AN INEXPENSIVE AND QUICK FLUID MECHANICS EXPERIMENT

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The first laboratory course in chemical engineering at the University of Alberta is taught to about seventy students in the first term of their junior year, and its primary objective is to improve and develop the students' writing skills. They are required to write three reports. The first two reports are each about ten pages in length and deal with technical material which is familiar to the student. Each of the first two experiments is performed and the reports written within a period of two weeks, and the corrected reports are returned to the students within another two weeks. The English construction and the presentation of the reports are brutally criticized by the academic staff responsible for the course. The students then do a standard engineering experiment and write their third (hopefully readable) report.

The key to this approach is to present simple and short experiments. Typically, the first two experiments should each take less than ten minutes to complete. Quick experiments have the advantage of conveying to the students that even though the technical aspects are easy, describing them in clear, understandable English is often very difficult.

For the last six years the first experiment we have used has been a computer simulation of a simple concept that the students should be able to understand. Usually, the simulated experiment is based on a fundamental principle that was taught to the students in the previous semester, *i.e.*, the vapor pressure of water as a function of temperature. The students run the program, specifying the temperature, with the simulator returning a slightly inaccurate value of the vapor pressure. The students are asked to compare the simulated vapor pressures with those in the steam tables and those calculated from a published correlation of vapor pressure and temperature. They then write a report about the simulated experiment.

Experience has taught us that both of the short experiments should not be simulations since, if they are, the students will write a simulated second report. Their attitude seems to be, "If you don't take the experiment seriously, why should we take the report seriously?" Clearly, this is not the objective of the course. So, we make the second experiment a real experiment—but still quick.

One experiment that is simple and fast is the filling and blowdown of a tank of air. In spite of its simplicity, this experiment is surprisingly rich in its technical content, involving ideal gas thermodynamics, unsteady state material balances, and simple fluid mechanics. All of these subjects have been covered in the preceding term or are being taken concurrently with this course. A further advantage is that the complexity of the data analysis can be adjusted to accommodate the technical skills of the students.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiment consists of two stages: 1) filling the tank with air at about 90 psig and 70°F, and 2) emptying the tank by venting the compressed air to the atmosphere. The equipment (shown in Figure 1)

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consists of a modified 30-pound propane bottle and a manifold mounted at the top of the bottle. The manifold has four nozzles, each of which is isolated with a quick-acting ball valve. Standard 1/2-inch copper tube and fittings are used for the manifold. The nozzles are brass plugs which have holes drilled to diameters ranging from 1/16 to 7/64 inch. These plugs are soldered into the outlet of 1/2-inch unions which are located above each of the ball valves. While it is not required because of the limited air supply pressure, a relief valve is installed on the tank for educational purposes. A cheaper alternative would be to incorporate the relief valve in the manifold. High-pressure air is supplied from the building service air through a flexible hose. Another quick-acting ball valve is installed on the tank at the hose connection.

The instrumentation consists of a fast response thermocouple, a pressure transducer, and associated signal conditioning equipment. The thermocouple is installed through the tank wall. Since one of the objectives of the experiment is to finish the experiment quickly, the data-logging was done with a microcomputer using an OPTO 22 interface.

The experimental procedure is straightforward:

With the pressure in the tank at atmospheric, close valves (V1 - V4). Start logging the pressure and temperature using a one-second sampling time. Open V5. The tank will reach the supply pressure in about ten seconds. Close V5 and open one or more valves in the manifold. The pressure in the tank will reduce to near atmospheric in about one minute, depending on which valve(s) is opened. At this time the data-logging can be stopped or the experiment repeated.

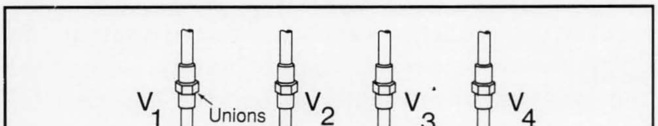

Figure 1.

The cost of the equipment is small (less than $100, not counting the instrumentation and microcomputer), and the total shop time required for machining and welding is less than two hours. Our advice is to purchase a new propane bottle and have the propane relief valve removed upon purchase since this valve is extremely difficult to remove. An entire class can perform the experiment in about two days by using scheduled 15-minute time slots, so the instrumentation and data-logging computer can usually be borrowed.

FILLING THE TANK

This part of the experiment focuses primarily on thermodynamics. As it is usually presented in thermodynamics texts, the theory for filling a tank with an ideal gas is correct—but the major assumption is wrong. The standard assumption is that the process is adiabatic. When the experiment is actually performed, however, the dominant influence is the heat transfer, not the thermodynamics. But, the students believe the textbooks and their professors—who also believe the textbooks!

Van Wylen and Sonntag[1] give the following equation as the appropriate form of the first law of thermodynamics over a control volume for a uniform state, uniform flow process:

$$\dot{Q}_{cv} + \sum m_i \left\{ h_i + \left(\tfrac{1}{2}\right) v_i^2 + gZ_i \right\} = \sum m_e \left\{ h_e + \left(\tfrac{1}{2}\right) v_e^2 + gZ_e \right\} + \dot{W}_{cv} + m_2 \left\{ h_2 + \left(\tfrac{1}{2}\right) v_2^2 + gZ_2 \right\} - m_1 \left\{ h_1 + \left(\tfrac{1}{2}\right) v_1^2 + gZ_1 \right\} \quad (1)$$

When we apply this equation to the filling of a tank, neglecting the heat transfer, work, potential energy changes, and all kinetic energy terms except for the input, we get

$$\dot{m}_i \left\{ h_i + \left(\tfrac{1}{2}\right) v_i^2 \right\} \cong m_2 u_2 - m_1 u_1 \quad (2)$$

When it is combined with the definitions of h and u for an ideal gas, this equation can be used to solve all the cases appropriate to this experiment. The simplest case occurs if the incoming kinetic energy and the initial mass of the system is neglected. Then

$$h_i \cong u_2 \quad (3)$$

If we assume constant heat capacities, then

$$T_2 \cong k\, T_i \quad (4)$$

where $k \equiv C_p / C_v$.

A better approximation of the final temperature can be made, *a priori*, by correcting for the initial mass of air in the tank. A further refinement can be made, after the experiment, by including an estimate of the incoming kinetic energy. Supposedly, the theory can be expanded or refined—however, our objective is to introduce the students to a simple laboratory experiment and to the difficulty of writing a technical report on such a simple experiment.

Table 1 shows a comparison of the measured final temperature and those calculated from Eq. (2), corrected for both the initial mass and kinetic energy of the incoming air but still assuming no heat transfer. The point to be made from this table is that all of the calculations are simply wrong and differ from the measurement by a minimum of approximately 200°F.

As soon as the students see the difference between the theory taught in thermodynamics lectures and the results of the experiment, they question the experiment. There is sufficient time to do multiple runs, but they find that the repeated experiments produce virtually the same results. The students are faced with an experiment which has precision but, in their minds, is of dubious accuracy. To explain the difference between the experiment and the temperature calculated from Eq. (4), many students do all the corrections indicated in Table 1. They work hard to prove that the theory is right and the experiment is wrong. Ultimately, they realize Eqs. (2) and (4) are simply not true and are based on a bad assumption, and finally they conclude that the process is not adiabatic and that the heat transfer is the dominant effect. A typical value of the heat transfer is 14 - 18 Btu/cycle.

VENTING THE TANK

At this point the students are convinced that experiments are worthwhile, but they are somewhat skeptical of theory. Venting of the tank is designed to resolve this conflict for them. The venting of air illustrates the polytropic decompression of an ideal gas and an unsteady state material balance.

The relationship between the temperature and the pressure of air in the tank, as it empties, must be established before the material balance is attempted. Analysis of the data is relatively simple and is covered in most introductory courses in engineering thermodynamics. The decompression of the air in the tank is taken to be a polytropic process. The appropriate equation in terms of the measured variables is

TABLE 1
Comparison of Experimental and Calculated Adiabatic, Final Air Temperatures

Method of Determination	Approximate Final Air Temperature
Experiment	100°F
Equation 4	315°F
Equation 2 (corrected for initial mass)	305°F
Equation 2	355°F

$$\frac{T}{T_0} \cong \frac{P^{(n-1)/n}}{P_0} \tag{5}$$

If the decompression is isothermal, n is equal to one. The process is isentropic when $n = k$. The value of n is found from the slope of the line through the measured pressure and temperature data when plotted on log-log paper. Many students use a regression program to estimate n from their data; however, we require the T versus P plot for educational reasons. Using the equipment and procedure described earlier, the typical experimental value of n was 1.04 ± 0.005. Even though the temperature of the air in the tank drops by approximately 85°F, the students conclude that the decompression process is better approximated by an isothermal process than by an isentropic one. This conclusion simplifies the mathematics of the material balance. A more accurate analysis is possible but is not worthwhile given the intent of the course.

After the students have established that the decompression process is approximately isothermal, the unsteady state material balance equation follows easily

$$\frac{dm}{dt} = -\dot{m}_N \tag{6}$$

Since the volume of the system is constant and the temperature is nearly constant, the material equation for the air in the tank is

$$\frac{dm}{dt} \cong \frac{V_{cv}}{RT}\frac{dP}{dt} \tag{7}$$

The mass flow rate of air through the nozzle(s) is

$$\dot{m}_N = \rho_N A_N v_N \tag{8}$$

Provided that the air pressure in the tank is above the critical pressure required for sonic velocity, the velocity of the air through the nozzle is equal to

$$v_N = \left(kRT_N\right)^{1/2} \tag{9}$$

The differential equation for the unsteady state material balance reduces to

$$\frac{V_{cv}}{RT}\frac{dP}{dt} \cong -\rho_N A_N \left(kRT_N\right)^{1/2} \tag{10}$$

The trick is to convert, in a simple way, the thermodynamic variables evaluated at nozzle conditions to those measured in the tank. The theory required for this transformation is fully developed in most mechanical engineering thermodynamics texts, though not in many chemical engineering texts. Our students do not deal with compressible flow in lecture courses until after the experiment. This problem is solved by simply stating that the thermodynamic variables at the nozzle can be evaluated at tank conditions by applying a correction factor. Students seem to like correction factors. The two relationships, shown below, are derived by Holman.[2]

$$T_N = T\frac{2}{k+1} \qquad (11)$$

$$P_N = P\left(\frac{2}{k+1}\right)^{\left(\frac{k}{k-1}\right)} \qquad (12)$$

When the change in mass inventory is equated to the mass flow rate out of the nozzle(s), the differential equation for the pressure in the tank is

$$\frac{dP}{dt} \cong -\frac{A_N}{V_{cv}}(kRT)^{1/2}\left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{2(k-1)}\right)}P \qquad (13)$$

Since the students previously established that the absolute temperature in the tank is approximately a constant, they can now write this differential equation in a short form as

$$\frac{dP}{dt} \cong -K_1P \qquad (14)$$

where K_1 is the constant term in Eq. (13). The solution, shown below, is simple; however, it is valid only until the critical pressure ratio is reached.

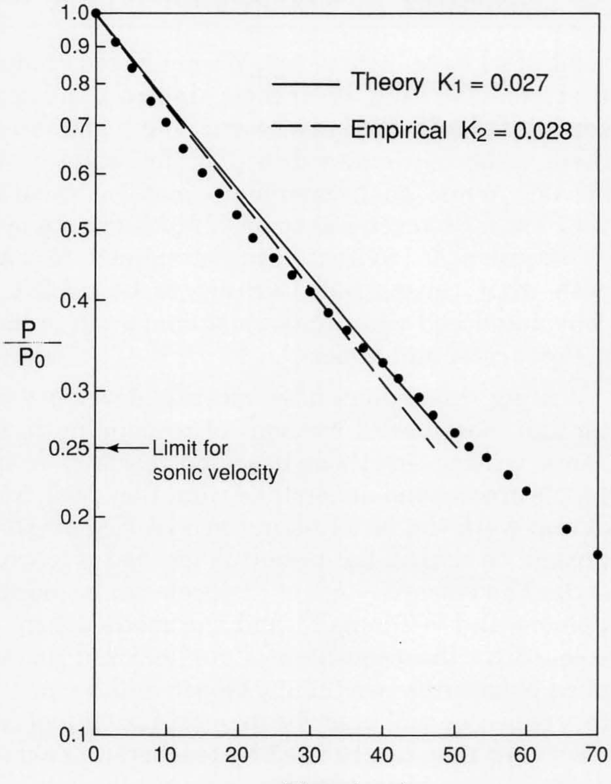

Figure 2.

$$\ell n\left(\frac{P}{P_0}\right) \cong -K_1t \qquad (15)$$

Another tack would be to regard this equation merely as the basis of a correlating equation. The equation would be the same as Eq. (15) but have a leading coefficient of K_2, as

$$\ell n\left(\frac{P}{P_0}\right) \cong -K_2t \qquad (16)$$

The students are required to plot the ratio of the measured pressure to the maximum pressure versus time on semi-logarithmic paper. They then compare the slope of the line determined by the data and Eq. (14) and that predicted by Eq. (13).

A typical set of data taken by the staff, a regression line based on Eq. (16), and the theoretical prediction from the solution of Eq. (15) are shown in Figure 2. A nozzle with an internal diameter of 3/32-inch was used. The empirical value of K_2 was found to be 0.028 based on ten runs with a total of 462 data points. The value of K_1 predicted by theory, at the average temperature, was 0.027.

About 90% of the students find that the difference in the slope of the lines between theory and data is 5% or less. This difference is not statistically significant given the inaccuracies in measurement of the nozzle diameters and the volume of the tank and manifold. Surprisingly, the other 10% of students, who predict differences of up to 200%, make the mistake of using the wrong nozzle(s) or recording the nozzle diameter(s) incorrectly. None of the students have difficulty with the mathematics, though some think that Eqs. (14) and (16) apply even when the velocity in the nozzle is subsonic.

CONCLUSION

The experiment described in this paper is ideal when the experiment requirements are a quick turnaround time, inexpensive equipment, and flexible technical content.

NOMENCLATURE

A = Area; ft^2
h = enthalpy; Btu/lb
K_1 = constant; defined by Eq. (14)
K_2 = constant; defined by Eq. (16)
ke = kinetic energy; Btu/lb
$\dot{m}$ = mass flow rate; lb/s
m = mass of system; lb
n = polytropic exponent
P = pressure; psia
$\dot{Q}$ = rate of heat transfer; BTU/cycle

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Fluid Mechanics Experiment

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R = gas constant; Btu/lb(R)
t = time; s
u = internal energy; Btu/lb
v = velocity; ft/s
V = volume; ft^3
$\dot{W}$ = power input; BTU/cycle
ρ = density; lb/ft^3

Subscripts

cv = control volume
e = exiting air
i = incoming air
N = nozzle
0 = time zero - valve opening
1 = initial state
2 = final state

REFERENCES

1. Van Wylen and Sonntag, *Fundamentals of Classical Thermodynamics,* 3rd ed., SI Version, John Wiley, New York (1985)
2. Holman, J.P., *Thermodynamics,* 4th ed., McGraw-Hill, New York (1988) ❒

REVIEW: Mass Transfer

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strongly recommended.

Chapter 17, on sorption processes, discusses fixed bed adsorption and ion exchange. The presentation on why loading and elution in ion exchange are not symmetrical is particularly clear and easy to understand. In general, the authors assume that the reader is familiar with these separation processes. Readers who are not (particularly electrodialysis) will find these chapters difficult, but readers familiar with the processes will gain deeper insight.

A third part of the book starts with Chapter 18, which compares the M-S, Fickian, and irreversible thermodynamics approaches to mass transfer. This is a very enlightening chapter, and sophisticated readers should read it following Chapter 2 or 3. Chapter 19 cites references. A rather complete list of symbols starts on page 160. I found myself referring to this list often and wish it were in a more prominent location.

The fourth part of the book consists of thirty-six worked exercises (pages 163 to 238) which consider some very interesting and challenging problems. Although the solutions are not polished, they are certainly sufficient to show how to attack the problems.

A major problem with this book is highlighted in the Guidelines to the Reader on page 11: "This text was written to accompany overhead transparencies in a course on multicomponent mass transfer. So the Figures are quite important." Unfortunately, many readers will not pay enough attention to this section and will find reading the book difficult until they have learned the proper way to read it. Also, since the figures are hand drawn, the reader needs to learn how to decipher the authors' script. The inclusion of equation numbers would be useful. Some of the examples are confusing since the problem statements are not clear (*e.g.,* Figure 6.2) and data or formulas are slipped into the solutions with little explanation (*e.g.,* Figures 3.7 and 5.5). Statements such as "Qualitatively the reasoning should be clear," (page 91) will unintentionally demotivate readers who are struggling, and they should be removed.

The basic ideas of the M-S approach are not summarized until pages 64 and 65. A much earlier exposition of this would help many readers. Also, since the authors assume considerable familiarity with mass transfer, Chapter 18 could appear earlier in the text. If a second edition is planned, the authors could aid readers by correcting these problems. One hopes that the authors will make this effort since the book presents a very important topic in a way which will be accessible to most readers.

Where can this book be used in the curriculum? The book is a curious mix of sophistication (M-S theory and challenging problems) and of approximations (difference solutions and overly simplified thermodynamics). Because of the subject matter and the assumed high degree of knowledge in mass transfer and separations, this text is appropriate at the graduate level. However, the approximations and some lack of rigor may cause difficulties. It book would be a very useful supplement in a graduate-level course, particularly if journal articles are used in most of the course. It is also a very good source of problems and examples for a graduate-level course.

Finally, for practicing professionals who missed the M-S theory in their formal education, this book would be very useful for self study. Wesselingh and Krishna will stimulate and frustrate, but the reader will never be bored. ❒