The object of this column is to enhance our readers' collection of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and which elucidate difficult concepts. Please submit them to Professors James O. Wilkes and Mark A. Burns, Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

WHEN IS A THEORETICAL STAGE NOT ALWAYS A THEORETICAL STAGE?

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The study of continuous distillation is one of the cornerstones of an undergraduate course in chemical engineering. Indeed, the presentation of the McCabe-Thiele construction is a well-rehearsed routine. Therefore an exercise which makes more experienced undergraduates reconsider widely used textbook asumptions and, in addition, to think more broadly about the subject is very useful.

BACKGROUND

The McCabe-Thiele analysis starts with a diagram much as the one shown in Figure 1.[1-4] The reboiler of Figure 1 must be a kettle (see Figure 2) in order to phase-separate vapor and liquid and meet the requirement that vapor return and bottoms be in equilibrium. Indeed, some undergraduate textbooks provide this detail.[5-7]

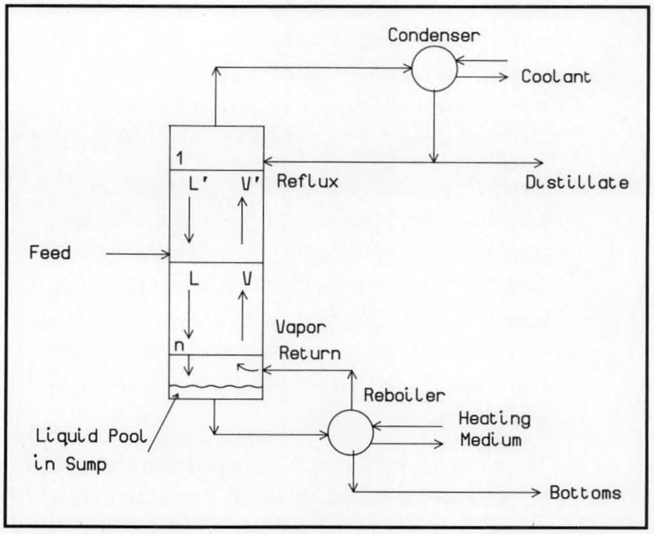
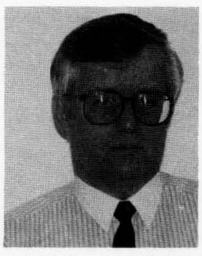


Figure 1

Kettle reboilers, however, are not widely used because they have some significant disadvantages. The most commonly cited disadvantages are:[8,9]

- The high cost of the large shell to permit vapor-liquid phase separation and provide bottoms surge volume.
- Their tendency to collect dirt and to foul.
- A liquid pool submerging the reboiler tubes, to a first approximation, operates uniformly at the bottoms composition and hence boils at the highest temperature; this narrows the temperature driving force and leads to increased heat transfer area.

It is important to note that kettle designs permit high vaporizations—up to 80% of the liquid from the bottom tray of the column. Columns can



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thus be designed with liquid/vapor traffic ratios as low as 1.25.

Thermosyphon reboilers are much more commonly used. They may be either vertical units with vaporization inside the tubes, or horizontal units with vaporization inside the shell. Thermosyphon designs rely on a head of liquid (achieved by precise positioning of the reboiler relative to the column) and careful piping design to force the return of a two-phase mixture to the column. Thermosyphons are popular because:[8,9]

- Their design normally allows for a high process flowrate through the unit; this is beneficial in reducing fouling and promoting a high heat transfer coefficient.
- Both phase separation and surge volume are moved to the column sump, where these functions are more easily managed.
- Boiling occurs over a range of temperatures, and hence there is an improved driving force for heat transfer.
- A cheaper overall construction is obtained because of the above factors.

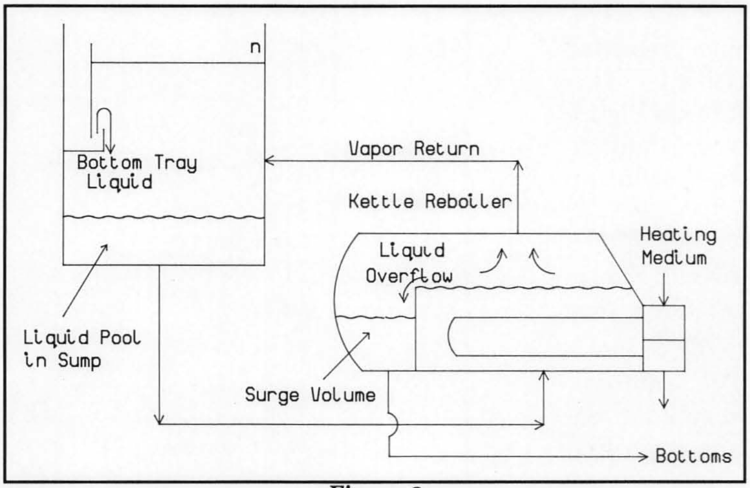

Figure 2

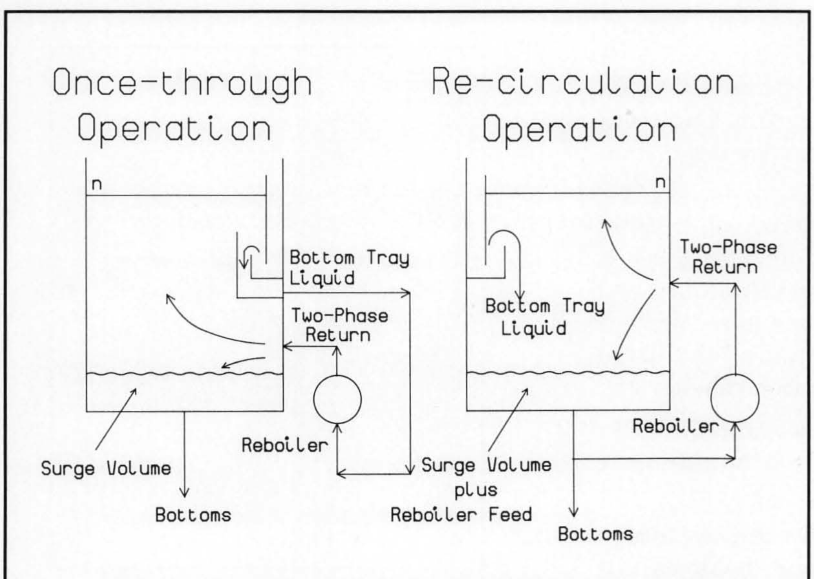

Figure 3a *Figure 3b*

One particular feature of thermosyphon design is a strict upper limit on the per-pass vaporization achievable, normally about 25% of the process liquid entering the unit with vaporization in the range of 5-15% being normal. Indeed, a high percentage of liquid in the two-phase return is an advantage because it ensures that the reboiler tubes are kept "wet," thus improving heat transfer and minimizing fouling. If a thermosyphon is connected to a column in an arrangement analogous to that for a kettle (contributing one theoretical stage with exiting vapor and liquid in equilibrium), then Figure 3a applies. But the upper limit on per-pass vaporization now poses a severe difficulty for the distillation design because liquid/vapor traffic ratios below 4.0 are not possible. Hence, once-through operation is only generally suitable for reboiled strippers.

Far more flexibility is achieved by recirculation operation (see Figure 3b). Now a low liquid/vapor traffic ratio can be achieved in the column because the process flowrate through the reboiler is boosted by recirculation of part of the returning liquid, keeping the per-pass vaporization to acceptable levels. There are two disadvantages, however:

1. The vapor rising from the reboiler toward the bottom tray is *not* in equilibrium with the bottoms liquid. Hence, the recirculation reboiler has a separation performance that is equivalent to less than one theoretical stage (this answers the question posed in the title of this article).
2. A portion of the bottoms liquid will experience a long residence time in the column sump, resulting in repeated contact with the hot tube surfaces of the reboiler—this may promote fouling.

Despite these disadvantages, recirculating reboilers are widely used, and adapting the McCabe-Thiele construction is a challenging exercise for more experienced undergraduates.

This exercise is useful, not only because it brings real industrial practice to the students' attention (often significantly different from the theory presented in textbooks), but also because it may be widened to include discussion of other aspects, such as

- Kettle versus thermosyphon reboilers
- Vertical versus horizontal thermosyphons
- Exchanger cleaning
- Thermosyphon operation
- Distillation column elevation
- Space requirements for different reboilers
- Detailed design of distillation column internals

This article concentrates on the McCabe-Thiele construction. Normally, it is helpful for the instructor to summarize some of the discussion contained in the "background" section above before students commence with the following "problem."

Problem Statement

The McCabe-Thiele construction for distillation is presented as taking credit for one theoretical stage in the reboiler. This is based on the traditional use of a kettle reboiler (see Figure 2). Industrial practice is often different, however, and involves using either a once-through or a recirculation thermosyphon (see Figure 3).

(a) Which of the two reboiler arrangements shown in Figure 3 is not equivalent to a theoretical stage? Give your reasons.

(b) The flows and more volatile component (MVC) compositions in the bottom section of a distillation column featuring a recirculation thermosyphon are shown in Figure 4. Derive the equation

$$x_b = \frac{Lx_n + (R - V)x_r}{(L + R - V)} \tag{1}$$

and use the result to assist sketching recirculation reboiler operation on a McCabe-Thiele diagram. Comment on the asymptotic behaviour when R is very large compared to L and V, and when R = V.

(c) A two-component mixture, having relative volatility of 2.0, is distilled to produce a bottom product with $x_b = 0.1$. If the liquid/vapor traffic ratio below the feed is 1.5, and 15% of the reboiler feed is vaporized per pass, draw a detailed McCabe-Thiele construction (making the usual simplifying assumption) for the recirculation reboiler and bottom tray.

(d) Express the separation performance as a liquid-phase Murphree efficiency.

(e) Devise column sump internals which give the advantage of recirculation but ensure that the reboiler behaves as a theoretical stage.

Solution

(a) The arrangement of Figure 3b is not equivalent to a theoretical stage as explained in the "background" section.

(b) Equation (1) is very simply derived as a MVC mass-balance around the column sump and may be interpreted as expressing bottom product composition as a blend of bottom tray liquid and reboiler return liquid. Figure 5 shows the required McCabe-Thiele construction where recirculation operation is represented as a partial step (rather like the construction taking into account tray efficiency) between operating and equilib-

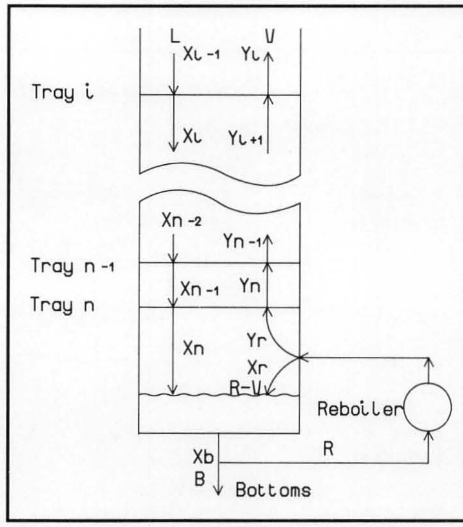

Figure 4

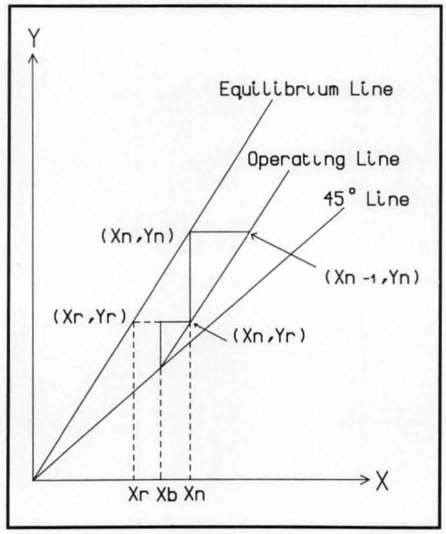

Figure 5

rium lines. Point (x_n, y_r) must lie on the operating line, and (x_r, y_r) on the equilibrium line; x_b lies between them in accordance with Eq. (1) and controls the step size.

When R is very large compared to L and V, then x_b approximates x_r and the system behaves as a theoretical stage.

When R = V (*i.e.*, the minimum reboiler feedrate), then x_b equals x_n and no separation is achieved.

(c) The general equation for the operating line, using the symbols of Figure 4, is

$$y_{i+1} = \frac{L}{V} x_i - \frac{B}{V} x_b \tag{2}$$

Equation (2) may be applied at any elevation below the feed. Hence

$$y_r = \frac{L}{V} x_n - \frac{B}{V} x_b \tag{3}$$

The reboiler return comprises vapour and liquid in equilibrium. Hence y_r and x_r are related by

$$y_r = \frac{\alpha x_r}{1+(\alpha-1)x_r} \tag{4}$$

Recirculating systems have an additional degree of freedom—namely the process flowrate, R, through the reboiler—and it is this variable that fixes the size of the partial step. An extra equation is required for the analysis, and it is obtained by a MVC mass-balance around the reboiler

$$R x_b = (R-V)x_r + V y_r \tag{5}$$

or

$$x_b = (1-f)x_r + f y_r \tag{6}$$

where the reboiler fraction vaporization, f = V/R, is a design parameter.

To perform the construction, x_b is known, but three compositions (x_n, x_r, y_r) are unknown. There are three equations in these unknowns, however—namely Eqs. (3), (4), and (6). Using the data in the Problem Statement,

$$\frac{L}{V} = 1.5, \quad \text{so} \quad \frac{B}{V} = 0.5, \quad \text{and} \quad f = 0.15$$

we obtain

$$y_r = 1.5\, x_n - 0.05 \tag{7}$$

$$y_r = \frac{2 x_r}{1+x_r} \tag{8}$$

$$0.1 = 0.85\, x_r + 0.15\, y_r \tag{9}$$

Eliminating y_r between Eqs. (8) and (9) gives a quadratic in x_r having feasible solution x_r = 0.08885. Figure 6 shows the completed construction where y_r = 0.1632 and x_n = 0.142.

(d) The normal convention is to express stage mass transfer efficiency in terms of vapor composition change. In this instance, however, there is no vapor feed to the stage, and the efficiency must be expressed in terms of liquid composition change,

$$E = \left(\frac{x_n - x_b}{x_n - x_r}\right) \times 100 = \left(\frac{0.142-0.1}{0.142-0.08885}\right) \times 100 = 79\% \tag{10}$$

Alternatively, if Eq. (1) is subtracted from $x_n = x_n$, the efficiency of the reboiler can be shown to be

$$\frac{E}{100} = \frac{R-V}{L+R-V} \tag{11}$$

Thus, knowing that L/V = 1.5 and V/R = 0.15, then L/R = 0.225, and we again obtain E = 79%. Equation (11) would be useful if the equilibrium data is not expressed in a simple form permitting analytical solution. Trial-and-error solution could be used to find x_r and x_n such that the predetermined efficiency is obtained.

(e) This part is a challenging exercise for students, since the problem is at two levels: first, finding a philosophy, and second, devising equipment

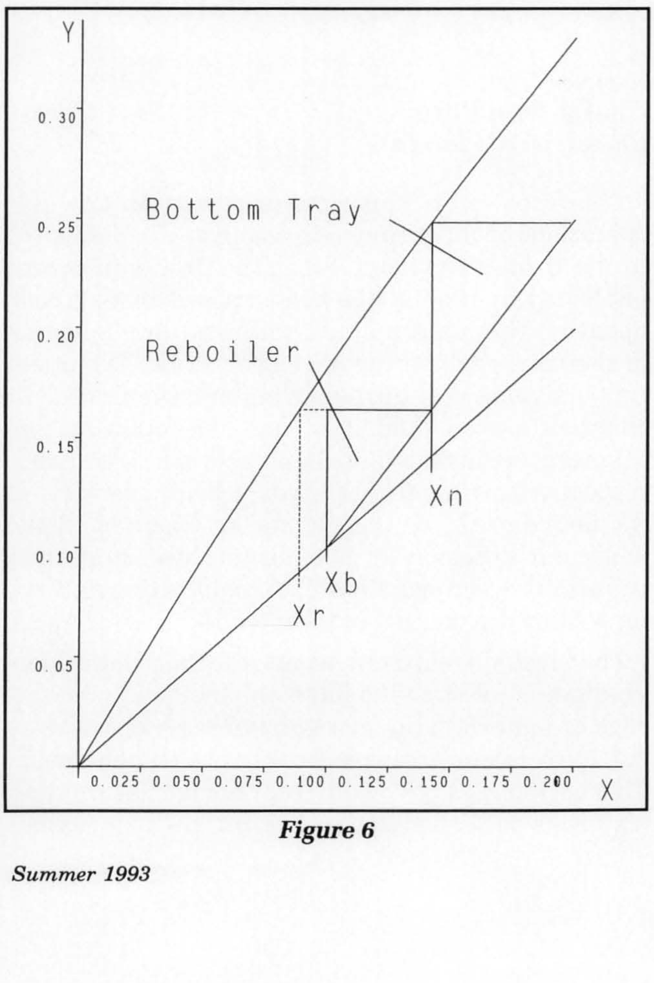

Figure 6

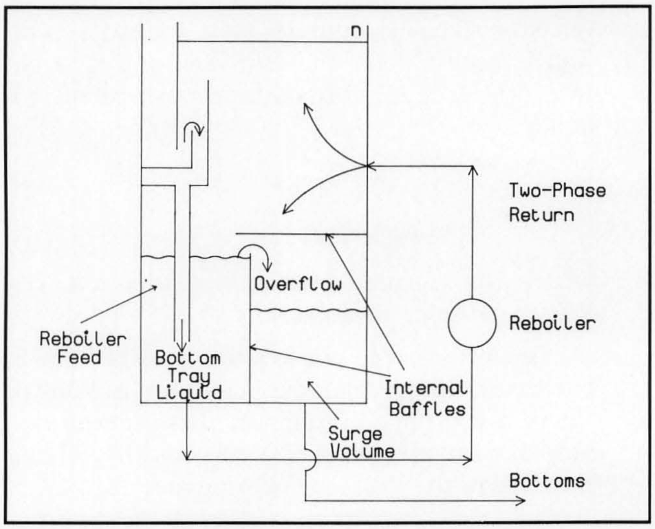

Figure 7

to implement the philosophy. In summary, the philosophy is to preferentially route the bottom tray liquid to the reboiler and sufficient reboiler return liquid to the bottoms. Excess reboiler return liquid (not needed as bottoms) joins the bottom tray liquid as reboiler feed. One way of implementing this on larger diameter columns is illustrated in Figure 7; note that the down-pipe from the trap-out mixes the bottom tray liquid into the excess reboiler return liquid well below the overflow level—this ensures the over-flow is predominantly reboiler return liquid. With this type of design, the vapor return and bottoms are in equilibrium so the system acts as one theoretical stage.

INDUSTRIAL PRACTICE

The loss of part of a theoretical stage is widely recognized in industry, but response varies considerably. At one extreme some companies do not take credit for the separation due to the reboiler (treating it as a safety factor) unless it is a kettle when full credit is taken for one theoretical stage. At the other extreme, others simply count the reboiler (irrespective of type) as one theoretical stage—but then a conservative tray efficiency applied to the column hides any shortcoming. Based on the approach in this article, it seems that a recirculating reboiler generally has a separation efficiency of at least 60%. Hence, even though one might not wish to actually perform the construction as described, it would be quite safe to credit the reboiler with a separation equivalent to an efficiency of 50%.

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ChE ***book review***

CHEMICAL ENGINEERING: *Vol. 1. Fluid Flow, Heat Transfer and Mass Transfer*

by J.M. Coulson and J.F. Richardson, with J.R. Backhurst and J.H. Harker

Pergamon Press, Headington Hill Hall, Oxford, OX3 0BW, United Kingdom; $48 (paperback) (1990)

Reviewed by
Chang-Won Park
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This book is an undergraduate text on the unit operations of chemical engineering and is published in the United Kingdom. Since the first edition was published in 1954 it has been revised three times, updating the material as significant developments in chemical engineering have been made. The material is divided into thirteen chapters: the first eight chapters are on fluid flow, and the following two chapters are devoted to heat and mass transfer, respectively; Chapter 11 gives a brief overview of the boundary layer theory, and in Chapter 12 the molecular diffusion in momentum, heat, and mass transfer is described; finally, humidification and water cooling are treated in Chapter 13.

The level of treatment seems adequate for undergraduate students who have an elementary knowledge of material and energy balances but who may not have taken a course in transport phenomena. This book uses slightly different nomenclature than textbooks published in the U.S., but not to an extent