AN INTERESTING AND INEXPENSIVE MODELING EXPERIMENT

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In the search for new laboratory experiments, a simple experiment that works well is always welcome. In this paper we describe an inexpensive apparatus, using simple and widely available components, that will help student understanding of process modeling. The equipment can be arranged in a variety of configurations to allow study of different models. Many chemical engineering departments carry out mixed-tank experiments, some with computer interfaces for data collection, that can be rearranged and modified to include the models suggested in this article.

THEORY

In a text by Levenspiel[1] several models are presented for long time scale behavior of real stirred tanks. The models examined here are Levenspiel's model L (which is described in more detail by Bischoff and Dedrick[2]) and a modification of that model.

In these models, shown in Figure 1, flow enters a perfectly mixed tank of volume aV, is interchanged at a rate bv with a second perfectly mixed tank of volume (1-a)V, and is discharged from the first tank. Nomenclature used here is consistent, where possible, with that used by Levenspiel. The total volume of the system is V.

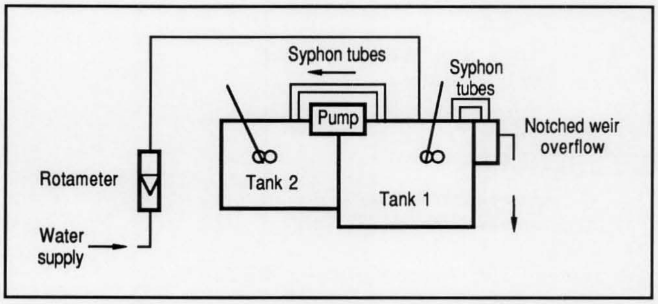

Figure 1. *Stirred tank model (Levenspiel's Model L)*

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In Levenspiel's model L, a unit impulse is imposed in the feed to tank 1. If the concentration in tank 1 is C_1 and the concentration in tank 2 is C_2, the material balances on the two tanks, assuming perfect mixing, yield for tank 1

$$v\delta(t) + bvC_2 - bvC_1 - C_1v_1 = \frac{d[aVC_1]}{dt} \tag{1}$$

and for tank 2

$$C_1bv - C_2bv = \frac{d[(1-a)VC_2]}{dt} \tag{2}$$

Initial conditions for each tank reflect no tracer in either tank with the initial condition in tank 1 a formal property assigned to the Dirac delta function as indicated by Churchill[3]

$$C_1(0) = 0 \qquad \text{and} \qquad C_2(0) = 0 \tag{3}$$

These equations yield to rather simple Laplace transform solution. The transformed equations are

$$C_1(s) = \frac{1 + bC_2(s)}{a\bar{t}s + (1+b)} \tag{4}$$

and

$$C_2(s) = \frac{bC_1(s)}{(1-a)\bar{t}s + b} \tag{5}$$

where $\bar{t}=V/v$. Inverse transformation of these equations yields the solutions in dimensionless time, θ, given by Levenspiel for the two tanks

$$E_\theta = C_{\theta 1} = \frac{1}{a(1-a)(m_1-m_2)}[m_1 - am_1 + b]e^{m_1\theta} - (m_2 - am_2 + b)e^{m_2\theta} \quad (6)$$

and

$$C_{\theta 2} = \frac{b}{a(1-a)(m_1-m_2)}\left[e^{m_1\theta} - e^{m_2\theta}\right] \quad (7)$$

where

$$m_1, m_2 = \frac{1-a+b}{2a(1-a)}\left(-1 \pm \sqrt{1 - \frac{4ab(1-a)}{(1-a+b)^2}}\right) \quad (8)$$

The discussion in Levenspiel is necessarily brief, and students need to be sure they understand the equations describing the model, Eqs. (6) and (7) above, and the procedures to reduce the data to a similar form (or to change the equation for the model to the data form). Fogler[4] also shows the development of the equations for Levenspiel's model L.

Most of the long time scale models presented by Levenspiel could be examined in the experimental apparatus with a little equipment rearrangement. A variation which has been tried by students in our laboratories is a modification of Levenspiel's model L in which the tracer or unit impulse is imposed in the "stagnant" compartment. The solution to this model as worked out with Jones[5] gives the following expressions:

$$E_\theta = C_{\theta 1} = \frac{b}{a(1-a)(m_1-m_2)}\left[e^{m_1\theta} - e^{m_2\theta}\right] \quad (9)$$

$$C_{\theta 2} = \frac{1}{a(1-a)(m_1-m_2)}\left[(am_1 + b + 1)e^{m_1\theta} - (am_2 + b + 1)e^{m_2\theta}\right] \quad (10)$$

EQUIPMENT

The stirred tanks for this experiment were one 5-gallon aquarium and one 10-gallon aquarium placed end-to-end. A schematic diagram of the apparatus is shown in Figure 2 and a photograph of the apparatus is shown in Figure 3. Water was fed to the larger aquarium through a small rotameter, and water was discharged from the larger tank via four syphon tubes into a small notched-weir overflow tank and then to the drain. Flow from the larger tank to the smaller tank was also enabled by using four syphon tubes. Return flow from the smaller tank to the larger tank was accomplished by using a simple aquarium pump/filter device without the filter.* The filter's intake was positioned in the smaller tank and the discharge, which was adjustable with an integral valve, was made to the larger tank. The filter used in this case included a one-liter hold-up tank which was filled with inert materials to eliminate a possible third mixed tank in the apparatus. Tracer selection could be dye, salt, or any tracer with detection capabilities available. In this work, the tracer selected for the quantitative work was sodium chloride because a YSI Scientific Model 35 conductance meter was available. This meter allowed for either a continuous record of conductance when used with a millivolt potentiometric recorder or an instantaneous reading. Total equipment cost excluding the conductance meter, rotameter, and stirrers was $85.00.

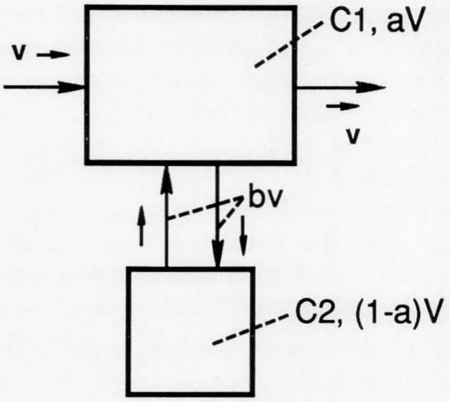

Figure 2. *Schematic diagram of apparatus*

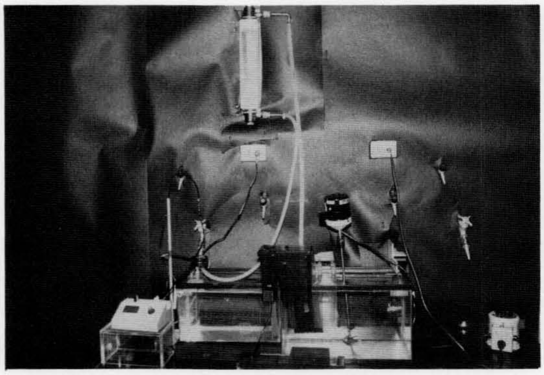

Figure 3. *The experimental apparatus*

PROCEDURE

Both tanks were initially filled with water. Then water flow at a rate of 3.1 liters/minute was initiated through the rotameter into the larger tank, and flow from the larger tank through the two syphon systems was started. Flow rates are typical and, of course, may be set at any reasonable level. Stirrers, placed in both tanks, were activated. Flow through the pump/filter was started and measured using the bucket-and-watch method on the outflow of the power filter after a period of time to allow steady-state flow. A return water flow rate to the larger tank of 4.8 liters/minute was measured. At steady-state the volume of liquid in tank 1 was 36.2 liters, and the volume in tank 2 was 16.5 liters. The above tank volumes and flow rates gave model

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* *In this case a Model 2 Secondnature Whisper Power Filter: catalog No. 6002, Willinger Bros., Inc., Wright Way, Oakland, NJ (201-337-0001).*

parameters of 0.678 for a and 1.55 for b. A one-molar solution of sodium chloride was prepared for use as tracer, and a calibration curve for the conductivity meter was prepared. Before an experimental run was made using the salt tracer, a run was made using dye as the tracer—this demonstrated the flow patterns in the system and gave some insight into the perfectly mixed tank assumptions. Dye, instead of salt, has also been used in separate experiments to monitor the tracer concentration.

When flows were properly established and steady-state conditions were obtained, one liter of the 1 M salt tracer was rapidly poured into the center of the larger tank over a short period of time to approximately replace the regular water flow in a pulse-shaped input. The concentration of the material in each tank was monitored alternately with the single conductivity probe, with the probe reinserted into the tanks in approximately the same location each time. Sampling was halted after approximately 37 minutes.

RESULTS AND DISCUSSION OF RESULTS

The experimental results are shown in Figure 4. The response of Model L to a unit impulse input was also determined by solving the equations numerically; these results are compared to the experimental results in Figure 4. The expected characteristic shapes were obtained and agreement between the experimental results and the model were within seven percent for tank 1 and within eleven percent for tank 2. The maximum concentration in tank 2 was eleven percent below the model and about two minutes late. No attempt was made to adjust model parameters.

Many variations of the experiment demonstrated here could be studied including other models, effect of tracer injection method, effect of adjusting the model parameters, and effect of mixing. Because of the flexibility derived from the ease of rearranging the system, individual laboratory participants or groups could study a number of different models or a few models in depth.

SUMMARY AND CONCLUSIONS

An interesting and inexpensive process-modeling experiment was demonstrated with qualitative and quantitative results. Other models could be examined by simple modifications to the experimental apparatus. The work integrates studies in chemical reaction engineering courses with process modeling and control courses and provides the students with some insight into problems in modeling systems.

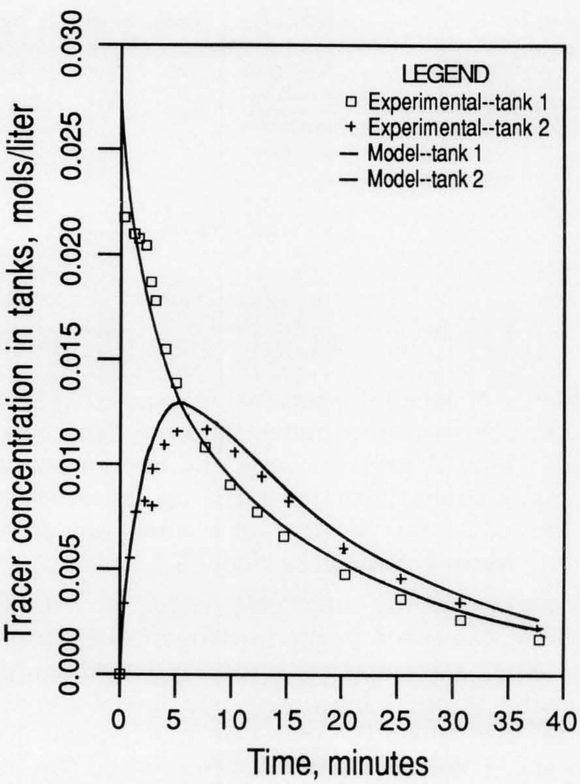

Figure 4. *Comparison of model with experiment*

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NOMENCLATURE

a = model parameter, fraction of total volume in feed tank
b = model parameter, fraction of feed volume flowing to "stagnant" region
C_1 = tracer concentration in tank 1
C_2 = tracer concentration in tank 2
$C_{\theta 1}$ = C-curve for tank 1 based on θ, $C_{\theta 1} = E_\theta = \bar{t}C_1$
$C_{\theta 2}$ = C-curve for tank 2 based on θ, $C_{\theta 2} = \bar{t}C_2$
E_θ = exit age distribution for tank 1 in dimensionless time
$\bar{t}$ = mean residence time, V/v
V = total system volume
v = volumetric feed rate
δ(t) = Dirac delta function for unit impulse
θ = dimensionless time $t/\bar{t}$

REFERENCES

1. Levenspiel, Octave, *Chemical Reaction Engineering,* 2nd ed., Wiley, New York (1972)
2. Bischoff, K.B., and R.L. Dedrick, *J. Theor. Biol.,* **29,** 63 (1970)
3. Churchill, Ruel V., *Operations Mathematics,* 2nd ed., McGraw-Hill, New York (1958)
4. Fogler, H. Scott, *Elements of Chemical Reaction Engineering,* 2nd ed., Prentice Hall, Englewood Cliffs, NJ (1992)
5. Jones, S., private communication (1990) ❐