industrial pollution prevention under the direction of Pete Counce.

The undergraduate curriculum allows some flexibility for specialization through the inclusion of four technical electives. Three 3-hour technical elective courses may be selected from a wide variety of advanced engineering, science, mathematics, or business courses, and the fourth 3-hour elective is taken in advanced chemistry or other advanced science (*e.g.*, microbiology, materials science). Possible areas of specialization include biotechnology, process control, industrial pollution prevention, and polymer engineering; a designated faculty member in each area can advise students on proper elective selection. Other areas of emphasis can also be developed in consultation with a faculty advisor.

The department participates actively in the Engineering college program of videotaped, off-campus graduate instruction. Core and elective courses in the Master's program are offered on a regular cycle to students at off-campus sites, including Oak Ridge, The Kingsport University Center, and other chemical plants in Tennessee. Additional videotaped graduate courses will be offered as dictated by demand. When student demand is sufficient, graduate courses are also taught "live" at Oak Ridge and Kingsport. Students may complete MS degrees at these locations (and the PhD at Oak Ridge) without a period of full-time residence in Knoxville.

### A CLOSING THOUGHT

Over the years, The University of Tennessee Chemical Engineering Department has enjoyed a happy combination of assets: faculty members with a wholeheared commitment to students in their roles as research directors, as graduate and undergraduate teachers, and as academic and career advisors; a highly trained technical support staff for both instrumentation/computers and mechanical systems, who provide invaluable support for experimental research and instruction laboratory development; and last but not least, a capable and caring secretarial staff whose motto is "how can I help you?" and not "go away and don't bother me!" Our chemical engineering graduates may soon grow hazy about the finer points of transport phenomena or the Second Law of Thermodynamics, but they will remember clearly how the departmental secretaries, Sancy Hail and Betty Frazier, created within a large and sometimes impersonal university a warm atmosphere of genuine care and concern for their welfare. 🗖



Thank you very much for your attention.

José Miguel Loureiro, Associate Professor Dept. de Engenharia Quimica • Univ. do Porto Rua dos Bragas • 4099 Porto Codex, Portugal

**Response to Letter to the Editor** 

To the Editor:

The letter of Prof. Loureiro contains several misconceptions which are very important to correct. These misconceptions are:

- 1. The results of a stability analysis for a linearized system are not valid for a nonlinear system.
- 2. A stable periodic solution is equivalent to a stable steady state.
- 3. The stability of a system can be deduced from a small initial response of the system to a disturbance.

These misconceptions will be discussed separately.

Relationship between stability of the nonlinear and the linearized system.

This relationship was established by the Liapunov theorem (see for example, reference 1). This theorem deals with the stability of a nonlinear system in the vicinity of a particular critical (steady state) point, and it states that "If the linearized solution is unstable, then the actual operation (as described by the nonlinear equations) will be unstable...." The meaning of Liapunov's theorem is that instability indicated by linear stability analysis is a sufficient condition for

Dear Sir:

This letter is motivated by the paper "Exothermic CSTRs—Just How Stable are the Multiple Steady States?" by Shacham, Brauner, and Cutlip, that appeared in the winter 1994 issue of *CEE*.

The authors argue that the upper steady state in an exothermic CSTR can be unstable; this is wrong and, as a matter of fact, it results from a misunderstanding of the mathematical stability concept.

Their conclusion stems from a linearized analysis of the problem which is not exact around that steady state; as they say in their paper, "...when it is integrated for a long enough time, the basic model will produce a limit cycle." As their basic model is, in reality, the nonlinearized model, we can believe that this is the correct answer to the problem; but, and this is the main mistake, a **limit cycle is a stable** periodic **solution**, as any standard textbook would teach them! Even if the oscillations were much larger than the ones observed in their Figure 6, where the "terrible oscillations" (which they incorrectly interpret as instabilities) are of the order of  $0.1^{\circ}R!$  Finally, one can say that one thing is the stability of the steady state, which can, in fact, be concluded from a plot with both the heats generated and removed (their Figure 1) and another is the nature of this stable steady state (limit cycle, in this case), which cannot be established from that plot.

Let's then restate that when three steady states result from the intersections of the lines of the heat generated and the heat removed, the upper and lower are stable (irrespective of the nature of these steady states), while the middle one is unstable. instability of the nonlinear system. Liapunov's theorem is the basis for the linear control theory which has been used successfully for decades in control system design, and it certainly cannot be dismissed in an offhand manner.

Relationship between stable periodic solution and stable steady state.

In order to clarify the behavior of the system in the vicinity of the upper steady state, we have prepared a figure which extends the simulation time scale for the basic model of the original paper. Figure 1 shows the three variables associated with the CSTR ( $C_A$  - outlet concentration; T - temperature inside the CSTR;  $T_j$  - temperature in the cooling jacket) after the model equations have been integrated for up to 150 hrs (using a powerful numerical integration package: DDASSL). Initially the variables begin to oscillate with monotonically increasing amplitudes, and after about 100 hours the major oscillations continue with constant amplitudes. These constant amplitudes oscillations provide what is called "stable limit cycle" in a phase plane diagram. This is indeed a stable periodic solution of the dynamic equations, as Prof. Loureiro refers to it, but it has nothing to do with a stable steady state.

Steady state is defined as a state where none of the variables change with time. A stable steady state is indicated where the



system returns to the original steady state after some time when a small perturbation is applied to the system. Clearly a reactor oscillating between 634°R and 674°R (as see in Figure 1) cannot be considered as being at steady state.

It should also be mentioned that the oscillations are not symmetrical around the upper steady state. The center of the oscillations is  $T \sim 654^{\circ}R$  where at the upper steady state it is T = $651^{\circ}R$ . Thus there is no direct con-

Figure 1. Simulation results to large time for a disturbance of the basic model.

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nection between the limit cycle and the upper steady state.

Small magnitude initial response as an indication of stability.

Prof. Loureiro notes that the magnitude of the initial response of the CSTR to disturbance is small  $(0.1^{\circ}R)$ , and as such it is judged as unimportant.

The disturbance that was introduced to the system in the paper was just due to the numerical values used as initial conditions in the numerical simulation. The upper steady state values of the temperatures were input with 5 decimal digits accuracy, and the concentration with 4 digits. These initial conditions were enough to perturb the system from the upper steady state. Thus this disturbance in the temperature was of the order of 10-3°R and the immediate system response was a hundred times larger. Obviously much bigger initial responses can be obtained by increasing the disturbance in many ways. For example, changing the steady state concentration from 0.0591 to 0.06 leads to an initial temperature oscillation with an amplitude of 4°R. The initial amplitude values are less important than the dynamic trends which develop after a disturbance. Figure 6 in the published paper<sup>[2]</sup> indicates growing oscillations, and Figure 1 clearly shows that the oscillations continue to increase dramatically until a stable limit cycle is reached.

In conclusion, we have shown using both a stability analysis of the linearized system and a numerical simulation of the nonlinear system that the upper steady state of this particular CSTR is unstable for the basic model. Prof. Loureiro's contentions that upper steady state is stable and is at steady state are just not correct, as we have demonstrated in this letter. An excellent summary of the mathematics of CSTR multiplicity and stability is given by Uppal, et al.<sup>[3]</sup>

#### Mordechai Shacham Neima Brauner Michael B. Cutlip

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