COMPRESSIBLE FLOW ANALYSIS

Discharging Vessels

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Compressible flows are usually observed for gases and characterized by a significant change in the fluid density with a change in either the pressure or the temperature. They represent an important topic within the undergraduate curriculum due to their common occurrence throughout the field of chemical engineering, *e.g.*, high-pressure gas jets used for mixing and chemical reaction. This paper reports on the second stage of an undergraduate laboratory experiment that was developed to illustrate some of the important concepts of compressible flow. The first stage of the experiment dealt with filling the vessels and was published previously in this journal.[1]

A schematic representation of the experimental rig is shown in Figure 1. It consists of an insulated pressure vessel, the inlet to which is connected to the air mains while the outlet is connected to a converging nozzle. The first stage of the experiment involves pressurizing the vessel from initial atmospheric conditions up to a predetermined elevated pressure and measuring the corresponding temperature change within the vessel.[1] The second stage of the experiment involves discharging the vessel to atmosphere via a converging nozzle and recording the pressure-versus-time relationship for this process. This paper considers only the second (or discharging) stage, and the principal objectives are

- *To measure the pressure-versus-time relationship as the vessel discharges to the atmosphere*
- *To develop a theory for this pressure variation for comparison with the experimental data*

The significance of this stage of the experiment concerns developing the theory for the time variations in vessel pressure. An important part of any theoretical analysis is specification of the simplifying assumptions on which the model is based, to enable the equations to be solved and ideally to produce an analytical solution. This study demonstrates that it is possible for an assumption to be justified by greatly reducing the complexity of a problem while generating a solution that adequately describes the experimental process, even though the assumption may appear to be invalid. This is the opposite situation to modeling the filling process, where the importance of correctly choosing the system boundary is highlighted.

THEORY FOR DISCHARGING THE VESSEL

The experiment described in this paper involves discharging a pressure vessel to the atmosphere through a convergent nozzle (as shown in Figure 1). In this section the theory for the discharging process is considered with the objective of determining an expression for the variation in vessel pressure with time.

The first step is to define the "system," or control volume, on which to perform the thermodynamic analysis; this is shown in Figure 1 and was described previously by Forrester and Evans.[1] For the discharge stage of the experiment, the system is initially isolated at some elevated pressure, P(0), while the experiment ends when the vessel pressure has dropped to that of the surroundings, *i.e.*, $P(t) = P_A$. The prin-

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ciple assumptions required in the analysis presented below include

- *The temperature and pressure within the vessel remain at stagnation conditions throughout the entire discharge process*
- *Isentropic flow through the nozzle*
- *The pressure vessel can be regarded as adiabatic*
- *The air behaves as a perfect gas*
- *The vessel temperature is constant throughout the experiment*
- *The nozzle is choked at all times*
- *The nozzle discharge coefficient is equal to one*

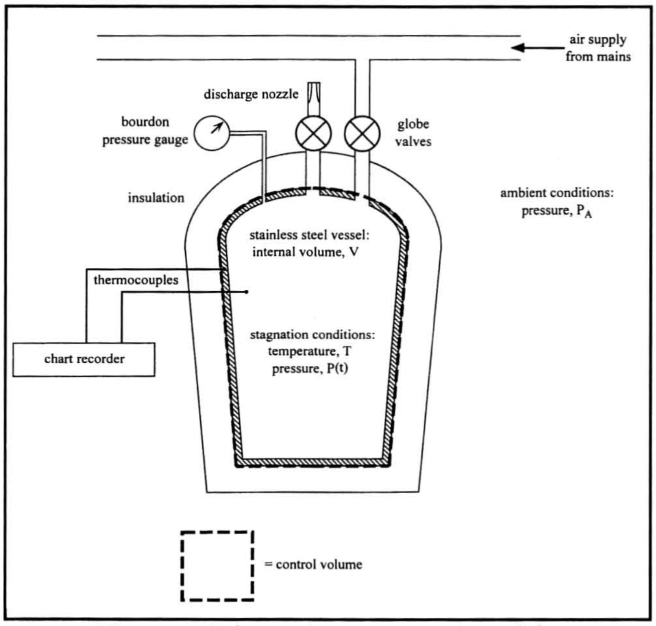

Figure 1. *Schematic representation of the experimental setup.*

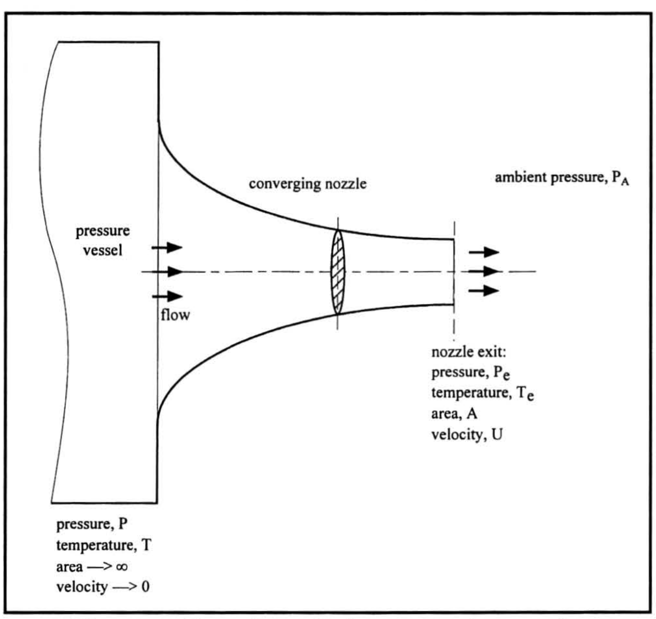

Figure 2. *Flow through the converging nozzle.*

Applying continuity to the system outlined in Figure 1 gives

$$\frac{dm}{dt} = -q(t) \tag{1}$$

where m is the mass of gas in the pressure vessel and q is the mass flow rate through the nozzle. Applying the perfect gas law, the mass of gas in the vessel is given by

$$m = \frac{P(t)V}{RT} \tag{2}$$

where P and T are the (stagnation) pressure and temperature inside the vessel, V is the external volume of the vessel, and R is the specific gas constant. The mass flow rate of gas exiting the vessel through the discharge nozzle, q, is given by

$$q(t) = \rho(t)AU \tag{3}$$

where ρ is the gas density, A is the cross-sectional area of the nozzle, and U is the mean gas velocity, all measured at the nozzle exit plane as shown in Figure 2. Substituting Eqs. (2) and (3) into Eq. (1) gives

$$\frac{V}{RT}\frac{dP}{dt} = -\rho(t)AU \tag{4}$$

Applying the perfect gas law gives

$$\rho(t) = \frac{P_e(t)}{RT_e} \tag{5}$$

where P_e and T_e are the pressure and temperature at the nozzle exit plane, respectively. For isentropic flow through the nozzle one has[2]

$$U = Ma\sqrt{\gamma RT_e} \tag{6}$$

where Ma is the Mach number and γ is the ratio of specific heats. Substituting Eqs. (5) and (6) into Eq. (4) gives

$$\frac{V}{RT}\frac{dP}{dt} = -\frac{P_e(t)}{T_e}AMa\sqrt{\gamma RT_e} \tag{7}$$

The next stage is to relate the pressure and temperature at the nozzle exit plane, P_e and T_e, to the stagnation conditions inside the vessel, P and T, respectively. This can be achieved using the following identities for isentropic flow:[2]

$$P_e = P\left\{1 + \frac{\gamma - 1}{2} Ma^2\right\}^{-\frac{\gamma}{\gamma-1}} \quad (8)$$

$$T_e = \frac{T}{\left\{1 + \frac{\gamma - 1}{2} Ma^2\right\}} \quad (9)$$

Substituting Eqs. (8) and (9) into Eq. (7) and rearranging gives

$$\frac{dP}{dt} = -P\left\{\frac{AMa}{V}\sqrt{\gamma RT}\left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{-\frac{\gamma+1}{2\gamma+2}}\right\} \quad (10)$$

Under conditions where the nozzle is choked, *i.e.,* Ma = 1, Eq. (10) can be simply integrated between the starting time, t = 0, up to some later time, t, giving

$$\ell n \frac{P(t)}{P(0)} = -kt \quad (11)$$

where k is a rate constant (with the dimension of reciprocal of time) described by

$$k = \frac{AMa}{V}\sqrt{\gamma RT}\left(1 + \frac{\gamma - 1}{2} Ma^2\right)^{-\frac{\gamma+1}{2\gamma+2}} \quad (12)$$

Eq. (11) suggests that a plot of $\ell n[P(t)/P(0)]$ against time, t, for the vessel discharge process is a straight line with the gradient equal to k.

The range of vessel pressures for which the nozzle is choked and Eq. (11) is valid can be determined by substituting Ma = 1 and $P_e = P_A$ into the isentropic flow relationship given by Eq. (8), leading to

$$P(t) \geq P_A\left(\frac{2}{\gamma + 1}\right)^{-\frac{\gamma}{\gamma-1}} \quad (13)$$

where P_A is the pressure at the nozzle exit as shown in Figure 2. For a convergent nozzle discharging air to the atmosphere, P_A = 101 kPa and γ = 1.4, thus Eq. (13) predicts that the nozzle will be choked for all P(t) ≥ 192 kPa.

EXPERIMENTAL

A schematic representation of the experimental rig is shown in Figure 1. Three different discharge nozzles, of diameters 2.5 mm, 3.0 mm, and 4.0 mm, can be attached to the pressure vessel outlet. The first stage of the experiment involves pressurizing the vessel from the air mains, the procedure for which is described elsewhere.[1] At the end of the pressurizing stage, both the inlet and exit globe valves are closed and the vessel is allowed to reach steady state (or equilibrium) conditions of temperature and pressure.

The second, or discharge, stage of the experiment is conducted by first setting the speed on the chart recorder according to which discharge nozzle is being used: 2 cm/min for the 2.5 mm nozzle, 6 cm/min for the 3.0 mm nozzle, and 20 cm/min for the 4.0 mm nozzle. The outlet valve is then opened and the vessel is allowed to discharge (with a muffler placed over the exit nozzle to reduce noise pollution). As the vessel pressure drops, the chart recorder switch is flicked every 35 kPa (5 psi) change in pressure, and this continues until the vessel pressure has dropped to the ambient conditions (which typically takes around 2 to 3 minutes). The vessel is then left for approximately 5 minutes to reach steady state before the next experimental run is commenced. The discharge stage is conducted for the three discharge nozzles defined above and a single value for the initial vessel pressure of approximately 730 kPa.

Important Note: *There are a number of inherent safety implications associated with this experiment that should be noted. First, although air is a benign material under atmospheric conditions, it can become hazardous at elevated pressures (4-10 atmospheres in this case). Second, the experiment should only be attempted using a properly certified vessel fitted with the appropriate pressure-relief system; this is absolutely essential when undertaking any experiment under pressurized conditions, but especially important when gases are involved since rupture can result in catastrophic explosion of the vessel.*

RESULTS AND DISCUSSION

Variation in the Vessel during Discharge

Typical experimental and theoretical results for the vessel pressure, plotted in the form $\ell n[P(t)/P(0)]$ as suggested by

TABLE 1
Input Parameters for the Theoretical Calculations

Parameter	*Value*	*Notes*
V	0.102 m^3	
R	287 J/kg/K	For air
γ	1.4	For air
P(0)	726 kPa	Nozzle diameter = 2.5 mm
T	295.9 K	
P(0)	733 kPa	Nozzle diameter = 3.0 mm
T	295.6 K	
P(0)	733 kPa	Nozzle diameter = 4.0 mm
T	289.0 K	

TABLE 2
Experimental and Theoretical Results for the Rate of Vessel Discharge

Nozzle diameter (mm)	k_{theory} *(1/s)*	$k_{exp,all}$ *(1/s)*	*Difference (%)*	$k_{exp. choke}$ *(1/s)*	*Difference (%)*
2.5	0.0096	0.0096	0.2	0.01	3.6
3	0.0138	0.0139	0.7	0.0144	4.3
4	0.0243	0.0247	1.6	0.0259	6.6

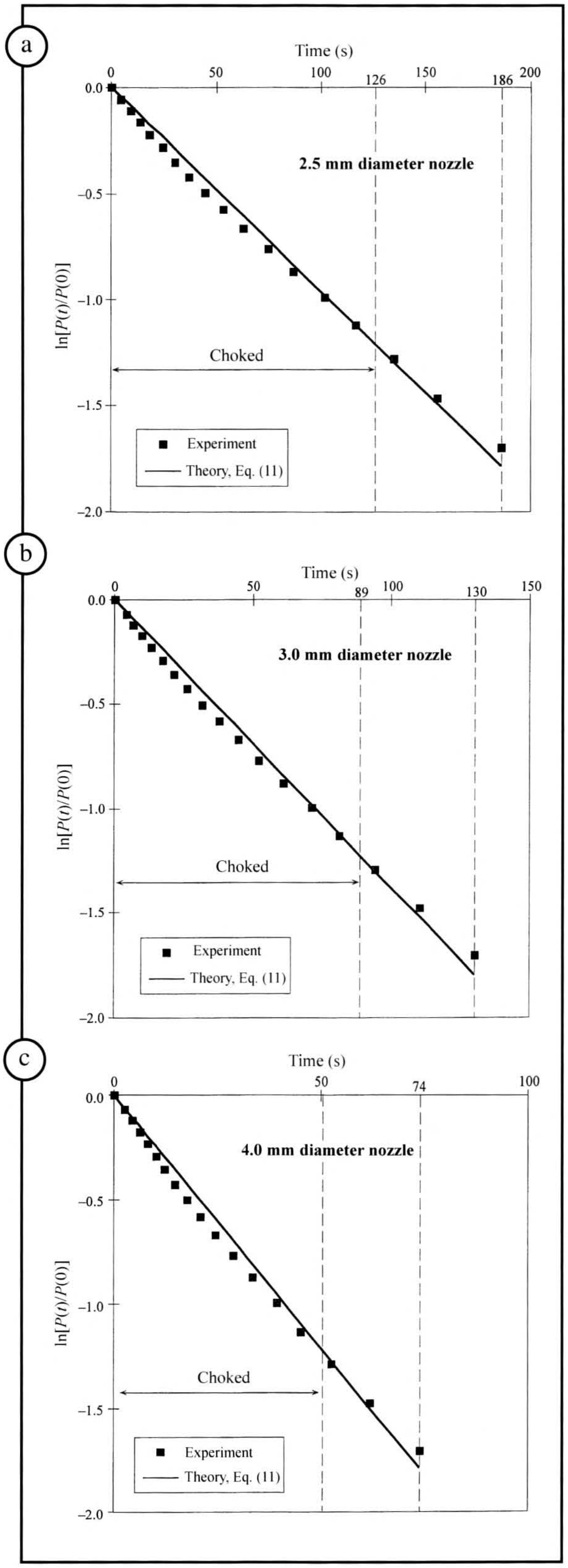

Figure 3. *Typical experimental and theoretical results for the variation in pressure inside the vessel with time during the discharge process at different nozzle diameters.*

Eq. (11), as a function of time are shown in Figure 3. The theoretical results are based on the input parameters listed in Table 1. These plots clearly demonstrate good absolute agreement in the variation of vessel pressure with time throughout the discharge process, confirming the applicability of the theoretical analysis presented above. In particular, the experimental data show a close-to-linear variation in $\ell n[P(t)/P(0)]$ with time as predicted by the theory.

It is possible to calculate the gradient of the experimental data points for comparison with the theoretical values of k determined from Eq. (12). In this study, two experimental gradients have been found, including

- Using all the experimental data points, $k_{exp, all}$
- Using only the experimental data points in the region over which the nozzle is choked, $k_{exp, choke}$

The results are given in Table 2 and confirm the very good agreement between the theoretical and experimental characteristic vessel discharge rates, the maximum deviation being 6 percent. The data in Table 2 also indicates that the absolute value of k increases as the nozzle diameter increases, corresponding to an increasing rate of vessel discharge. This is confirmed in Figure 3, which illustrates that for the 2.5-mm nozzle it takes 186 s for the vessel to discharge to ambient pressure conditions, for the 3.0-mm nozzle it takes 130 s and for the 4.0-mm nozzle it takes only 74 s. In addition, as the nozzle diameter increases, the slight discrepancy between the experimental and theoretical values for the discharge gradient also increases. As the nozzle diameter increases, the rate at which the vessel discharges also increases, leading to larger experimental error in the manual operation of recording the time for each 35 kPA (5 psi) drop in vessel pressure. The final point to note from Table 2 is that the experimental discharge gradients based on all data points, $k_{exp, all}$, give slightly better agreement with the theoretical values derived assuming choked flow throughout, k_{theory}, compared to those based only on the data points in the choked flow region, $k_{exp, choke}$. This is unlikely to be of physical significance, but is simply a result of experimental error and the assumptions used in deriving theoretical expression.

EXPERIMENTAL ASSESSMENT OF THE THEORETICAL ASSUMPTIONS

The results shown in Figure 3 clearly illustrate very good agreement between experiment and theory, indicating the applicability of the model described above to predict the pressure-versus-time relationship for a pressure vessel discharging to atmosphere. It is also of interest, however, to use the experimental data to examine the applicability of the principal assumptions underlying the theoretical analysis.

In particular, two of the assumptions used in the model for the discharging vessel can be investigated based on the experimental results. First it is assumed that the nozzle is choked throughout the entire discharge process, which (as discussed earlier) is only true under conditions where $P(t) \geq 192$ kPa. The actual region over which the nozzle is choked is illustrated in each of the plots in Figure 3; all the nozzles are choked for the first two-thirds (68 percent) of the discharge process, but not for the final third (32 percent). Clearly, the experimental data points give good agreement with the theory of Eq. (12) throughout the entire discharge process, under both choked and unchoked conditions. There may be a very slight drift in the experimental data away from the theory as the nozzle enters the unchoked region—however, the difference remains insignificant even when the discharge process reaches completion. Therefore, even though the nozzles are choked for only the first two-thirds of the discharge process, it can reasonably be assumed they remain close to being choked throughout the remaining period of the discharge process in determining the pressure-versus-time relationship for the vessel.

Second, the assumption that the vessel (stagnation) temperature, T, remains constant throughout the discharge process can be assessed. The chart-recorder output, used to measure the pressure-versus-time discharge relationship, indicates a significant drop in the internal vessel temperature, typically by about 10K over the duration of the experiment. To determine the impact of this temperature change on the theory for the rate of vessel discharge, the following relationship between the discharge gradient and the stagnation temperature can be obtained from Eq. (11)

$$k \alpha T^{1/2} \qquad (14)$$

Hence a ΔT change in the value of T over the duration of an experiment can be related to a Δk change in the value of k by

$$\frac{\Delta k}{k^*} = \left(1 + \frac{\Delta T}{T^*}\right)^{0.5} - 1 \qquad (15)$$

where T^* and k^* represent the conditions at the start of the discharge process. Eq. (15) predicts that for a temperature drop, ΔT, of 10 K based on an initial stagnation temperature, T = 300 K, the corresponding value of k decreases by only 1.7%, which is clearly insignificant. Hence, even though the stagnation temperature may change significantly during the vessel discharge, this will have a negligible impact on the rate of discharge, and it is therefore reasonable to assume that T is constant in determining the pressure-versus-time relationship for the discharge process.

If, however, one really wants to solve the pressure equation for the variation in the stagnation temperature with time, the expression is described by

$$\frac{dP}{dt} = -P\left\{k\sqrt{\theta} + \frac{1}{\theta}\frac{d\theta}{dt}\right\} \qquad (16)$$

where the discharge rate constant, k, is determined by Eq. (12) with T being replaced by the stagnation temperature at time, t = 0, and the scaled variable, θ, is a function of time, which is defined by $\theta = T(t)/T(0)$. For small variations in the stagnation temperature with time, θ is of the order of unity, and Eq. (16) reduces to Eq. (10), as expected. In the general case, Eq. (16) is a nonlinear differential equation of the first order for the stagnation pressure, P, with respect to t, which has to be numerically integrated using, for example, the four-step Runge-Kutta scheme. The initial condition includes $P = P(0)$, $\theta = 1$, and time t = 0. Therefore, this topic can be suitably extended to advanced undergraduate students.

CONCLUDING REMARKS

In this paper, we have described an undergraduate experiment on compressible flow, based on the discharge of an adiabatic pressure vessel through a converging nozzle. In particular, the vessel is emptied from an initial pressure of approximately 730 kPa to ambient conditions, and the variation in vessel pressure with time is recorded; the process is repeated for three different nozzles of diameter 2.5 mm, 3.0 mm, and 4.0 mm. Discussion of the experimental results has involved a qualitative description of the variation of internal vessel pressure as a function of time, the development of a theoretical model for the process, and a comparison of the resulting model predictions with the experimental data.

Both the experimental and theoretical results show a linear relationship between $\ell n[P(t)/P(0)]$ and time, with the absolute value of the gradient increasing with increasing nozzle diameter, corresponding to an increasing rate of vessel discharge. Furthermore, there is good absolute agreement between the experimental and theoretical gradients for all three nozzles used, the maximum discrepancy being 6%, confirming the applicability of the theoretical analysis. The theory predicts a discharge gradient of -0.00960/s for the 2.5-mm nozzle, -0.0138/s for the 3.0-mm nozzle, and -0.0243/s for the 4.0-mm nozzle.

The experimental results also allow the applicability of some of the principal assumptions used in the theoretical development to be assessed. First, it is assumed that the nozzle is choked throughout the discharge while experimentally the nozzle is observed to be choked for only the first two-thirds of the experiment. Second, it is assumed that the vessel stagnation temperature remains constant throughout the discharge, while experimentally it is observed to drop by around 10K during the experiment. Although these assumptions have been shown to not apply strictly, they are justified in allowing the development of a simple analytical model describing the pressure variation during discharge, which provides an excellent fit to the experimental data. This is the opposite situation to the case of filling the vessel where it was very important to correctly define the system boundary and its initial conditions.

NOMENCLATURE

- A cross-sectional area of nozzle at the exit plane [m^2]
- q mass flowrate of gas through the nozzle [kg/s]
- m mass of gas in the pressure vessel [kg]
- Ma Mach number [–]
- P stagnation pressure [N/m^2]
- P_A ambient pressure [N/m^2]
- P_e pressure at the nozzle exit plane [N/m^2]
- R specific gas constant [J/kg/K]

t time [s]
T stagnation temperature [K]
T_e temperature at the nozzle exit plane [K]
U gas velocity at the nozzle exit plane [m/s]
V internal volume of pressure vessel [m^3]
k discharge rate constant defined [1/s]
γ ratio of specific heats [–]
ρ gas density at the nozzle exit plane [kg/m3]
θ dimensionless temperature, defined as $\theta = T(t) / T(0)$

REFERENCES

1. Forrester, S.E. and G.M. Evans, "The Importance of System Selection on Compressible Flow Analysis: Filling Vessels," *Chem. Eng. Ed.*, 308-313(1998)
2. Kay, J.M. and R.M. Nedderman, *Fluid Mechanics and Transfer Processes*, Cambridge University Press, Cambridge, United Kingdom (1985) ❐

DUST EXPLOSION APPARATUS

Continued from page 189.

ratus. In this experiment, the time from ignition to full involvement was on the order of (200 - 70) ms, or 130 ms for propagation through about 20 cm. This corresponds to a propagation rate of roughly 1.5 meters per second, extremely slow by explosive standards. For example, black gunpowder propagates at a rate of about 400 meters per second, while typical high explosives such as TNT propagate at about 4000 meters per second.[11]

Flammable dusts rarely, if ever, constitute a hazard in the open air. Operations capable of creating dust explosion hazards are usually conducted inside buildings such as flour mills and grain elevators, as well as in facilities associated with the manufacture and/or use of such products as edible flours, powdered sugar, metallic pigments, etc. Dust concentrations capable of ignition are reported to contain on the order of at least 30 g/m^3.[10] This is much higher in solids content than could be tolerated by human operators. For example, it has been noted that minimum flammable concentrations of most dusts would limit visibility to a meter or so. Accordingly, flammable dust-air compositions are usually found in closed processing containers or in isolated areas within a manufacturing facility. An ignition source is also required—perhaps a pilot flame, a welding spark, an electrical fault, or the like.

The original explosion may be too small to cause appreciable damage. The resulting shock wave may, however, dislodge additional dust from horizontal surfaces, cracks and crevices, storage areas, and the like. A new and perhaps larger dust cloud is formed and may be ignited by the original source or by hot embers. This cycle, typical of dust explosions, may repeat itself four or five times or more and culminate in complete destruction of the facility. Cleanliness counts in keeping control of dust explosions.

Dust explosions in closed containers are reported to generate pressure on the order of 3 to 7 atmospheres.[2] Buildings housing ordinary manufacturing facilities will not support such internal overpressures. Quite modest excess pressure, on the order of a fraction of an atmosphere, may cause roofs to rise and walls to bulge, leading to a complete collapse of the structure.[12] This collapse represents most of the energy released during the incident. Keep in mind that the initial dust explosion had only a small fraction of that energy. The dust explosion energy probably served only to move or distort structural elements upon which the building was supported. A little can do a lot.

CONCLUSION

We have provided a simple system to demonstrate the explosiveness of dusts. Students witnessing these experiments are always impressed and tend to remember this demonstration for many years thereafter. The experience creates an awareness of the explosiveness of dust and of the necessity to prevent such experiences from happening inadvertently.

AUTHOR'S NOTE

As we were preparing this paper, a high school teacher, Mr. David Barr, Cranston High School West, pointed out to us a similar experiment used during Halloween that is described on the internet.[13,14]

REFERENCES

1. Simpson, J., and E. Weiner, *The Oxford English Dictionary*, 2nd ed., Oxford University Press, Corby, UK (1989)
2. *Hazard Prevention and Control in the Work Environment: Airborne Dust*, World Health Organization, <http://whqlibdoc.who.int/hq/1999/WHO_SDE_OEH_99.14.pdf> (1999)
3. Eckhoff, R.K., *Dust Explosions in the Process Industries*, Butterworth-Heinemann, Boston, MA (1997)
4. *West Pharmaceutical Services Plant Explosion and Fire, Kinston, NC*, Chemical Safety Board, <http://www.chemsafety.gov/completed_investigations/info.cfm?ID=34> (2003)
5. Louvar, J.F., and R.W. Schoeff, *Dust Explosion Control*, Center for Chemical Process Safety, AIChE, New York, NY (1995)
6. *Explosions*, Center for Chemical Process Safety, AIChE, New York, NY (1999)
7. *NFPA 68—Guide for Venting of Deflagrations*, NFPA, 1 Batterymarch Park, Quincy, MA (2002)
8. ASTM Standard, E1226, ANSI, Washington, DC (from Dr. Joe Sencal Kidde-Fenwal, Inc.)
9. Crowl, D.A., and J.F. Louvar, *Chemical Process Safety: Fundamentals with Applications*, 2nd ed., PR Prentice Hall, Engelwood Cliffs, NJ, p. 259 (2002)
10. NFPA 68, "Venting of Deflagrations," National Fire Protection Association, Batherymarch Park, Quincy, MA, App. D (1988)
11. Cook, M.A., ACS Monograph No. 139, *The Science of High Explosives*, Reinhold Pub. Corp., New York, NY, p. 45 (1985)
12. Crowl, D.A., and J.F. Louvar, *Chemical Process Safety: Fundamentals with Applications*, 2nd ed., PTR Prentice Hall, Engelwood Cliffs, NJ, Table 6.9, p. 267) (2002)
13. Geyer, M., *Subject: A Halloween Story*, found at <http://www.thecatalyst.org/forum.halloween/halloween1.html> (2003)
14. Flinn Scientific, Inc., *Lycopodium Powder, The Mini Grain Elevator Explosion*, found at <http://www.thecatalyst.org/download/demos/lycopodium1.pdf> (2003) ❐