The object of this column is to enhance our readers' collections of interesting and novel problems in chemical engineering. Problems of the type that can be used to motivate the student by presenting a particular principle in class, or in a new light, or that can be assigned as a novel home problem, are requested, as well as those that are more traditional in nature and that elucidate difficult concepts. Manuscripts should not exceed 14 double-spaced pages and should be accompanied by the originals of any figures or photographs. Please submit them to Professor James O. Wilkes (e-mail: wilkes@umich.edu), Chemical Engineering Department, University of Michigan, Ann Arbor, MI 48109-2136.

TEACHING TRANSPORT PHENOMENA AROUND A CUP OF COFFEE

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We are all aware that teaching scientific matter is much more accepted by students when it can be related to situations they can experience in their everyday life. A good example is the cooling of a cup of coffee, whose scientific analysis is much more instructive than we could have thought at first sight. Indeed, we will see that all heat transfer mechanisms (conduction, convection, and radiation), as well as those of mass transfer, (because of the evaporation of the coffee) are involved. This problem was often addressed as "leisure in science" or "first approach of science," and a quick search on the Web shows that this problem has been proposed at all levels of education, from beginning to university. The approach presented here is aimed at being rigorous, but because we do not intend to use very powerful numerical modeling, simplifications will be made. An important quality for an engineer is to make the "right" simplification, *i.e.*, which results only in slight inaccuracies, while respecting the correct hierarchy for the parameters. In the case chosen here no chemical reaction is present, but the coupling of heat and mass transfer in a nonstationary process is a common situation in chemical engineering. It can be encountered, for instance, in small industrial units when a tank, after a batch transformation, is let to cool freely before discharge. Another very important characteristic of the study is that experiments to assess the modeling are easy to perform with very simple tools, such as a thermometer, a stopwatch, and a balance (to estimate the loss by evaporation). Such experiments could even be done in a kitchen, in full accordance with the "everyday life" aspect of the situation. The method to approach the problem, and the reflection about transport phenomena that it induces, make it a good basis for discussion between students and teachers. To avoid a lengthy paper, all equations given here are not discussed deeply and, for a student, may deserve an additional look into textbooks or, better, a discussion with teachers.

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PRESENTATION OF THE PROBLEM AND HYPOTHESES

The problem: we put a cup of hot coffee on a table. Its initial temperature, θ_0, is around 80 °C, and ambient air is at temperature θ_a, with, for instance, a relative humidity of 50% (that means half-saturated). What is the temperature of the coffee after 10 min. for instance? Or, more widely, when will I be able to drink it safely and what parameters influence this duration? A scheme of the situation is given in Figure 1 and the different fluxes will be discussed in paragraph 2. To solve the problem we have to make a list of simplifying hypotheses:

1. *Temperature θ is homogeneous through the liquid in the cup. There is no temperature gradient in the liquid, and the inner wall temperature of the cup is equal to that of the liquid, because internal free convection is sufficiently high. These are very important hypotheses and we will devote a specific paragraph to assess them.*

2. *Even if our system is time dependent, we will use steady-state equations to model the heat and mass transfer fluxes. This "pseudo steady-state approximation" is very often proposed and is fully justified here, because establishment of transfers is more rapid than evolution of temperature of the liquid. It is always difficult to demonstrate this statement, and intuition is often the only indicator. Such ambiguity is rarely addressed, but it has been discussed by Cussler in his book about mass transfer.[1]*

3. *There is no heat loss through the bottom of the cup, because the table blocks the heat flux. Nevertheless, we may foresee that putting the cup on a massive metallic surface will speed up the cooling. In this case, the bottom heat flux would not obey the steady-state law, (see, in textbooks, the chapter devoted to conductive transfer in semi-infinite medium). This will not be considered here.*

4. *At the vertical cylindrical wall of the cup and at the surface of the liquid, heat loss occurs by free convection and radiation. Moreover, at the liquid surface, evaporation of the liquid simultaneously takes place. This evaporation induces an extra heat loss, corresponding to the heat needed for vaporization of water, that is provided from a decrease in the internal energy of the liquid and the cup. Forced convection by blowing air is not considered here, although it could be very easily implemented through adapted computation of the coefficients of convection. It is important to mention here that, at temperatures below 200 °C, chemical engineering calculations usually neglect radiation fluxes because they are competing with forced convection fluxes, which are much larger. When dealing only with free convection, this omission would lead to significant errors, even at low temperature.*

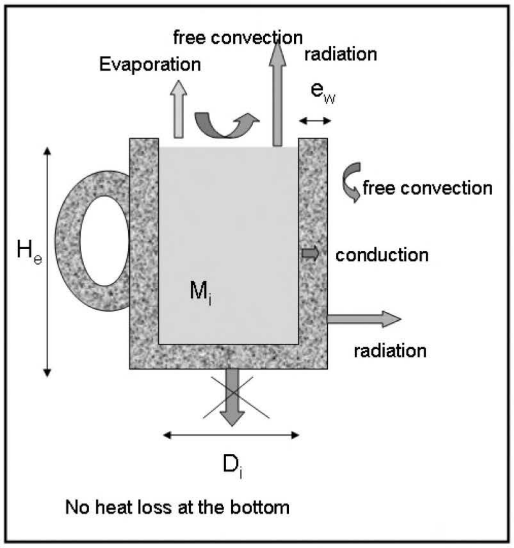


***Figure 1**. Schematics of the fluxes.*

5. *The coffee cup is simulated by a cylinder, external height H_e, internal diameter D_i, with constant wall thickness e_w, and with a thermal conductivity λ_w. The area of the external vertical wall surface is A_{we} and that of the horizontal liquid surface is A_s. Also, this hypothesis upon the geometry of the cup is not very restrictive and can be adapted for other cases. Liquid is supposed to fill the cup almost entirely.*

6. *Coffee is similar to water, and properties are evaluated at 60 °C.*

Description of the Equations for Modeling

The heat loss through the wall and at the liquid surface results in a temperature decrease that may be described by the instantaneous heat balance equation, where accumulation of internal energy in the water and the cup (considering homogeneous temperature) equals the sum of all instantaneous heat losses. Because water evaporates, we also need an instantaneous mass balance:

$$(\mathrm{mCp_c} + \mathrm{MCp_{wat}})\frac{d\theta}{dt} = -\sum \text{heat losses} \qquad (1)$$

$$\frac{dM}{dt} = -\text{evaporative flux} \qquad (2)$$

where m and Cp_c refer, respectively, to the mass and specific heat of the cup, and M and Cp_{wat}, to mass and specific heat of the water. We can now describe the different heat losses and express them using steady-state equations of heat and mass transfer, as stated in hypothesis 2.

Heat Loss at the Vertical Wall of the Cup, Q_w

This is a transfer, in series, by conduction through the wall, then, in parallel, free convection and radiation to ambient air. As stated in hypothesis 1, internal convection at the inner wall is not considered. This global transfer is accounted for by a global coefficient U_{we}, referred to the external area, given by:

$$\text{area, given by}: U_{we} = \left(\frac{1}{h_{nv} + h_R} + \frac{e_w}{\lambda_w \dfrac{D_e + D_i}{2D_e}} \right)^{-1} \qquad (3)$$

and

$$Q_w = U_{we} A_{we} (\theta - \theta_a) \qquad (4)$$

We need to evaluate the coefficient h_{nv} for free convection at the vertical wall. An equation for such a heat transfer coefficient can be found in:[2]

$$h_{nv} = 1.35\left(\frac{\theta_w - \theta_a}{H_e}\right)^{1/4} \qquad (5)$$

where Eq. (5) is adapted to be used directly for free convection in air. SI units are used throughout.

Radiation transfer is accounted for by a radiation coefficient h_R. To estimate h_R, we can approximate our case by a situation in which a small gray surface at θ radiates toward a large gray enclosure, the room at θ_a. In this case, and if θ and θ_a are not very different, it can be shown (see any heat transfer textbook, for instance Reference 3), that h_R is proportional to the third power of the mean absolute temperature:

$$h_R \cong 4\sigma\varepsilon\left(\frac{(\theta + 273) + (\theta_a + 273)}{2}\right)^3 \qquad (6)$$

where ε represents the emissivity of the surface and σ is the Stefan-Bolzman constant. This linearization of radiation fluxes is very convenient and is a great help to account for the radiation without adding complex equations.

An important quality for an engineer is to make the "right" simplification, i.e., which results only in slight inaccuracies . . . .

Note that the convection coefficient, as well as the radiation coefficient, depends on the outer wall temperature θ_w. Indeed, it is not convenient in the computation to evaluate the outer wall temperature, so, for estimation of these coefficients, we will equate the outer wall temperature to that of the liquid. It results in some inaccuracy for h_{nv} and h_R. Eventually, this inaccuracy is likely to be weak because, for usual materials and thicknesses, the thermal resistance of the wall is low in respect to the outer thermal resistance, and the outer wall temperature is actually not very different from the inner wall temperature. It does not mean that the thermal resistance of the wall is neglected here, because it does appear in the equation of U [Eq. (3)]. The extreme case of an insulating wall (as for an expanded polystyrene cup, see paragraph 4) where the inaccuracy is maximum is well described because the "inaccurate" term has a weak numerical influence in the computation of U [Eq. (3)].

Heat loss, by Heat Transfer Only, at the Surface of the Liquid, Q_s

It also occurs by free convection and radiation, in parallel, and is accounted for by a global coefficient h_s with:

$$h_s = h_{ns} + h_{Rs} \qquad (7)$$

$$Q_s = h_s A_{se}(\theta - \theta_a) \qquad (8)$$

h_{ns} represents the coefficient for free convection at a horizontal surface. From Reference 2, for air, it is given by:

$$h_{ns} = 1.31\left(\frac{\theta - \theta_a}{D_e}\right)^{1/4} \qquad (9)$$

Heat loss resulting from the evaporation, Q_{evap}

We must first estimate the evaporative molar density of flux, N_{wat}. At the interface, air is saturated at the surface liquid temperature θ, and water partial pressure is equal to its vapor pressure $P_v(\theta)$ at this temperature. Far from the surface, for half-saturated ambient air, the water partial pressure is $0.5P_v(\theta_a)$. In the case of a single component we can find explanations in mass transfer textbooks (see for instance Reference 5):

$$N_{wat} = k_c C_T \frac{1}{F}\left(P_v(\theta) - 0.5P_v(\theta_a)\right) \qquad (10)$$

k_c is the mass transfer coefficient referred to a molar concentration difference at low or equimolar transfer fluxes. F is the logarithmic mean of the partial pressure of air, $P_{air} = P_T - P_{wat}$ at the surface and far from the surface, and it accounts for the influence of the bulk flow of air. So:

$$F = \frac{\left(P_T - 0.5P_v(\theta_a)\right) - \left(P_T - P_v(\theta_a)\right)}{\ln\frac{P_T - 0.5P_v(\theta_a)}{P_T - P_v(\theta_a)}} \qquad (11)$$

C_T is the total molar concentration. $P_v(\theta)$ can be computed from a vapor pressure law for water, such as Clapeyron's or Antoine's law. Here we have used, from Reference 4:

$$P_v(\theta) = \frac{10^5}{760}10^{7.9668 - \frac{1.668.21}{228+\theta}} \text{ where } P_v \text{ is in Pa}, \theta \text{ in } ^\circ\text{C} \qquad (12)$$

An important feature is now to estimate the mass transfer coefficient k_c. This can be done using the analogy between heat and mass transfer, as first proposed by Chilton and Colburn.[6] For the air-water system, because the Lewis number, Le, is close to 1, it gives[7]:

$$k_c = \frac{h_{ns}}{\rho_{air} Cp_{air}} \qquad (13)$$

The molar density of the flux is then

$$N_{wat} = \frac{h_{ns}}{\mathfrak{M}_{air} Cp_{air}} \frac{1}{F}\left(P_v(\theta) - 0.5P_v(\theta_a)\right) \qquad (14)$$

After some rearrangements, using the perfect gas law, mass flux is:

$$W_{wat} = \frac{h_{ns}\mathfrak{M}_{wat}}{\mathfrak{M}_{air} Cp_{air}} \frac{1}{F} A_{si}\left(P_v(\theta) - 0.5P_v(\theta_a)\right) \qquad (15)$$

Now, knowing the evaporative mass flux, the heat loss by evaporation, Q_{evap}, is given by:

$$Q_{evap} = W_{wat}\Delta H_v \qquad (16)$$

where ΔH_v is the heat of vaporization of water per kg of water.

Finally the system of differential equations to solve is:

$$\left(mCp_c + MCp_{wat}\right)\frac{d\theta}{dt} = -h_s\left(\theta\right)A_{se}\left(\theta-\theta_a\right) - U_w\left(\theta\right)A_w\left(\theta-\theta_a\right) - \Delta H_v\frac{h_{ns}\mathfrak{M}_{wat}}{\mathfrak{M}_{air}Cp_{air}}\frac{1}{F}A_{si}\left(P_v\left(\theta\right)-0.5P_v\left(\theta_a\right)\right) \tag{17}$$

$$\frac{dM}{dt} = -\frac{h_{ns}\mathfrak{M}_{wat}}{\mathfrak{M}_{air}Cp_{air}}\frac{1}{F}A_{si}\left(P_v\left(\theta\right)-0.5P_v\left(\theta_a\right)\right) \tag{18}$$

with initial conditions at $t = 0$, $\theta = \theta_0$, and $M = M_0$. This system can be solved numerically by the variable step Runge-Kutta method, for instance. For all our computations, we have used a very convenient commercial software, Mathcad 13, where automatic resolution of such system of equations is implemented. Listing of the program can be found at <http://lgc.inp-toulouse.fr/internet/pers/condoret.htm>.

RESULTS OF THE MODELING AND COMPARISON WITH EXPERIMENTS

The experimental apparatus, including a numerical thermometer, a balance, and a stopwatch, is seen in Figure 2. Three different porcelain cups (No.'s 1, 2, and 3) were used filled with water. The cups were put on the balance plate, hot water from an electric kettle was poured in, and the temperature and mass variation of the liquid were recorded. A piece of insulating material was set under the cups to prevent direct contact with the balance plate. It proved to be useful with respect to hypothesis 3. Physical and geometrical data are given in Table 1. Figures 3 a, b, and c presents the comparison between experimental temperature and the modeling as described above. The modeling appears very good, although it slightly underestimates the cooling rates in all cases. A simple explanation could be that area of the handle was not taken into account in the computations (indeed, cup No. 2, which gave the best results, had a small handle). Figure 4 also presents good agreement between experimental and modeled mass variation. As an example, Table 2 gives computed values of different terms of the equation for experiments of cup No. 2, and relative contribution of each flux can be appreciated. It can be seen, for instance, that the evaporative flux, except at the end, is quite significant (see below, paragraph 4). Also note that radiation and free convection coefficients are in the same range (around 7 W m^{-2} °C^{-1})

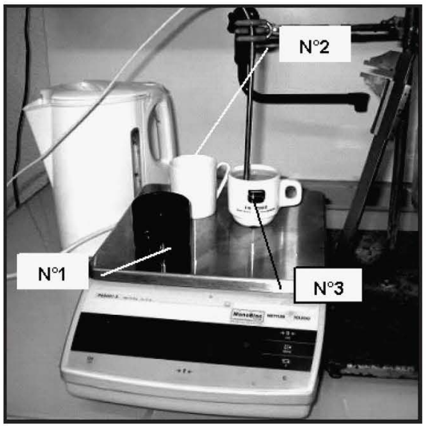
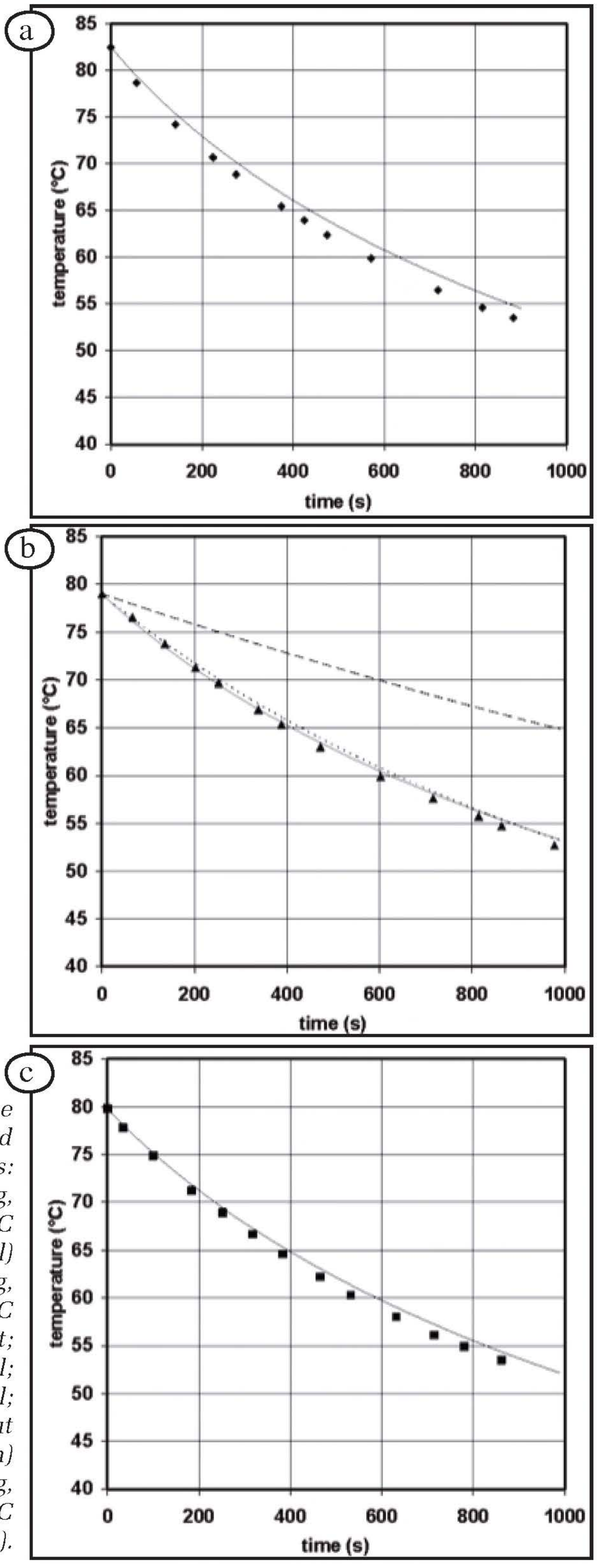


Figure 2. *Experimental apparatus.*



Figures 3. *Variation of the temperature of the liquid for the 3 different cups:* ***a)*** *cup No. 1, M_0 = 78.6 g, θ_a = 22.3 °C, θ_0 = 82.5 °C (◆: experiment; ——: model)* ***b)*** *cup No. 2, M_0 = 102.9 g, θ_a = 21.8 °C, θ_0 = 79 °C (▲: experiment; ——: model; -------: simplified model; – – – –: model without evaporation)* ***c)*** *cup No. 3, M = 87.2 g, θ_a = 21.1 °C, θ_0 = 80 °C (■: experiment; ——: model).*

USE OF A SIMPLIFIED ANALYTICAL SOLUTION

One may feel frustrated to need a numerical solution for the system of differential equations. First note that the mass of water varies only slightly (less than 3%), so we can suppress the mass balance equation and consider the mass of water as a constant, equal to M_0. Furthermore, Table 2 shows that variation, in respect to the temperature, of heat and mass transfer coefficients is not very large. We have found that use of parameter values computed at the mean temperature leads to very similar results. Eventually, we can only consider one simplified differential equation, using averaged values:

$$\left(M_0 Cp_{wat} + mCp_c\right)\frac{d\theta}{dt} = -h_{sav}A_{se}\left(\theta-\theta_a\right) - U_{wav}A_{we}\left(\theta-\theta_a\right) - \frac{h_{ns_{av}}\mathfrak{M}_{wat}}{\mathfrak{M}_{air}Cp_{air}}\frac{1}{F_{av}}A_{si}\left(P_v\left(\theta\right)-0.5P_v\left(\theta_a\right)\right)\Delta H_v \tag{19}$$

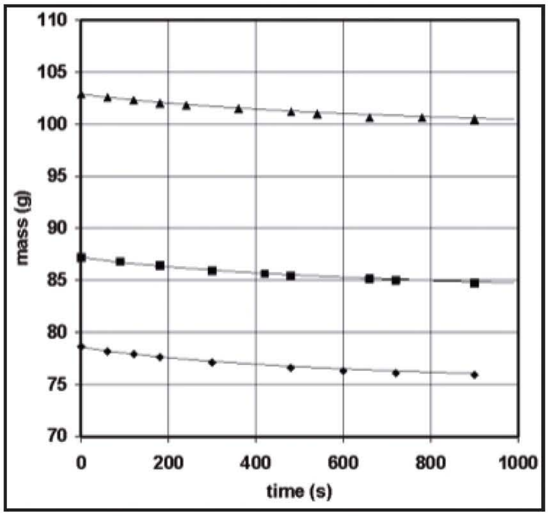


Figure 4. Loss of mass (g) for the 3 different cups:
a) cup No. 1, M_o = 78.6 g, θ_a = 22.3 °C, θ_0 = 82.5 °C (◆: experiment; ——: model)
b) cup No. 2, M_o = 102.9 g, θ_a = 21.8, °C, θ_0 = 79 °C (▲: experiment; ——: model)
c) cup No. 3, M_o = 87.2 g, θ_a = 21.1 °C, θ_0 = 80 °C (■: experiment; ——: model)

Nevertheless, even with the proposed averaging, Eq. (19) has still no obvious analytical solution, due to the exponential term in the expression of $P_v(\theta)$. But, if the function $P_v(\theta)$ is approximated by a parabolic equation, $P_v(\theta) = b\theta^2 + c\theta + d$, we can propose an analytical solution. We found by numerical fitting, that

b =18.367   c =-1237.2   d = 27753   in the range 40 °C to 80 °C

In this case, Eq. (19) is a differential equation with separated variables, whose solution is:

$$t\left(\theta\right) = \left(M_0 Cp_{wat} + mCp_c\right)\left(\frac{2}{\omega}\arctan\left(\frac{2bB\theta + A + cB}{\omega}\right) - \frac{2}{\omega}\arctan\left(\frac{2bB\theta + A + cB}{\omega}\right)\right) \tag{20}$$

with

$$\omega = \sqrt{-4bBA\theta_a + 4bB^2dm - A^2 + 2AcB - c^2B^2} \tag{21}$$

$$A = -h_{sav}A_{se} - U_{wav}A_{we} \tag{22}$$

TABLE 1

Numerical Values of the Parameters of the Three Different Cups

Values are in SI units as given in the nomenclature.

	D_i	H_e	e_w	m	λ_w	Cp	ε
cup N°1	0.0520	0.0495	0.0040	0.1092	1	970	0.924
cup N°2	0.0512	0.0610	0.0020	0.0642	1	970	0.924
cup N°3	0.0520	0.0635	0.0040	0.1278	1	970	0.924

TABLE 2

Numerical Values Given by the Model for Experiment of Cup No. 2 (M_o = 0.1029 kg, θ_a = 21.8 °C)

Values are in SI units as given in the nomenclature.

time	temp.	mass(g)	Qevap	Qw	Qs	hnw	hns	hRw	hRs	Uwe
0	79.0	102.9	12.0	8.6	2.1	7.5	7.4	7.1	7.6	14.1
90	75.2	102.5	9.5	7.8	1.9	7.3	7.3	7.0	7.5	13.9
180	71.9	102.1	7.8	7.2	1.7	7.2	7.2	6.9	7.3	13.7
270	69.0	101.8	6.6	6.7	1.6	7.1	7.1	6.8	7.2	13.5
360	66.3	101.5	5.6	6.3	1.5	7.0	7.0	6.7	7.2	13.3
450	64.0	101.3	4.9	5.9	1.4	6.9	6.9	6.6	7.1	13.2
540	61.8	101.1	4.3	5.5	1.3	6.8	6.8	6.5	7.0	13.0
630	59.8	101.0	3.8	5.2	1.2	6.7	6.7	6.5	6.9	12.9
720	58.0	100.8	3.4	4.9	1.2	6.7	6.6	6.4	6.9	12.7
810	56.3	100.7	3.1	4.6	1.1	6.6	6.6	6.4	6.8	12.6
900	54.7	100.6	2.8	4.4	1.0	6.5	6.5	6.3	6.8	12.5

$$B = -\frac{h_{ns_{av}} \mathfrak{M}_{wat}}{\mathfrak{M}_{air} Cp_{air}} \frac{1}{F_{av}} A_{si} \Delta H_v \tag{23}$$

$$dm = d - 0.5 P_v (\theta_a) \tag{24}$$

Figure 3b compares the numerical and the analytical solution of the equation, and shows that the simplification is quite valid.

The case may be further simplified if we consider that evaporation does not occur (insulating cover on the cup). In this case Eq. (18) becomes:

$$(M_0 Cp_{wat} + mCp_c) \frac{d\theta}{dt} = -U_{wav} A_{we} (\theta - \theta_a) \tag{25}$$

which is very easily integrated to

$$\frac{\theta_i - \theta}{\theta_i - \theta_a} = e^{-\frac{U_{wav} A_{we}}{(M_0 Cp_{wat} + mCp_c)} t} \tag{26}$$

Results of this analytical solution are presented in Figure 3b, showing that the final temperature is significantly higher in this case. This situation exists in real life. It corresponds to the "fast food coffee," which is served in expanded polystyrene cups with a cover that insulates and blocks evaporation. This absence of evaporation combined with an increased thermal resistance of the wall (expanded polystyrene has a very low conductivity) results in very slow cooling. This explains why we often burn our lips at the end of a fast food meal when we drink our coffee without precaution, as we cannot imagine it is still so hot after the duration of the meal!

COMMENTS ON THE HYPOTHESIS OF HOMOGENEOUS LIQUID TEMPERATURE

We can use the analogy with the well known case of heating or cooling of a solid. The homogeneity of the solid temperature is usually assessed by considering the Biot number,

$$Bi = \frac{hL}{\lambda} \tag{27}$$

where L is a characteristic length of the system. The Biot number evaluates the ratio between inner conductive transfer and outer convective transfer. When the Biot number is much smaller than 1, homogeneity of the solid temperature is insured. In our case, the Biot number can be written as:

$$Bi = \frac{U_w D / 2}{\lambda_{wat}} \tag{28}$$

So with $Uw \approx 14\ Wm^{-2}\ {}^\circ C^{-1}$, $D = 4 \times 10^{-2}$ m, and $\lambda_{wat} = 0.67\ W\ m^{-1}\ {}^\circ C^{-1}$, we obtain $Bi = 0.8$. This value is not "much" smaller than 1, but we have considered here that only thermal conduction occurs in the liquid, while free convection is actually present, and greatly increases the inner transfer. For instance, we can estimate the enhancement of the "apparent" conductivity by the value of the Nusselt number, Nu. To evaluate this value, we can use a simplified sketch and consider inner free convection in a horizontal cell. Equations for such situation can be found in Reference 9:

$$Nu = \frac{hL}{\lambda} = 0.069 Ra^{0.33} Pr^{0.074}$$
$$\text{with } 3x10^5 < Ra = \frac{H^3 g \beta \Delta\theta}{\alpha \nu} < 7x10^9$$
$$\text{and } Pr = \frac{Cp\mu}{\lambda} \tag{29}$$

In our case, if we want to accept a temperature difference $\Delta\theta = 1\ {}^\circ C$, between bottom and surface of the liquid, Eq. (29) predicts a conductivity enhancement of around sevenfold that now allows a better fulfillment of the Biot criterion. Remember that this very simplified approach aims only at estimating if we are in the acceptable range. If we now consider the case of the industrial tank with a characteristic length of 1 m, Eq. (29)—which gives a conductivity enhancement of 120-fold—allows maintaining the Biot number at a low value, and the hypothesis of homogeneity is still valid.

CONCLUSION

The agreement between modeling and experiments (Figures 3 and 4) was surprisingly good. Indeed, every experienced researcher knows that a totally predictive model is often disappointing and parameter adjustment is common practice (conversely, students are very confident in these predictive models!). Nevertheless, be aware of the numerous simplifications we used that here proved to be reasonable. As a practical conclusion, note that when preparing a cup of coffee another scenario is possible: hot coffee from the pot is poured into the cup. In this case there is first cooling of the coffee by exchange of enthalpy with the cup. The cup and the liquid quickly reach an equilibrium temperature, θ_{eq}, given by the equation:

$$\theta_{eq} = \frac{M_0 Cp_{wat} \theta_i + mCp_c \theta_a}{M_0 Cp_{wat} + mCp_c} \tag{30}$$

Indeed, the temperature decrease is significant and this speeds up considerably the desired cooling. Evaluation of the kinetics of this process is not easy, but is useless because its rapidity (a few tens of seconds) can be easily demonstrated. So, an even more efficient cooling process would be to pour the coffee again into a new cup (as massive as possible), and repeat if necessary. Because everyday life situations are an unlimited source of scientific questions, what will happen if we add sugar to the liquid? Will this influence the cooling rate? This is another story, worth being discussed—around a cup of coffee!

NOMENCLATURE

A	area (m^2)
A	term defined by Eq. (22)
B	term defined by Eq. (23)
Bi	Biot number, Eq. (27)
Cp	specific heat ($J\ kg^{-1}\ {}^\circ C^{-1}$)

C_T	total concentration (Mol m^{-3})
D	diameter (m)
e	thickness (m)
F	logarithmic mean of partial pressures of air (Pa)
H	height (m)
h_n	free convection heat transfer coefficient ($Wm^{-2}\,^{\circ}C^{-1}$)
h_R	radiation heat transfer coefficient ($Wm^{-2}\,^{\circ}C^{-1}$)
k_c	low or equimolar flux mass transfer coeff. (kg $s^{-1}m^{-2}$)
L	characteristic length (m)
Le	Lewis number= ratio of thermal and massic diffusivities
M	mass of water (kg)
m	mass of the cup (kg)
$\mathfrak{M}$	molecule weight (kg Mol^{-1})
N	molar density of flux (Mol $s^{-1}m^{-2}$)
Nu	Nusselt number, Eq. (29)
Q	heat flux (W m^{-2})
Pr	Prandt number, Eq. (29)
P_v	vapor pressure (Pa)
Q_{evap}	evaporative heat flux (W m^{-2})
Ra	Rayleigh number, Eq. (29)
t	time (s)
U	global heat exchange coefficient (W $m^{-2}\,^{\circ}C^{-1}$)
W	mass flux (kg s^{-1})
ΔH_v	massic latent heat of water (J kg^{-1})
α	thermal diffusivity ($m^2\,s^{-1}$)
β	thermal expansion coefficient (K^{-1})
ε	emissivity of the surface
λ	thermal conductivity
ν	kinematic diffusivity ($m^2\,s^{-1}$)
ϱ	density (kg m^{-3})
σ	Stefan Boltzman constant = 5.67 x 10^{-8} (W $m^{-2}\,K^{-4}$)
θ	temperature ($^{\circ}C$)

subscripts

0	initial
a	ambient
air	air
av	average
c	cup
e	external
i	internal
s	surface
v	vertical
w	wall
wat	water

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