concepts. Please submit them to Professor James O. Wilkes and Professor T. C. Papanastasiou, ChE Department, University of Michigan, Ann Arbor, MI 48109.

DRAINAGE OF CONICAL TANKS WITH PIPING

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Applied mathematics problems in chemical engineering, useful to educators, often appear in various trade journals. Many of these applications have been in the area of fluid dynamics, specifically concerning the time requirements to drain process vessels, which come in a variety of geometrical shapes. Thus, formulas have been summarized [1] to compute the times required to empty vessels of four different shapes: vertical cylinder, cone, horizontal cylinder, and sphere. Later articles gave similar formulas for draining elliptical vessel heads [2] and elliptical saturator troughs [3]. An approximate method for estimating fluid level changes in vertical cylindrical tanks with a multiplicity of outlets (or leaks), of various sizes and at different elevations, has also been presented [4].

All of the above results are based upon the assumption of orifice-type drains, *e.g.*, short tubes, and ignore any associated piping. One article [5] derived a formula for computing the time to drain a vertical cylindrical tank, considering drain piping. Later works [6,7] gave analogous formulas for draining spherical tanks and elliptical dished heads, respectively, with drain piping. Another (not uncommon) shape of process vessel in the chemical industry is conical. Computation of the time required to empty such a vessel through associated drain piping is also amenable to analytical solution, as shown below. A sketch of this configuration is given in Figure 1.

DERIVATION

From the Bernoulli equation, applied to points 1 and 2

$$\frac{P_1}{\rho}\cdot\frac{g_c}{g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho}\cdot\frac{g_c}{g}+\frac{V_2^2}{2g}+Z_2+h_\ell \tag{1}$$

With the conventional assumptions that $P_1 = P_2$ and $V_1 \approx 0$, we have

$$Z_1=\frac{V_2^2}{2g}+Z_2+h_\ell \tag{2}$$

Introducing the Moody friction factor (f) for the drain piping

$$h_\ell=\frac{fL}{d}\cdot\frac{V_2^2}{2g} \tag{3}$$

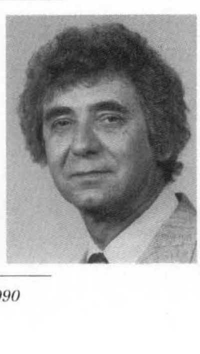
and noting that $Z_1 - Z_2 = H$, Eq. (2) can be solved for the drain pipe velocity:

$$V_2=\left[\frac{2gH}{1+\frac{fL}{d}}\right]^{\frac{1}{2}} \tag{4}$$

A dynamic material balance for the liquid in the tank yields

$$A\frac{dh}{dt}=-aV_2 \tag{5}$$

The cross-sectional area of the liquid level in the tank at any time is merely the circular area described by the radius r at the current level h, or πr^2.



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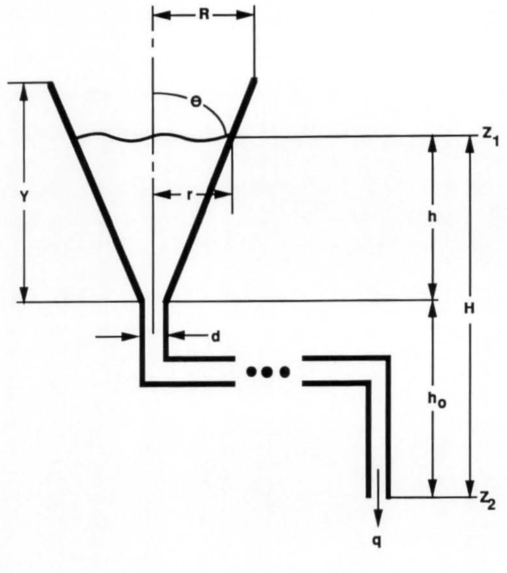

FIGURE 1. Sketch of a conical tank with associated drain piping.

From similar triangles we have

$$\frac{r}{h}=\frac{R}{Y}\equiv\alpha(=\mathrm{TAN}\,\Theta) \qquad (6)$$

and

$$A=\pi\alpha^2h^2 \qquad (7)$$

Recognizing that $a=\pi d^2/4$ and after inserting Eqs. (4) and (7) into Eq. (5), we have

$$\alpha^2h^2\frac{dh}{dt}=\frac{-d^2}{4}\left[\frac{2gH}{1+\frac{fL}{d}}\right]^{\frac{1}{2}} \qquad (8)$$

Lastly, since

$$h=H-h_o \qquad (9)$$

the differential equation to be integrated becomes

$$\left(H-h_o\right)^2\frac{dH}{dt}=-\left(\frac{d}{2\alpha}\right)^2\left[\frac{2gH}{1+\frac{fL}{d}}\right]^{\frac{1}{2}} \qquad (10)$$

Integrating Eq. (10) from an initial (t = 0) liquid level elevation of H_o to some final level, H_f, then yields the following expression for the time required

$$t=C\left[\left(\frac{2}{5}H_o^2-\frac{4h_oH_o}{3}+2h_o^2\right)\sqrt{H_o}-\left(\frac{2H_f^2}{5}-\frac{4h_oH_f}{3}+2h_o^2\right)\sqrt{H_f}\right] \qquad (11)$$

where

$$C=\left(\frac{2\alpha}{d}\right)^2\sqrt{\frac{1}{2g}\left(1+\frac{fL}{d}\right)} \qquad (12)$$

SPECIAL CASES

Two special cases are of interest. The first of these corresponds to complete draining of a partially filled conical tank. In this case, $H_f = h_o$, and Eq. (11) becomes

$$t=C\left[\left(\frac{2H_o^2}{5}-\frac{4h_oH_o}{3}+2h_o^2\right)\sqrt{H_o}-\frac{16}{15}h_o^{5/2}\right] \qquad (13)$$

The second special case is concerned with complete draining of a completely filled cone. In this case, $H_o = Y + h_o$, $H_f = h_o$, and there results

$$t=\frac{2C}{15}\left[\left(3Y^2-4Yh_o+8h_o^2\right)\sqrt{Y+h_o}-8h_o^{5/2}\right] \qquad (14)$$

EXAMPLE

A conical tank with a height of 3m and a top diameter of 1.2m is initially filled with water to a level of 2.4m. How long will it take to drain this liquid through a drain pipe system with 150m of equivalent length and with an inside pipe diameter of 1.5cm? The Moody friction factor for this piping system is equal to 0.0185, and the elevation of the outlet from this drain system is one meter below the bottom of the conical tank.

From the problem data

$$H_o=3.4\mathrm{m}, \qquad H_f=h_o=1.0\mathrm{m},$$

$$\alpha=0.2, \qquad C=2190\,\mathrm{s/m}^{5/2}$$

Inserting these values into either Eq. (11) or Eq. (13), we find a drainage time requirement of t = 6108s, or 1.70hr.

By way of comparison, from the earlier work [1] the time required to drain the same amount of liquid out of an identical conical tank through a short pipe with the same diameter of 1.5cm, assuming an orifice discharge coefficient equal to 0.80, is 716s.

NOMENCLATURE

A = cross-sectional area of the liquid level in the tank at any time

a = cross-sectional area of the drain piping

$$C = \left(\frac{2\alpha}{d}\right)^2 \left[\frac{1}{2g}\left(1+\frac{fL}{d}\right)\right]^{1/2}$$

D = upper diameter of the conical tank

d = inside diameter of the drain piping

f = Moody friction factor

g = acceleration due to gravity

g_c = conversion factor

H = liquid height above the drain pipe outlet at any time

h = liquid level in the tank at any time

h_ℓ = head loss in the piping

h_o = elevation of the tank bottom above the drain pipe outlet

L = equivalent length of the piping

P = pressure

q = liquid flow rate out of the tank

R = upper radius of the conical tank

r = radius of the liquid level at any time

t = time

V = liquid velocity

Y = height of the conical tank

Z = vertical elevation

Greek Letters

α = R/Y

Θ = angle formed by the cone with the vertical axis

π = number pi (3.14159...)

ρ = liquid density

Subscripts

f = final condition

o = initial condition

1 = liquid surface in the tank at any time

2 = drain pipe outlet

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ChE book review

HAZARDOUS WASTE MANAGEMENT

by Charles A. Wentz

McGraw-Hill Book Company, 1221 Avenue of the Americas, New York 10020; $46.95 (1989)

Reviewed by
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The nation's need for educated and trained professionals in hazardous materials and waste management is enormous and growing [1,2]. In a recent survey paper, my colleagues and I concluded that universities were beginning to respond to the need, albeit slowly [3]. We were able to identify 113 universities offering credit courses related to hazardous waste management (HWM), and 52 universities providing non-credit short courses at the professional level, for a total of 130 universities providing some kind of HWM education. This new area of knowledge is being studied by a very wide array of practitioners, from traditional chemical and civil engineers and chemists to environmental scientists, environmental health professionals, and medical technologists. It appears that a whole new graduate profession is emerging, since there is plenty of conventional chemical and civil engineering to be accomplished, but the additional role of interdisciplinary management must be implemented. There is a clear need for such new managers at (almost) the entry level, and the career path leads up to the vice-presidential level when environment, health, and safety aspects are combined.

In this context, the pioneering text, Hazardous Waste Management, by Charles A. Wentz, fills an enormous need as the first teaching textbook on the market. I expect this book to enable virtually all chemical, civil, and applied science departments to introduce a survey course in HWM. The author is particularly well-qualified to have undertaken this task, having a rare blend of industrial, university

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