## AN AGITATION EXPERIMENT WITH MULTIPLE ASPECTS

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gitation and mixing are important in a wide variety of areas in both the traditional and modern process industries, [1] and it is appropriate that chemical engineering students see related experiments in the unit operations laboratory. This paper describes a teaching experiment involving both agitation and mixing that illustrates—using a quite simple and relatively inexpensive apparatus—a number of aspects of this field. In particular the experiment involves not only simple and direct measurements, for example of torque and power as functions of stirring speed, but also more sophisticated data-acquisition and processing methods, for example indirect measurement involving the use of a model and a parameter estimation method.

#### **APPARATUS**

The apparatus is shown schematically in Figure 1 (next page). The major components of the apparatus are:

A torque table, consisting of a 12-inch-diameter aluminum circle mounted on a tapered roller bearing set in a 12-inch-square aluminum base plate. Even with a load of about 20 kg, the torque needed to set the upper plate in motion is less than 0.00706 N m (1 oz inch). An arm attached to the upper plate bears a load cell (Omega Engineering, Model LCGC) connected to a panel meter (Omega Engineering). The data are acquired by a LabView program at 0.2-second intervals.

- o A variable speed DC motor (Cole-Parmer) with speed controller and torque indication. The speed can be varied from 60 to 2400 RPM, and the maximum torque is 45 oz-in (0.318 N m).
- o Two six-bladed turbines (of diameter 14.4 and 7.5 cm and blade width 3.32 and 1.83 cm), and two three-bladed propellers of diameter 14.4 and 7.5 cm. All are mounted on 3/8-inch glass-epoxy shafts inserted in a chuck on the motor shaft.
- O A polycarbonate tank of ID 29.2 cm and volume about 18 L. This tank is equipped with four removable stainless baffles of width 2.43 cm, mounted on an acrylic top plate. A stainless funnel is mounted near the top of the tank, to be used for adding a conductive tracer, for example NaCl or KCl at 30 g/L. A platinum electrode conductivity cell is mounted near the bottom of the tank 180 degrees from the funnel. The cell is connected to a conductivity meter (Amber Sciences) that sends a 0- to

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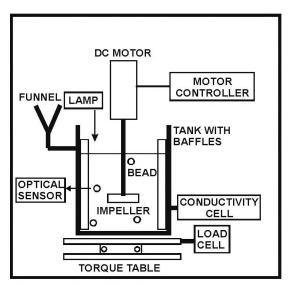


Figure 1. Schematic diagram of the apparatus, showing baffled tank on torque table with load cell, motor and impeller, conductivity probe, funnel for tracer addition, and optical sensor and lamp for bead detection.

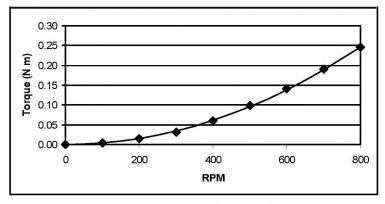


Figure 2. Torque as a function of turbine speed for water in baffled 18-liter tank. Also shown is a curve of the form Torque: k RPM.

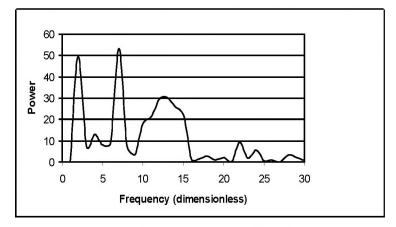


Figure 3. Power spectrum of torque signal for 7.5-cm turbine in 18-liter baffled tank at 800 RPM.

1-volt signal to the computer. A phototransistor (OPF-703) mounted on the side of the tank 3 cm from the bottom is used to detect the approach of almost neutrally buoyant 0.9 cm toroidal plastic beads. The fluid near the sensor is illuminated by a 20 Watt desk lamp.

- A second polycarbonate tank of inner diameter 17.5 cm, also equipped with four stainless baffles, and with volume about 4 L. All dimensions of the smaller tank/impeller combination are proportional to the dimensions of the larger tank/impeller combination, with a factor of 0.551.
- A heat transfer probe constructed by inserting a 90 W resistance heater in the center of a 5-cm-diameter by 7.5-cm-long aluminum cylinder. A transistor-based temperature sensor (LM35-CAZ) is also mounted, off-center, in the cylinder. The probe could be suspended near the middle of the 20 L tank, about 5 cm from the wall.

#### **RESULTS**

Presented below are some typical results, with comments related to what the results illustrate for the student.

#### **Torque and Power**

The 18-liter tank was filled with water to a depth of 25 cm. With the baffles in place and the 7.5-cm-diameter turbine mounted in the motor, the

stirring speed was varied over a range of RPMs. The torque was measured by the load cell and averaged by a LabView program. Typical results are shown in Figure 2, where the torque (N m) is plotted as a function of RPM. Also shown is a curve of the form Torque = k RPM<sup>2</sup>, which fits the data almost perfectly, as expected.<sup>[1]</sup>

Since the torque signal was found to vary with time (with an amplitude about 10% of the average torque), especially at high RPM where the flow in the tank is quite turbulent, it was of interest to look at the power spectra of the data. A typical spectrum (Figure 3) shows that most of the power is at low frequencies, less than 0.15 cycles/sec. It is probable that the fluctuations reflect the presence of eddies or vortices generated by the impeller.

#### Non-Newtonian and High Viscosity Fluids

In order to examine the behavior of a non-Newtonian liquid, the 4-liter tank (with baffles) was filled to a depth of 15 cm with commercial ketchup. The 7.5-cm-diameter flat-bladed turbine was used to stir the ketchup. At 200 RPM the surface of the ketchup was stationary, clearly a non-Newtonian behavior. At 400 and 800 RPM the ketchup flowed smoothly at the surface. The data were well, but not perfectly, fitted by a curve of the form Torque = k RPM². Data were also collected under the same conditions using corn syrup (Karo), a Newtonian fluid with a viscosity about 2,500 times that of water. For these runs the torque was a linear function of the RPM, as expected.

### Tracer Studies of Mixing Times—Modeling and Parameter Estimation

Mixing times have been estimated<sup>[1,2]</sup> by visual observation following the addition of a dye or conductive tracer to the agitated liquid. This method, while satisfactory in some cases, has the disadvantage that it is inherently subjective, and cannot be used with cloudy or opaque fluids. A more objective method is based on acquiring and processing conductivity data following the addition of a conductive tracer (*e.g.*, NaCl solution) to the agitated fluid in the baffled or unbaffled 18-liter tank. The data are acquired by a LabView program, typically 100 baseline points in 20 seconds, followed by 400 conductivity points in 40 seconds. The tracer data are saved from Excel as a space-delimited (.prn) file readable by a QuickBASIC parameter estimation program.

The model used to fit the data, shown in Figure 4, consists of six well-mixed tanks. Three, of equal volume, correspond to downward-moving fluid in the core of the tank. The other three tanks (all of the same volume) correspond to the fluid moving upward along the walls of the tank. Symmetry around the propeller shaft is assumed, so that only three shell tanks are shown. Salt solution injection is assumed to take place at the upper right, and a conductivity probe is located at the lower left. The model contains two undetermined parameters, denoted b<sub>1</sub> and b<sub>2</sub>, with b<sub>1</sub> the fraction of the known tank volume in the three core tanks, and b the volumetric flow rate (L/s) downward through the core tanks and upward through the shell tanks. The volume of a core tank is  $b_1V/3$ , and the volume of a shell tank is  $(1 - b_1)V/3$ . An objective mixing time is three times the tank volume, denoted V, divided by b<sub>2</sub>.

Figure 5 shows the normalized conductivity vs. time data for salt solution injection into the 18 L tank, with agitation provided by a 7.5-cm-diameter downward-driving propeller rotating at 60 RPM. Also shown is the best-fit curve corresponding to the model discussed above. The best-fit parameter values were  $b_1 = 0.484$  and  $b_2 = 1.368$  L/s. The conductivity does not rise for about five seconds, corresponding to the time needed for the salt solution to move from the wall of the tank to the center, down to the bottom of the tank, and then over to the conductivity probe located opposite the injection point. Then the conductivity rises and drops rapidly as the bolus of salt solution passes the probe. After some further oscillations the conductivity reaches a constant value. The curve based on the six-pool model fits the data fairly well, but certainly not perfectly. This is because Summer 2006

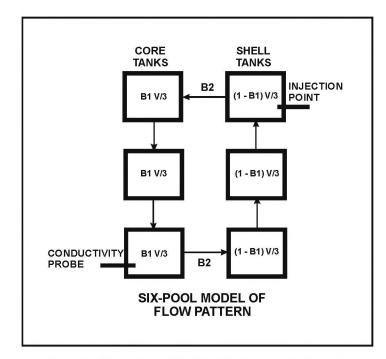
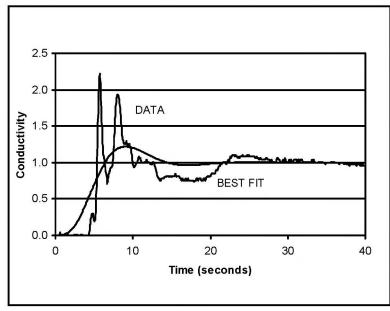


Figure 4. Six-pool model of the flow pattern in the tank, used for analysis of conductive tracer-injection data. The core pools (tanks) represent the downward-flowing liquid in the center of the tank, the shell pools represent the upward-flowing fluid near the tank wall. Parameter  $b_1$  is the fraction of the tank volume V in the core, and  $b_2$  is the volumentric flow rate (L/s) through the array of well-mixed tanks. The state variables are the tracer concentrations in the six tanks.



**Figure 5.** Conductivity data vs. time, and best-fit response from model, for tracer injection into 18-liter baffled tank at 60 RPM using 7.5 cm propeller. Parameters:  $b_1$ =0.484,  $b_2$ =1.368 L/s.

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the model is much too simple to correspond exactly to the highly turbulent, three-dimensional, and complex flow pattern actually existing in the baffled tank, even at low RPM. The parameter  $b_1$  (the fraction of tank volume) was 0.484, and  $b_2$  (the circulation rate) was 1.368 L/s. When the stirring speed was doubled to 120 RPM the conductivity rose much earlier, overshot less, and settled out about three times more rapidly—corresponding to more vigorous mixing. The value for the circulation rate parameter  $b_2$  was 2.765, more than double that for the 60 RPM run.

#### Similitude

In theory, for tanks that are geometrically similar, at the same Reynolds number the power numbers should also be equal. As a test of this theory, the torque is measured at 600 RPM using water and a flat-bladed turbine in the 4-liter baffled tank, and the Reynolds number is calculated. Then the torque is measured in the geometrically similar 18-liter tank, at an RPM corresponding to the same Reynolds number. When the experiments were performed, the calculated and measured torques typically agreed within 20%.

#### **Heat Transfer Coefficient**

It is well documented<sup>[1, 2]</sup> that heat transfer coefficients in agitated tanks depend strongly on the intensity of agitation. In order to demonstrate this, the heat transfer probe was immersed in water in the baffled 18 L tank. The power to the probe was turned on, the stirring speed was set, and the probe temperature was allowed to come to steady state, which occurred in a few minutes. The difference between the probe and water temperature was plotted as a function of RPM. As expected, the temperature difference was highest at 0 RPM, dropped monotonically as RPM increased, and approached a nonzero constant as the RPM ap-

proached high values. This reflects the fact that the resistance to heat transfer is the sum of a constant resistance due to the aluminum wall of the probe, and a film resistance at the probe surface that varies with the 2/3 power of RPM. From the high RPM asymptote the first resistance can be calculated, and from a second point the heat transfer coefficient can be found as a function of RPM. In general the results were consistent with Eq. (1):

$$k = a N^{0.67} = a N^{2/3}$$
 (1)

where k is the heat transfer coefficient (Wm<sup>2</sup>/ $^{\circ}$ C), a is a constant, and N is the stirring speed (s<sup>-1</sup>).

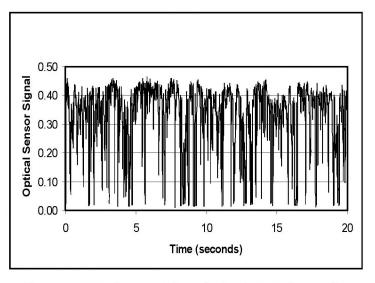


Figure 6. Optical sensor voltage for beads in 18-liter tank at 400 RPM. Each drop in voltage corresponds to the entry of a bead into the field of view of the phototransistor. Only drops below a selected voltage are counted as events.

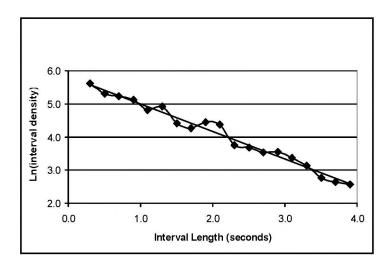


Figure 7. Semilog plot of distribution of bead arrival intervals. The straight line corresponds to a Poisson distribution.

# Note that all real-world measurements involve both signal and noise. In most cases the information is contained in the signal and we attempt to minimize the noise. But in some cases the noise also contains useful information.

The results . . . illustrate this.

#### **Particle Dynamics**

At high Reynolds numbers the flow in the tank is, at least in some sense, strongly turbulent. This implies that particles suspended in the tank move in a chaotic and uncorrelated way, and thus move independently. In order to test this prediction about 200 plastic toroidal beads of approximate diameter 0.9 cm were added to the tank. The 7.5-cm turbine was run at 400 and 800 RPM. A typical sensor signal is shown in Figure 6, corresponding to a sample rate of 100 per second. Each drop in voltage represents the entries of one or more beads into the illuminated region. The program that acquired the data also used simple logic to identify close-bead approaches, to calculate the time interval between entry, and to construct an interval distribution function. If the beads in fact move independently, this is expected to be a Poisson distribution. Typical results, shown in Figure 7 in a semilog plot, correspond reasonably well to a straight line and thus to a Poisson distribution. The slope of the line is related to the average frequency of bead events, but also depends on the efficiency of detection of bead approaches, which is not easily known. The lower values at low time intervals probably correspond to the fact that the data acquisition program is not able to differentiate between two or more bead entries that occur at almost the same time.

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#### Salt Crystal Dissolution

When a crystal of a soluble salt, for example NaCl, sits in water that is not moving, it dissolves relatively slowly. But the opposite holds for a crystal suspended in strongly turbulent water. Approximately one gram of a coarse (about 150 crystals per gram, the crystals being of variable size) kitchen salt was added to water in the baffled 18 L tank. At 0 RPM full dissolution required more than 60 minutes. The signal from the conductivity probe recorded by a LabView program is shown in Figure 8. Using the 7.5-cm-diameter turbine, dissolution was almost twice as fast at 800 RPM as at 400 RPM. These results demonstrate that the apparatus described above can be used in a wide variety of studies of the effect of stirring speed, impeller design, and other parameters on the rate of dissolution of (or extraction from) solids, as long as the solids release a conductive tracer.

#### CONCLUSIONS

The agitation and mixing experiment described above is based on a relatively simple and inexpensive apparatus. But it illustrates a number of aspects of the subject, including the

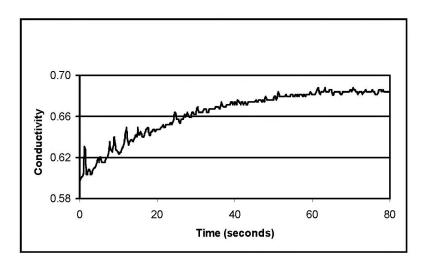


Figure 8. Conductivity as a function of time following addition of 1 gram of NaCl crystals to the agitated 18-liter baffled tank at 400 RPM.

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dependence of torque on stirrer speed, impeller design, baffle design, and nature of the fluid involved. The principle of similitude can be tested. The experiment also illustrates very general and more sophisticated concepts, including the use of modern data acquisition software and nonlinear regression methods to estimate the parameters of a model of the flow pattern in an agitated vessel. The apparatus is well adapted to studying the rate of dissolution of salt crystals, and the effect of agitation intensity on heat transfer from a solid surface.

Finally, students are able to acquire and process essentially stochastic data to obtain some information on the turbulent flow in the vessel.

#### **REFERENCES**

- McCabe, W.L., J.C. Smith, and P. Harriot, Unit Operations of Chemical Engineering, 4th Ed., McGraw-Hill, New York (1985)
- 2. Uhl, V.W., and J.B. Gray, *Mixing, Theory, and Practice*, Vol. 1, Academic Press, New York (1966) □