

# COOKING POTATOES

## *Experimentation and Mathematical Modeling*

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During the past ten years, application of established chemical engineering principles has increasingly become an important branch of this traditional discipline. Many of today's chemical engineering graduates are employed by the food industries in our part of the world. Through the ever-increasing understanding of the unique chemical and microstructural nature of foods, and the principles and theories that are written in the language known to food technologists or food scientists, certain advancement of the chemical engineering discipline has also occurred.

There have been a number of emerging technologies, specially developed for food processing, that provide highly stimulating grounds for study by chemical engineers. Today, the boundaries between the related disciplines are dissolving and it is not at all uncommon for a chemical engineering department to be involved in industrial food microbiology, material-science-type research on food structures, etc.<sup>[1]</sup>

There is increased interest in introducing chemical engineering students to food concepts. The food industry is one of the prime industries in New Zealand, so understanding the key aspects of food processing is advantageous to undergraduates because it improves their prospects for employment. On the other side of the Tasman Sea, Australia has a large agri-food business worth some \$60 billion per year. The food manufacturing industries turn over approximately \$40 billion per annum. Indeed, there is a genuine push in the industries to have a greater number of qualified process engineers who are familiar with food processing.

Over the years our degree program (BE in chemical and materials engineering) has shifted from the traditional British-origin curriculum to a more local-industry orientation. A food process engineering course as a final-year elective has become very popular. Over the two years we have offered the course, however, the challenge has been to "compress" the vast amount of information on food science, technology, and practice into a mere 36-lecture-hour course.

Laboratory practice is crucial where foods are dealt with. The intention for such labs in our situation is generally to

minimize the cost and food-storage requirement while maintaining effectiveness and good reproducibility.

Potatoes are a fairly robust product and are genuinely cheap. The experiment suggested by Fraser<sup>[2]</sup> fits well with our requirements, which include cost effectiveness, less complication in data interpretation, and using a common food material that is a good representative of a starchy food. Fraser proposed a simple mathematical model of cooking potatoes, which can be solved analytically. This makes it easier for the undergraduate student to carry out data analysis without the use of numerical methods. For this reason, this experiment has been chosen as one of three that are run in conjunction with our formal lectures.

The original model analysis described by Fraser,<sup>[2]</sup> however, was not directly related to physical and chemical changes in the food. This part of the experiment needed to be enhanced since the primary aim of the student labs is to link engineering principles to food technology, and cooking is one of the main processes used in food manufacturing. In particular, the heat of reaction, obtained by fitting the model to the experimental data on cooked thickness, etc., has been found in our labs to be much larger than that reported in the literature. It was necessary for us to be able to explain this discrepancy.

This paper describes a modified approach to the same lab. In particular, the microstructure aspects of the experiment have been highlighted. The key aspects of the results of



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detailed background readings and the revised laboratory procedure and data analyses are given here.

In addition, a more consistent mathematical model that compares well with experimental data and published kinetics parameters is presented. The model provides a good understanding of the process, which students have found useful. In particular, the laboratory conveys a strong message that applying chemical engineering principles to food processing is not just a one-way approach—it requires combining knowledge from both engineering and food science.

## THE EXPERIMENT AND AN ANALYTICAL MODEL

The experimental procedure is simple. Essentially, students are asked to heat the potatoes in a temperature-controlled and mechanically-stirred water bath that is held at 98°-99°C. They are given ready-made thermocouples ( $\pm 0.5^\circ\text{C}$ ) (calibrated beforehand by technicians) and corresponding digital meters in order to assess certain assumptions made in the lab manual and to monitor the process closely.

The whole potato needs to be washed and cleaned before cooking in boiling water. The procedure should be straightforward. Students are expected to draw up an experimental plan, generate their own standard data-recording sheets for their group/team of three to four students, and discuss their intentions with the lab demonstrator prior to the start of the lab. They must investigate the validity of the model proposed by Fraser in light of the fact that the heat of the reaction has been found to be much greater than the reported value. The following tasks are assigned:

- ❶ Measure the cooked thickness and the temperature at the interface between the cooked and uncooked regions.
- ❷ Document all the raw data and suggest the likely uncertainties in such experiments.
- ❸ Write a brief error analysis.
- ❹ Plot the necessary diagrams showing the experimental results and the theoretical predictions, and suggest why there may be large discrepancies.
- ❺ Is there an alternative approach to modeling the process? (This is a teamwork problem.)
- ❻ Is there another geometry that you would employ to avoid certain physical limitations of the procedure for testing the whole potato?
- ❼ Calculate the heat of reaction and examine the validity of the assumptions used in the simple analytical model.
- ❽ Calculate how much heat is required to cook one kilogram of potatoes.
- ❾ Calculate how long it will take to cook the potato, assuming room temperature is 20°C.

A peripheral question also asked in the laboratory was, “Should the skin be intact? Can you give a reason why people tend to leave the skin on when boiling?” A few students suggested that it prevents the nutrients from being leached out, which is a reasonable answer.

The laboratory is reasonably interesting because it deals

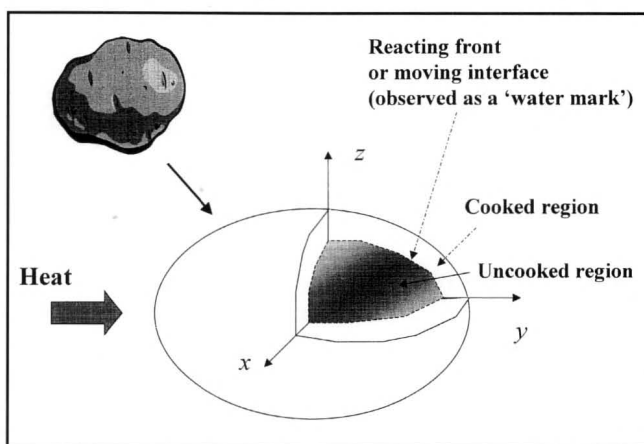
with a normal household material that is not usually the object in other labs, and because students are surprised by how distinct the “water mark” is between the cooked and uncooked regions (see Figure 1). Most of them never paid any attention to this phenomenon before. After these labs, it became apparent to the students that you can, in fact, use the basic principles of chemical engineering to adequately deal with food processing problems, provided you have the literature information or have measured the reaction and thermal properties of the material in question.

In any case, let us look first at the model approach as proposed by Fraser. This simple model is very useful as it provides a manageable formula to use in correlating the data. In order to model the process, a number of simplifying assumptions have been made:

- A potato can be treated as a uniform spherical body. (It was a difficult task to select potatoes which were actually spherical!)
- The rate of cooking is determined by the rate at which heat arrives at the cooking interface.
- All the heat conducted through the shell to the interface is consumed by the interface cooking reaction. (This assumption is perhaps based on the heat of reaction being much larger than normal heating effects.)
- The driving force for heat transfer can be considered constant.

Previous experience (in Fraser) indicates that temperatures of the outside of the potato and the cooking interface stay roughly constant at 98°C and 65°C, respectively. Our own results validate this argument. The reacting surface was found to be at  $65 \pm 1^\circ\text{C}$ .

In order to develop the differential equation for this system, an expression for conduction through a spherical shell is required (see Figure 2). According to Fraser,<sup>[2]</sup> this, plus all the assumptions mentioned above, leads to a differential equation that is a function of the outside radius and the radius of uncooked potato at any particular time (refer to Figure 3).



**Figure 1.** An illustration of the “water mark” between the cooked and uncooked regions of a potato after being heated in water for a few minutes.

$$\frac{dr_i}{dt} = \left( \frac{4\pi k\Delta T}{\Delta H_R \rho} \right) \left( \frac{1}{r_i^2 \left( \frac{1}{r_o} - \frac{1}{r_i} \right)} \right) \quad (1)$$

where

$r_o$  = the outer radius of the potato (m)

$r_i$  = the radius of the uncooked potato (m), *i.e.* the location of the reacting front or interface

$k$  = the thermal conductivity of the potato ( $\text{Wm}^{-1}\text{K}^{-1}$ )

$\Delta H_R$  = the potato heat of reaction based on the total mass,  $\text{Jkg}^{-1}$  (note: in Fraser's<sup>[2]</sup> original paper, this was defined as the molar heat of reaction—his approach then requires a molecular weight of potato to finally calculate the heat in our procedure)

$\rho$  = the potato density ( $\text{kgm}^{-3}$ )

$\Delta T$  = the temperature difference between the outside of the potato and the cooking zone (K)

$\Delta H_R$  in Joules per kg of potato (flesh) is more practical than Joules per mole of potato.

In fact,  $4\pi$  should not be in the constant term in Eq. 1. As such, this equation is solved analytically, giving a cubic relation between the uncooked radius and time

$$\frac{1}{3r_o} r_i^3 - \frac{1}{2} r_i^2 + \left( \frac{1}{6} r_o^2 - \left( \frac{k\Delta T}{\Delta H_R \rho} \right) t \right) = 0 \quad (2)$$

If the assumption is reasonable, the value in the bracket in front of  $t$  would be more or less a constant

$$\phi = \left( \frac{k\Delta T}{\Delta H_R \rho} \right) = \text{constant} \quad (3)$$

After the experimental results of  $r_i$  at time  $t$  has been obtained, a value for  $\phi$  can be generated. The results can then be illustrated by plotting  $r_i$  versus  $t$  for the experimental and modeled (based on the  $\phi$  calculated).

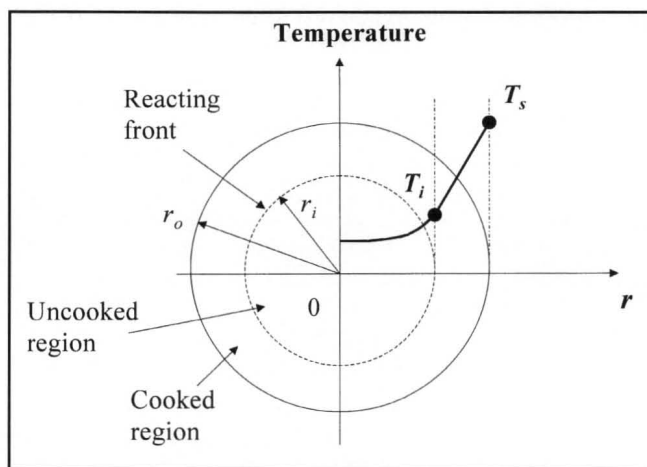
The students in fact produced good fits of the results to such a model (see Figure 4 for an example), but the heat of reaction calculated must be much larger than the reported values as measured by differential scanning calorimetry (DSC). The reported values for potato starch is between 21 and 23  $\text{kJ}\cdot\text{kg}^{-1}$  potato starch. Since potato starch takes up about 18 to 22% of the weight of potato flesh, one expects that about 20% of the value for the pure starch would be the true value for  $\Delta H_R$  for the potato flesh. As mentioned in the procedure, the students were asked to examine the assumptions made before the solution was obtained. The strongest was suggested to be the third assumption, as it neglected the heat conduction from the "reacting front" into the core region.

Since there is definitely a thermal gradient next to the interface (in the uncooked region) there must be a heat "loss" into the core region. The simple model (Eq. 2) does

not account for that. A more realistic approach, to be outlined later in this paper, was then necessary. In particular, it was necessary to show that, for the food experiments, the existing chemical engineering principles plus the basic quantitative information on the physical properties can be used to quantify such a process accurately. Furthermore, when looking closely at the data reported by Fraser (see Figure 3), there is an upward trend of the uncooked thickness versus time curves. This is, in fact, captured by the analytical solution (Eq. 2), illustrating that it is the inherent property of the spherical geometry (see Figure 4 also). Because there is no information in Fraser's paper about the initial potato sizes used in obtaining this data, it was not possible to combine this with Figure 4.

### PHYSICAL AND CHEMICAL CHANGES DURING THE POTATO-COOKING PROCESS

Potatoes have a water content of about 80wt% while solids make up the rest. Of the solids, the primary content is starch (perhaps more than 16wt%) and the rest is cellulosic material.<sup>[3]</sup> Only 1% of the volume is taken up by air,<sup>[4]</sup> so you would expect that boiling itself does not greatly alter the volume and the weight of the whole potato. Rahman<sup>[5]</sup> documented the data on the water content of fresh and cooked potatoes at an increase of only 2wt% for the boiled potato. This corresponds well to the low air space as mentioned above. This makes boiling potatoes a good example to use in developing a cooking model, since they are of constant density and water content throughout the cooking process. The interface temperature of approximately 65°C is, in fact, a property of the starchy material undergoing gelatinization. From the differential scanning calorimetry (DSC) studies, it has been found that the onset temperature of gelatinization is 57 to 59°C, peak temperature is 71°C, and conclusion temperature is 94 to 95°C.<sup>[6]</sup> The 65°C can be viewed as a



**Figure 2.** Schematic diagram of the partially-cooked potato (note that Fraser<sup>[1]</sup> assumed that all the heat transferred to the interface from the outside is consumed by the reaction).

“critical” temperature beyond which the reaction becomes much more vigorous.

Whether or not a potato is cooked is conventionally judged by how soft the material is after a certain period of boiling (or other means of cooking). Such softness is a direct result of starch gelatinization in the presence of excess water content (say a minimum of 63wt%, corresponding to a ratio of a minimum of 14 water molecules to one anhydrous glucose unit).<sup>[6,7]</sup>

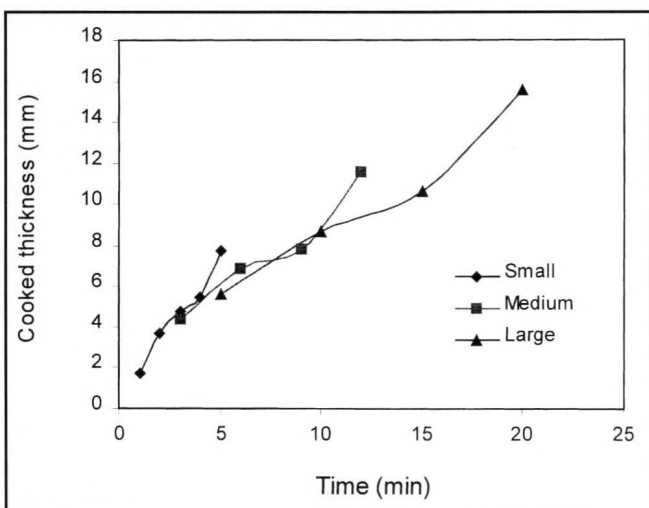
Starch is composed of two polymeric units—a linear form, amylose, and the highly-branched amylopectin. Plants lay down starch granules, normally 10 to 15  $\mu\text{m}$  in size, in which molecules are organized into a radially-anisotropic, semi-crystalline unit (e.g. as high as 45% crystalline). The

center of the cross is at the hilum, the origin of growth of the granule. The semi-crystalline state becomes apparent when studied under x-ray diffraction.<sup>[4]</sup> There are several levels of structural complexity in starch granules. The first level is the “cluster arrangement” of the amylopectin branches where alternate regions of ordered, tightly-packed, parallel glucan chains alternate with less-ordered regions corresponding to branch points. Thus, the starch granule appears to be formed by alternating concentric rings or clusters of amorphous (branching points) and crystalline (glucan chains) lamellae.<sup>[4]</sup> The size of each amylopectin cluster is about 9 to 10 nm. The amylose fraction is assumed to exist in native granules randomly interspersed among the amylopectin molecules in both the amorphous and crystalline regions. As a result, small amylose molecules located at the periphery are free to leach out of the granule, forming a gel structure (gelatinization).

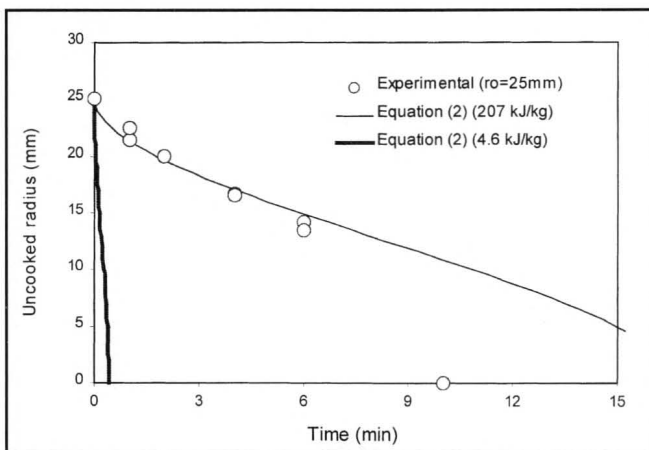
Starch granules are insoluble in cold water. When heated in the presence of excess water, the amorphous regions in the granule swell to form a continuous gel phase. As the temperature exceeds a value typical for each plant species (roughly between 50°C and 80°C), the crystalline structure is also disrupted by gelatinization. If the water content is reduced or solutes are added, the gelatinization temperature is increased. Because starch can be considered a polymer, the gelatinization process can be viewed as a melting process and as corresponding to an order-disorder transition. The individual linear macromolecules (amylose) diffuse into the aqueous medium, increasing its viscosity and forming a “wet-looking” region. With further heating and/or shearing, a starch paste consisting of a continuous phase of granule remnants is formed. After cooling, a dilute solution of starch will precipitate, but concentrated dispersions may form a firm, viscoelastic gel having crystallites as junction zones. During aging, starch molecules can reassociate into crystalline segments (retrograde) to an extent that depends upon factors such as the source of the starch, amylose/amylopectin ratio, the molecule weight, the linearity of the molecules, and the time and temperature of cooling.

#### A MORE DETAILED MATHEMATICAL MODEL

As mentioned earlier, Eq. 2 did not yield a realistic heat of reaction as reported in experimental thermodynamics studies. One important reason for the large discrepancy is that the simple model does not account for the heat conduction from the reacting interface (the boundary between the cooked and uncooked regions) toward the uncooked core region. This might have neglected a large amount of heat, which is conducted away from that interface—thus the high “apparent” heat of reaction (which is the “retarding” factor to the cooking process as the reaction is endothermic) was obtained. Here, a mathematical model is described, which assumes a reacting moving front (for starch gelatinization) during the cooking process but also considers the heat con-



**Figure 3.** The experimental results on three potato sizes, produced from the table of results by Fraser.<sup>[2]</sup>



**Figure 4.** Predictions made using Fraser’s model to the data points collected for a potato with the mean diameter of  $25 \pm 1$  mm. (The physical properties will be referred to in the main modeling section described later.) Case 1: The heat of the reaction used here is  $23 \times 0.2 \text{ kJ/kg}^{-1}$  (the reported value). Case 2: The heat of the reaction is 45 times greater than the reported value.

duction and sensible heating in the uncooked region. As mentioned earlier, experiments have been carried out to measure this moving front, which is also visible in the cross-section obtained at each cooking time.

### Basic Model Formulation

Region I (cooked):

$$\rho_I C_{pI} \frac{\partial T}{\partial t} = \frac{1}{r^j} \frac{\partial}{\partial r} \left( r^j k_I \frac{\partial T}{\partial r} \right) \quad (4)$$

Region II (uncooked):

$$\rho_{II} C_{pII} \frac{\partial T}{\partial t} = \frac{1}{r^j} \frac{\partial}{\partial r} \left( r^j k_{II} \frac{\partial T}{\partial r} \right) \quad (5)$$

Where I and II denote the cooked and uncooked regions respectively and  $j$  is the shape factor (which is equal to 2 for spherical geometry). At the interface of regions I and II, (*i.e.*  $r = r_i$ ) the net heat influx is balanced by the heat of reaction

$$T_i = T_c \quad (6a)$$

$$-k_I \left. \frac{\partial T}{\partial r} \right|_I + k_{II} \left. \frac{\partial T}{\partial r} \right|_{II} = \Delta H_R \rho \frac{dr_i}{dt} \quad (6b)$$

This leads to the rate of advancing reacting front

$$\frac{dr_i}{dt} = \frac{k_{II} \left. \frac{\partial T}{\partial r} \right|_{II} - k_I \left. \frac{\partial T}{\partial r} \right|_I}{\Delta H_R \rho} \quad (7)$$

Note here that  $k_I$  is assumed to be the same as  $k_{II}$ . The outer boundary condition is usually Newton's law of heating (or cooling)

$$-k_I \left. \frac{\partial T}{\partial r} \right|_I = h(T_s - T_\infty) \quad \text{at } r = r_o \quad (8)$$

Based on the lab observation, however, the convection is strong and the boundary temperature can be assumed to be constant, *i.e.*

$$T_s = T_\infty \quad \text{at } r = r_o \quad (9)$$

At the geometrical center, *i.e.*,  $r = 0$ , the symmetry condition prevails

$$\frac{\partial T}{\partial r} = 0 \quad \text{at } r = 0 \quad (10)$$

The initial condition for the potato is

$$T(r) = T_o \quad \text{at } t = 0 \quad (11)$$

The physical and chemical properties involved are

Density of potato:

$$\rho = 0.2 \rho_{\text{starch}} + 0.8 \rho_{\text{H}_2\text{O}} \quad (\text{kg} \cdot \text{m}^{-3})$$

Density of starch:<sup>[5]</sup>

$$\rho_{\text{starch}} = 1500 \quad (\text{kg} \cdot \text{m}^{-3})$$

Thermal conductivity of potato:<sup>[5]</sup>

$$k = 0.624 + 1.19 \times 10^{-3} T(^\circ\text{C}) \quad (\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1})$$

Specific heat capacity of potato:<sup>[5]</sup>

$$C_p = 3500 \quad (\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1})$$

Heat of reaction of potato starch:<sup>[6]</sup>

$$\Delta H_R = 23 \quad (\text{kJ} \cdot \text{kg}^{-1})$$

Heat of reaction (based on potato flesh):

$$\Delta H_R \approx 23 \times 0.2 \quad (\text{kJ} \cdot \text{kg}^{-1})$$

Reacting interface temperature (as in the current lab):

$$T_i = 65 \quad (^\circ\text{C})$$

The heat transfer coefficient is assumed to be fairly large, giving a constant boundary temperature as mentioned earlier.

The solution procedure has been simplified by maintaining the linear temperature gradient in the cooked region, just as Fraser did, but also by solving numerically the detailed partial differential equation for the uncooked core region. Both spherical and planar cooked regions have been tested. The procedure is similar to a typical moving-boundary-type modeling.<sup>[8,9]</sup> The time increment of 0.001 s and the number of 20 uniform divisions for the core region have been found satisfactory in the usual manner. Further refinements (by a factor of 2 in the distance increment and by a factor of 10 in the time increment) did not yield deviations of more than 3% in the cooking time prediction.

### Model Predictions and Discussion

To make the comparison meaningful, the experimental results from potatoes of similar average radius ( $25 \pm 1$  mm) have been summarized and plotted together with the model predictions in Figure 5. The initial temperature was  $17^\circ\text{C}$ . Good agreement has been found, indicating the model is reasonable.

Using the model established in this work, Figure 6 is produced to show the progressive temperature-distance profiles for spherical shell approach under the same conditions as those in Figure 5. As heating proceeds, the center temperature increases and the temperature gradient of the uncooked region, immediately next to the reacting interface, is reduced. This accelerates the cooking process as the interface approaches the geometrical center of the potato.

We must note that the calculations here may be suitable for a more advanced paper (such as a post-graduate paper). Fraser was able to offer a useful analytical solution to undergraduate students. The results obtained in this study are intended to provide a good foundation for further deriving an analytical solution.

### WHAT IS NOT COVERED

Although the model does a good job of predicting the reaction front during cooking, it does not account for texture development. This development of the texture is clearly one of the most important properties governing people's perception of how "cooked" the potato is. Further work is underway to incorporate this aspect. It may involve a gelatiniza-

tion “reaction” model, which uses a texture parameter (such as hardness) as the primary quality.

The change in hardness is a direct result of starch gelatinization in the presence of excess water content. Preview work has indicated that the compression behavior for cooked and uncooked potato tissues is very different.<sup>[4]</sup> Defining whether a starchy food is cooked or not (or defining the degree of cooking) is actually a difficult task. In one approach, it is judged by testing the food structure with a human tooth or an engineering probe. With an engineering probe, a response (a signal) can be recorded. A knowledge of the structure and its relation to force is necessary. Using mechanical means to define “cooked or not cooked” or “the degree of cooking” is a simplification, but is nevertheless a sensible approach.

More fundamentally, judging whether material is cooked or not is based not only on the mechanical ease with which the solid chunks are broken down, but also by the level of aroma released. This is influenced largely by the action of saliva during mastication (chewing) and the strength and

frequency of mastication. All these aspects have been explained to the students in class.

## CONCLUSIONS

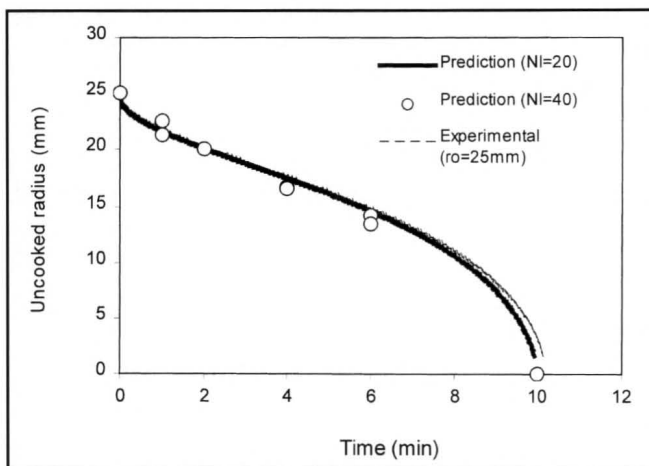
Cooking a potato in hot water, as proposed by Fraser, is a good experiment for introducing some basic concepts of food processing to chemical engineering undergraduates. The experiment is simple and economical. To address the heat-of-reaction issue, as mentioned in the text, a more detailed mathematical model has been proposed, based on the principle of a moving reacting front. The current model predicts the experimental measurement of the cooked thickness of a potato very well. This has provided a sound basis for obtaining a more realistic, approximate, and analytical solution to the problem, so that the same aim as Fraser’s original work can be achieved. In the future, it may also be possible to address the effect of the non-spherical shape of the “normal” potato using one of the existing principles in literature.

## ACKNOWLEDGEMENTS

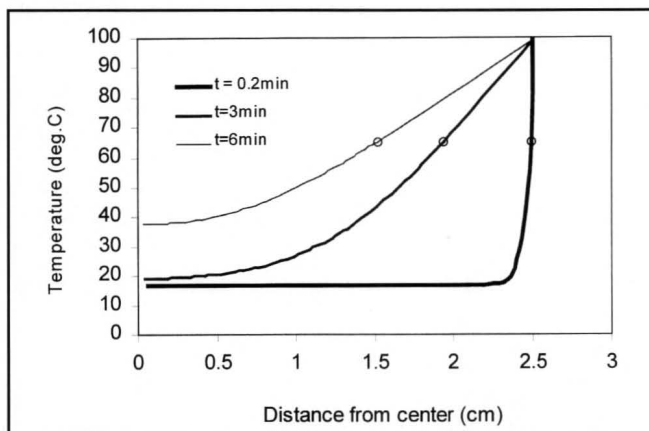
The author acknowledges that Professor Fraser proposed an excellent lab, which has been used well in our university. Thanks also go to Dr. Andrew Russell (chemical and materials engineering, University of Auckland) and Dr. Ian Wilson (chemical engineering, Cambridge University), who contributed significantly to the development of the food process engineering paper. Associate Professor Mohammed Farid (chemical and materials engineering, University of Auckland) has provided stimulating discussions on moving boundary modeling.

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**Figure 5.** A comparison between the predicted results (using two sets of distance increments) and the experimental results (conditions the same as in Figure 3).



**Figure 6.** Temperature-distance profiles at different cooking times (potato diameter =25 mm and the initial temperature =17°C).