A SIMPLE PROCESS DYNAMICS EXPERIMENT

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Process dynamics is concerned with analyzing the time-dependent behavior of a process in response to an input change. Understanding the *dynamic* behavior is essential for process design, for selection of optimal operating conditions, and for implementing process control strategies, but a majority of experiments in a typical unit operations laboratory focus on the *steady-state* behavior of chemical processes. In this paper, we will describe a simple, low-cost experiment for analyzing the dynamic behavior of a second-order system. While the experiment is easy to perform, it requires the student to combine analytical as well as computational skills to analyze experimental data.

EXPERIMENTAL SET-UP

A schematic of the experimental setup is shown in Figure 1. There are two tanks connected in series, with each tank having a cross-sectional area of 48.65 cm^2. Water is fed to Tank 1 from an overhead reservoir. The reservoir has an overflow pipe near the top and is continuously supplied with water. The overflow pipe ensures that a constant flow rate is maintained at the inlet of the first tank. A strip of masking

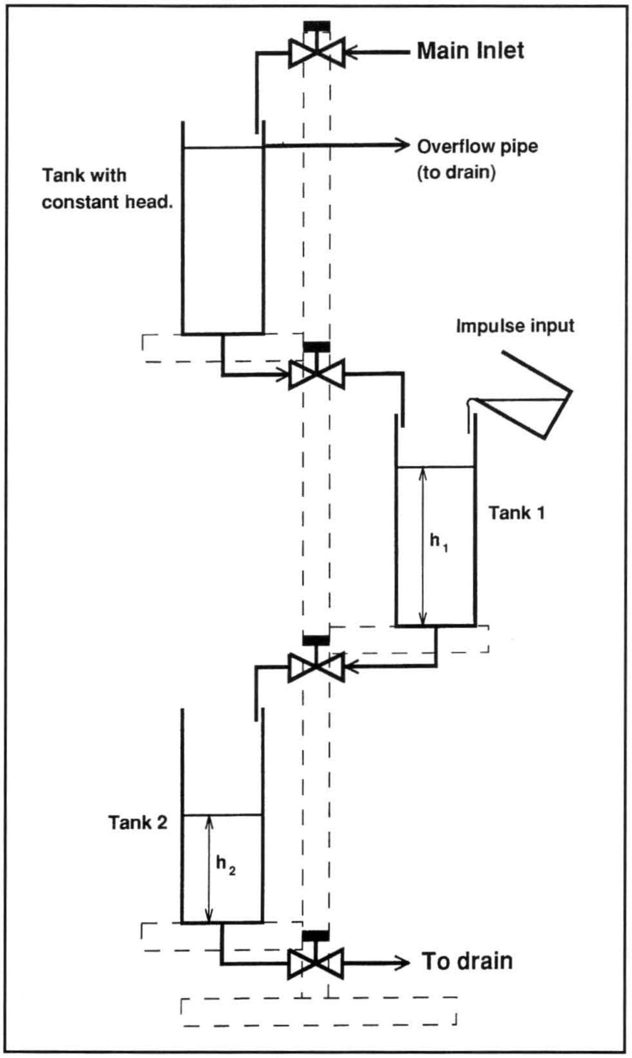
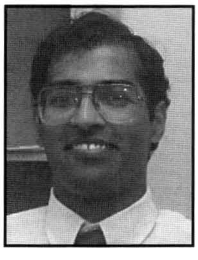


Figure 1. *Schematic representation of the experimental set-up.*



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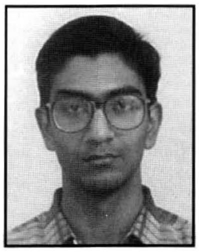

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tape is attached to the side of each tank so that the water level at any given time can be marked off on the tape. The flow rates to the first tank and the second tank can be adjusted by ball valves.

Initially, the water levels in the two tanks are at steady state. The objective of the experiment is to predict how the water level in the two tanks changes with time when a beaker full of water is suddenly added to the first tank, and to verify this prediction experimentally.

THEORY

Assuming that the cross-sectional area of both tanks is uniform and equal to A, and that the density of water is constant, an unsteady-state mass balance around each individual tank results in the following equations:

$$A\frac{dh_1}{dt} = q_{in} - q_1 \qquad \text{and} \qquad A\frac{dh_2}{dt} = q_1 - q_2 \tag{1}$$

where q_{in} and q_1 are the volumetric flow rates of the inlet and outlet streams of Tank 1, and q_2 is the volumetric flow rate of the outlet stream of Tank 2. The flow rates in the exit lines of the two tanks depend on the pressure drop, which in turn depends on the water levels h_1 and h_2 in the two tanks.

LINEAR MODEL

If a linear head-flow relationship is assumed, we get

$$q_1 = \frac{h_1}{R_1} \qquad \text{and} \qquad q_2 = \frac{h_2}{R_2} \tag{2}$$

where R_1 and R_2 are the flow resistance terms of the pipes exiting from Tank 1 and Tank 2, respectively.

Substituting Eq. (2) into Eq. (1), and putting in matrix form, we get

$$\frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{AR_1} & 0 \\ \frac{1}{AR_1} & -\frac{1}{AR_2} \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} q_{in} \tag{3}$$

At steady state

$$\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$$

Thus, from Eq. (3)

$$R_1 = \frac{h_{1s}}{q_{ins}} \qquad \text{and} \qquad R_2 = \frac{h_{2s}}{h_{1s}} R_1 \tag{4}$$

where h_{1s} and h_{2s} are the steady-state values of the water level in Tank 1 and Tank 2, respectively, and q_{ins} is the steady-state flow rate.

It can be easily shown[1] that when q_{in} is subjected to an impulse change of strength M, and $R_1 \neq R_2$, the impulse response is given by

$$h_1 = h_{1s} + \frac{M}{A}\exp\left(-\frac{t}{AR_1}\right)$$

$$h_2 = h_{2s} + \frac{MR_1}{A(R_1 - R_2)}\left(\exp\left(-\frac{t}{AR_1}\right) - \exp\left(-\frac{t}{AR_2}\right)\right) \tag{5}$$

Eq. (5) provides analytical expressions that predict the water levels h_1 and h_2 with time.

A simple low-cost dynamics experiment is described in this paper. The apparatus costs less than \$100 to build. The experiment introduces the concept of process dynamics and illustrates the differences between linear and nonlinear model prediction. The students are tested on their analytical as well as their computational skills.

NONLINEAR MODEL

The valve discharge rate in each tank can be modeled by the following square-root law:[2]

$$q_1 = C_1\sqrt{h_1}$$

$$q_2 = C_2\sqrt{h_2} \tag{6}$$

where C_1 and C_2 are the valve coefficients of valves 1 and 2, respectively, and depend on the valve openings.

Substituting Eq. (6) into Eq. (1), the following nonlinear model is obtained:

$$A\frac{d}{dt}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -C_1\sqrt{h_1} \\ C_1\sqrt{h_1} - C_2\sqrt{h_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} q_{in} \tag{7}$$

At steady state,

$$\frac{dh_1}{dt} = \frac{dh_2}{dt} = 0$$

Thus, from Eq. (7)

$$C_1 = \frac{q_{ins}}{\sqrt{h_{1s}}}$$

$$C_2 = C_1\sqrt{\frac{h_{1s}}{h_{2s}}} \tag{8}$$

When a beaker of water of volume M is suddenly added to Tank 1, the initial conditions of the system represented by Eq. (7) are given by

$$h_1(0) = h_{1s} + \frac{M}{A}$$

$$h_2(0) = h_{2s} \tag{9}$$

The nonlinear model represented by Eq. (7) can be integrated numerically using the initial conditions given by Eq. (9) to predict how the water level in the two tanks changes with time.

EXPERIMENTAL PROCEDURE

1. Set up the apparatus as shown in Figure 1.
2. Start the flow rate of water into the reservoir and wait until steady state is reached. Note the area of cross section of the two tanks.
3. Measure the steady-state heights in the two tanks.
4. Measure the steady-state flow rate of water coming out of the second tank with the help of a graduated cylinder and stop watch.
5. Take a beaker of water (about 400 ml) and measure the volume of water in a graduated cylinder. Add this beaker of water to the first tank. At the same time, start the stop watch and mark off the level of water in both tanks.
6. Mark off the level of water in both tanks in 10-second intervals for 120 seconds.
7. Measure the water level recorded with time on the masking tape attached to each tank.
8. Plot the water level in each tank versus time.
9. Using the steady-state values of the water level in each tank and the steady-state flow rate, calculate the flow resistance R_1 and R_2 in the linear model using Eq. (4).
10. Plot the analytical solution given by Eq. (5) on the same graph as the experimental data.
11. Using the steady-state values of the water level in

Table 1
Steady State Measurements

A	=	48.65 cm^2
h_{1s}	=	24.5 cm
h_{2s}	=	16.6 cm
M	=	400 cm^3

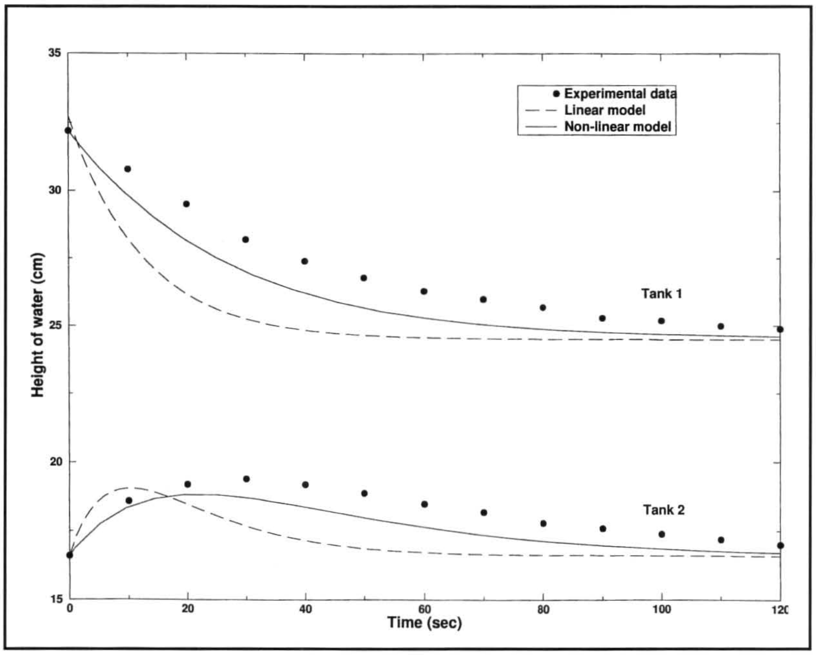

Figure 2. *Graph showing comparison between the two models and their fit with the experimental data.*

each tank and the steady-state flow rate, calculate the valve coefficients C_1 and C_2 in the nonlinear model using Eq. (8).

12. Integrate the nonlinear model numerically using the initial conditions given in Eq. (9).
13. Plot the numerical solution given of the nonlinear model on the same graph as the experimental data.
14. Comment on the accuracy of the linear model and the nonlinear model.

TYPICAL RESULTS AND DISCUSSION

Table 1 shows the experimental values obtained in a typical experimental run. Figure 2 shows the prediction of the linear model and the nonlinear model along with the experimental data points. A program in MATLAB™ [3] to generate Figure 2 is shown in Table 2. We note that the nonlinear model prediction of the experimental data is better than the linear model prediction.

A comparison of the initial volume of water in Tank 1 and Tank 2 with the volume of water added at the initial time shows that there is a 33.6% deviation from the initial steady state of Tank 1. This deviation may be too large for the linear model to predict dynamic behavior accurately.

It is assumed that the beaker of water is added instantaneously. This addition takes finite time, however, and so the use of an impulse response is an idealization of the actual pulse response that might lead to some discrepancy between predicted and experimental values. Errors in the experiment could also result due to lack of coordination between the student who is keeping time and the students who are marking off the water level in the two tanks.

TABLE 2
MATLAB Program

```
% Numerical Integration of Nonlinear Model
t0 = 0;
tf = 120;
x0 = [32.7 16.6];
[t,x] = ode23('ode', t0, tf, x0);

% Analytical Solution of Linear Model
t1 = linspace(0, 120, 100);
y1 = 24.5 + 8.222 * exp(-t1/12.6);
y2 = 16.6 + 17.435 * (exp(-t1/12.6) - exp(-t1/8.562));

% Experimental Data
t2 = [0; 10; 20; 30; 40; 50; 60; 70; 80; 90; 100; 110; 120];
h1 = [32.7; 30.8; 29.5; 28.2; 27.4; 26.8; 26.3; 26; 25.7; 25.3; 25.2; 25; 24.9]
h2 = [16.6; 18.6; 19.2; 19.4; 19.2; 18.9; 18.5; 18.2; 17.8; 17.6; 17.4; 17.2; 17]

% Comparison between solutions
plot(t, x, t1, y1, '--' t1, y2, '--', t2, h1,'+',t2, h2, 'o'

function xdot = ode (t,x)
xdot(1) = 1.942 - 0.392 • sqrt(x(1));
xdot(2) = 0.392 * sqrt(x(1)) - 0.477 * sqrt(x(2));
```

HOMEWORK EXERCISE

1. Derive Eq. (5).
2. The derivation of Eq. (5) assumes that $R_1 \neq R_2$. Under what physical conditions would $R_1 = R_2$? Derive the impulse response when $R_1 = R_2$.
3. For what magnitude of the impulse input M is the linear model prediction close to the nonlinear model prediction?
4. If the system is subjected to a step input of magnitude M, what are the predictions of the linear and nonlinear models?

CONCLUSIONS

A simple low-cost dynamics experiment is described in this paper. The apparatus costs less than $100 to build. The experiment introduces the concept of process dynamics and illustrates the differences between linear and nonlinear model prediction. The students are tested on their analytical as well as their computational skills.

NOMENCLATURE

A	=	area of cross section of Tank 1 and Tank 2, cm^2
C_1	=	valve coefficient of Valve 1
C_2	=	valve coefficient of Valve 2
h_1	=	water level in Tank 1, cm
h_2	=	water level in Tank 2, cm
q_{in}	=	volumetric flow rate into Tank 1, cm^3/sec
q_1	=	volumetric flow rate out of Tank 1, cm^3/sec
q_2	=	volumetric flow rate out of Tank 2, cm^3/sec
R_1	=	flow resistance of Pipe 1
R_2	=	flow resistance of Pipe 2
t	=	time, sec

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