MIXING HOT AND COLD WATER STREAMS AT A T-JUNCTION

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Chemical Engineering students at the University of Alberta are taught how to write technical reports in the first term of their third year. At this point they have a very limited background, having only taken introductory thermodynamics and a mass and energy balance course. The objectives of this course are to provide a bridge between theoretical study and practical applications, and to apply these principles to critical analysis of real experimental data in a professional, clearly written, and concise format. Furthermore, the experiment described in this paper exposes many students to their first look at real equipment and measuring devices.

To teach entry-level chemical engineering students with limited theoretical and statistical analysis background to write technical reports and apply material and energy balance principles to a critical analysis of real data, it is necessary to use a simple experiment. Most previous studies involving the mixing of heated water require dynamic analysis of stirred-tank heaters;[1, 2] however, students with a limited background would have trouble with the theory of such systems. A simpler experiment is the mixing of hot and cold water at a T-junction. This experiment can be used to demonstrate how to use steady-state material and energy balances as a troubleshooting tool to predict flow rates and temperatures of the mixed water stream. Also, this experiment emphasizes the importance of properly placing process measurement devices, *i.e.*, the thermocouples in the current experiment. Furthermore, material and energy balances can also be used to check flow rate calibration equations of the orifice meters for each of the experimental streams. It is important for students to learn the importance of proper calibration in process measuring devices since calibration equations can change over time due to corrosion, erosion, or scale buildup during its use.

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THEORY

For any given continuous, nonreactive process at steady-state, the general material and energy balances can be written as (Felder and Rousseau[3]):

$$\sum m_{out} - \sum m_{in} = 0 \tag{1}$$

$$\dot{Q} - \dot{W} = \sum m_{out}\left(h_{out} + \frac{U_{out}^2}{2} + gz_{out}\right) - \sum m_{in}\left(h_{in} + \frac{U_{in}^2}{2} + gz_{in}\right) \tag{2}$$

In the present study, hot- and cold-water streams are mixed at a T-junction to produce one mixed stream. Assuming the hot- and cold-water streams are completely mixed at the T-junction, the system is adiabatic, work is neither done by or to the system, there are no frictional losses, and that kinetic and potential energy changes are negligible, Eq. (1) and Eq. (2) can be reduced to

$$m_{mix,pred} = m_{cold} + m_{hot} \tag{3}$$

$$m_{mix,meas} h_{mix} = m_{cold} h_{cold} + m_{hot} h_{hot} \tag{4}$$

where h is the enthalpy of each stream and can be defined as (Cengel and Boles[4]):

$$h = C_P (T - T_0) \tag{5}$$

By setting the reference temperature to 0°C and assuming the heat capacity, C_p, is constant for water in the temperature range investigated in the experiment (15 to 50°C), the temperature of the mixed stream could be predicted by combining Eq. (4) and Eq. (5) to give

$$T_{pred} = \frac{m_{cold} T_{cold} + m_{hot} T_{hot}}{m_{mix,meas}} \tag{6}$$

The predicted temperature of the mixed stream using Eq. (6) is dependent upon measured information of the streams before and after the T-junction. In order to determine which (if any) mass flow rate is wrong, Eq. (3) and Eq. (6) can be solved simultaneously to predict the mixed stream temperature. For example, if the mass flow rate of the cold stream is assumed to be wrong, the temperature of the mixed stream can be predicted by replacing the cold stream flow rate using Eq. (3) to give the predicted temperature as

$$T_{pred,no\,m_{cold}} = \frac{\left(m_{mix,meas} - m_{hot}\right) T_{cold} + m_{hot} T_{hot}}{m_{mix,meas}} \tag{7}$$

Similarly, if the mass flow rate of the hot or mixed stream is assumed to be wrong, the predicted mixed stream temperature can be calculated by

$$T_{pred,\ no\ m_{hot}} = \frac{m_{cold} T_{cold} + (m_{mix,meas} - m_{cold})\ T_{hot}}{m_{mix,meas}} \tag{8}$$

$$T_{pred,\ no\ m_{mix}} = \frac{m_{cold} T_{cold} + m_{hot} T_{hot}}{(m_{cold} + m_{hot})} \tag{9}$$

If all three measured mass-flow rates are correct, then Eq. (7), Eq. (8), and Eq. (9) will all give the same value for $T_{pred,mix}$. If one calibration equation is incorrect, however, then only one of those three equations will accurately predict the mixed stream temperature, which will agree with the measured temperatures after the T-junction and therefore indicate that the mass flow rate of the missing stream in the equation is incorrect.

EXPERIMENTAL SETUP AND PROCEDURE

The experimental setup is shown in Figure 1. The entire setup is constructed using half-inch nominal copper pipe, fittings, and brass valves. The feed lines are connected to the domestic hot and cold water supply lines. The hot and cold water streams are mixed at a T-junction before exiting into the drain. As can been seen from this figure the experiment can be done with the hot and cold water streams flowing through either of the inlet lines by setting the appropriate valve combination at the inlet manifold. The flow rate of the water is controlled using globe valves and can be roughly set using

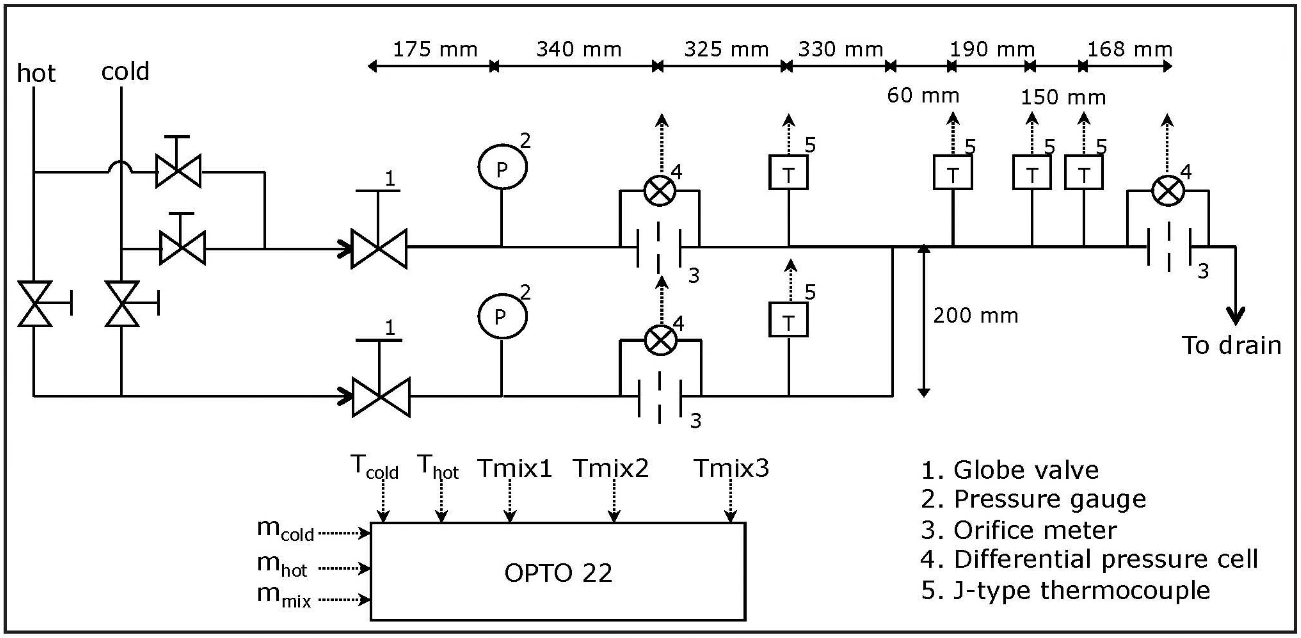


Figure 1. *Schematic Diagram.*

the pressure gauges (Wika). The flow rates of the hot, cold, and mixed streams are measured using a combination of an orifice meter and differential pressure cell (Validyne). The fluid temperatures are measured using J-type thermocouples. The temperature of the mixed stream is measured at 60 mm (Tmix1), 250 mm (Tmix2), and 400 mm (Tmix3) after the T-junction. This is done to determine the proper location for the thermocouple in order to measure the correct temperature of the mixed stream. The detailed locations of other thermocouples, orifice meters, pressure gauges, and globes valves are also shown in Figure 1. Analog signals from the differential pressure cells and thermocouples are converted to digital signals using an OPTO 22 system. These signals are sent to the personal computer, where they are stored and displayed using LabView (version7.1) software. In this experiment, the mass flow rates of the hot, cold, and mixed streams are recorded in terms of volts and temperatures are recorded in terms of degrees Celsius. The following orifice calibration equations are needed to convert volt readings to mass flow rates.

$$m_{cold} = 0.0265\sqrt{V_{cold}} \tag{10}$$

$$m_{hot} = 0.0251\sqrt{V_{hot}} \tag{11}$$

$$m_{mix,meas} = 0.0473\sqrt{V_{mix}} \tag{12}$$

All three calibration equations have a systematic error (accuracy) of ±5 %. To run the experiment, the flow rates of the hot and cold water must be set using the globe valves in conjunction with the pressure gauges. The data acquisition system must be initiated to record the five temperature and three flow rate readings at a set time interval (usually set at either 2 or 5 seconds). When the flow rate and temperature profiles shown by the software remain constant, steady-state can be assumed for that run. At least 100 seconds worth of data for each run should be recorded to ensure the system is at steady-state and to get sufficient sample points for a reasonable analysis. The same experimental procedure is then repeated at different flow rate settings as many times as possible (this depends on class size and laboratory availability), and if reproducibility is to be examined then at least one flow rate setting must be repeated multiple times. The entire experiment can be completed in 10 to 30 minutes, depending on the number of runs students conduct. The short time span of the experiment and large number of flow rate combinations enables even large classes to do individual experiments in a rather short time period. The data is recorded in a Microsoft Excel file that contains the hot, cold, and mixed stream voltage readings from the DP/cell's and the temperatures in °C from the thermocouples.

RESULTS AND DISCUSSION

For this example, five separate runs at various hot- and cold-water flow rates were conducted to illustrate the principles of material and energy balances. Additionally, four more runs were conducted to show the reproducibility of the data. When comparing experimental and predicted results it is necessary to do an error analysis on the variables being compared. The total error for an experimental value can be determined from the sum of the systematic (accuracy) and random (precision) errors of the data. The accuracy error comes from the maximum absolute error in calibration of the measuring device. For this study, the maximum absolute errors in the calibration equations for the measured mass flow rates and temperatures are ± 5% and ± 0.3°C, respectively. The precision error can be obtained directly from the standard deviation, σ, of the measured values. To determine the precision error, Coleman and Steele[5] state that when the number of data points for one time series is equal to or greater than 10, two times the standard deviation gives a good approximation for the 95 % confidence interval. Therefore, the total error in the measured mass flow rates and temperatures is calculated by

$$\omega_m = \pm\left[0.05m + 2\sigma\right] \tag{13}$$

$$\omega_T = \pm\left[0.3 + 2\sigma\right] \tag{14}$$

In order to illustrate how material and energy balances can be used to determine an incorrect calibration and to predict exit stream temperature, orifice calibration Eq. (10) for the cold stream was changed to (without the students knowledge)

$$m_{cold} = 0.0168\sqrt{V_{cold}} \tag{15}$$

The students are then expected to find the incorrect calibration equation and to develop a new orifice calibration equation to fit the data they have.

Figures 2 and 3 show the measured mass flow rates and temperatures as well as the predicted values for all five runs as a function of time, respectively. The measured cold, hot and mixed stream flow rates were calculated using the calibration Eq. (15), Eq. (11), and Eq. (12), respectively. The predicted mass flow rate for the mixed stream was calculated using Eq. (3). For these calculations the average hot- and cold-water flow rates over each steady-state period were used. The predicted temperature of the mixed stream was calculated using Eq. (6) and time-averaged mass flow rates and temperatures for the hot, cold, and mixed stream water streams. The uncertainty for the predicted mass flow rate and temperature of the mixed stream were calculated using the method described by Coleman and Steele[5] and Holman.[6] The uncertainty in the predicted mass flow rate is based on experimental errors in the measured hot- and cold-water stream flow rates and can be calculated by

$$\omega_{m_{mix,pred}} = \pm\left[\left(\frac{\partial m_{mix,pred}}{\partial m_{cold}}\omega_{m_{cold}}\right)^2 + \left(\frac{\partial m_{mix,pred}}{\partial m_{hot}}\omega_{m_{hot}}\right)^2\right]^{1/2} \tag{16}$$

Similarly, the uncertainty in the predicted temperatures obtained from Eq. (6) to Eq. (9) can be determined from the errors in the measured hot, cold, and mixed stream flow rates as well as the measured hot and cold stream temperatures. The uncertainty in the predicted temperature is given by

$$\omega_{T_{pred}} = \pm \left[\left(\frac{\partial T_{pred}}{\partial m_{hot}} \omega_{m_{hot}} \right)^2 + \left(\frac{\partial T_{pred}}{\partial m_{cold}} \omega_{m_{cold}} \right)^2 + \left(\frac{\partial T_{pred}}{\partial m_{mix,meas}} \omega_{m_{mix,meas}} \right)^2 + \left(\frac{\partial T_{pred}}{\partial m_{hot}} \omega_{T_{hot}} \right)^2 + \left(\frac{\partial T_{pred}}{\partial m_{hot}} \omega_{T_{hot}} \right)^2 \right]^{1/2} \quad (17)$$

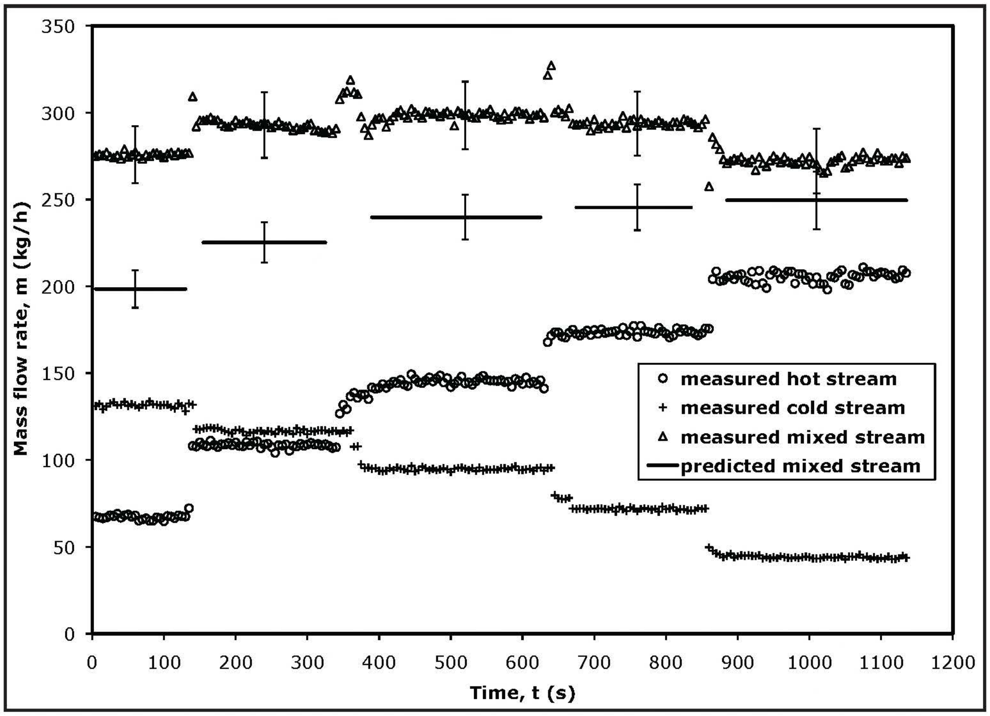
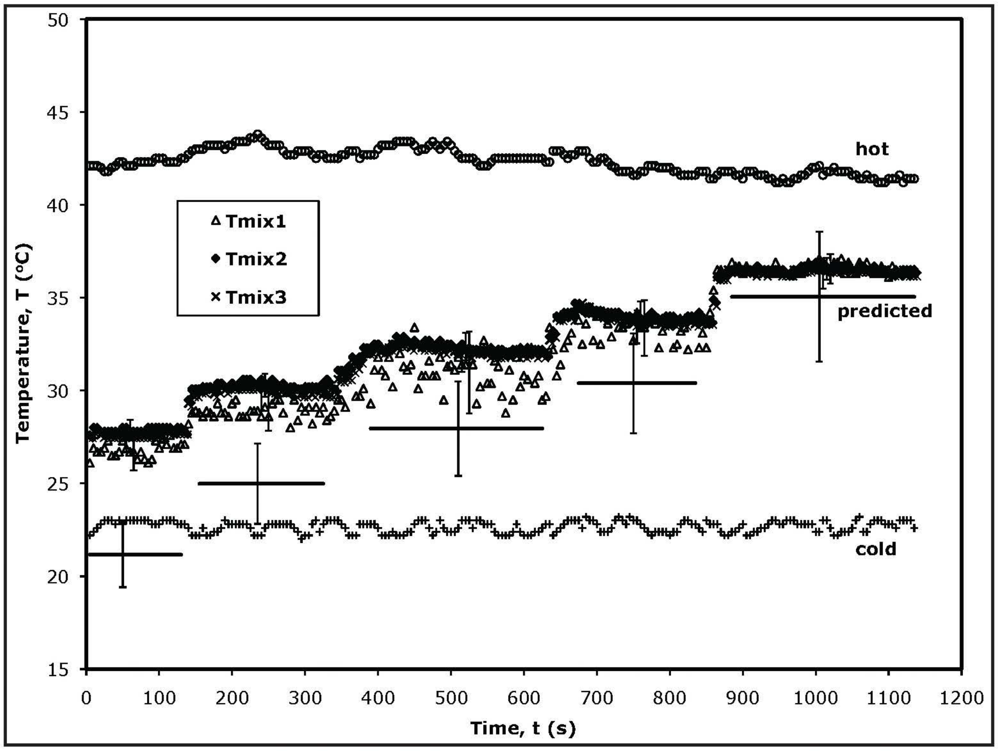


Figure 2. *Mass flow rates of hot, cold, and measured and predicted mixed streams for five steady-state runs.*



◀ **Figure 3.** *Temperatures of hot, cold, measured mixed and predicted mixed water streams for five steady-state runs.*

From these figures it is evident that there were five distinct steady-state periods having time ranges of 0 to 130 s, 155 to 325 s, 390 to 625 s, 675 to 835 s, and 885 to 1135 s. The constant flow rate profiles of the measured values in Figure 2 indicate that the system was indeed behaving at steady-state for each of the five runs. This is also evident in Figure 3 where the measured temperatures before and after the T-junction remain constant for each stream and steady-state period. Since the system is non-reactive and is at steady-state, the material balance equation, shown in Eq. (3) should be valid. Figure 2, however, clearly shows that the predicted flow rate of the mixed stream does not agree, within error, with the measured mixed stream flow rate for the first four runs—leading to the conclusion that one, or more, of the orifice calibration equations must be incorrect.

The mixed stream temperature when measured only 60 mm (Tmix1) downstream of the T-junction has larger error bars and is always approximately 1°C lower compared to the temperatures measured at 270 mm (Tmix2) and 400 mm (Tmix3) downstream of the T-junction, as shown in Figure 3. The error bars were calculated by assuming each thermocouple has the same systematic (accuracy) error of ±0.3 °C. Therefore, the larger total error determined at the Tmix1 location must be only due to higher standard deviations. This, combined with the fact that the measured average temperature reading at the Tmix1 location is lower than the temperatures measured at the Tmix2 and Tmix3 locations, indicates that at the thermocouple closest to the T-junction the two streams have not completely mixed. Furthermore, the measured temperatures at the Tmix2 and Tmix3 locations are not only very similar, but also have very similar size error bars, which indicates that a thermocouple needs to be placed a minimum of 270 mm downstream of the T-junction to ensure complete mixing. Figure 3 also shows that the predicted temperature does not agree within error with the measured values at Tmix2 or Tmix3 for the first four runs, again indicating that there is likely an incorrect calibration equation. For each run, the difference between the measured and predicted temperatures is decreasing until the fifth run, where the measured and predicted values agree within experimental error. This agreement is likely due to the decreasing cold stream flow rate, which has less of an effect on the predicted temperature of the mixed stream.

TABLE 1
Average mixed stream temperatures after the T-junction for all five steady-state trials

Run	Tmix2 (°C)	$T_{pred,nom_{cold}}$ (°C)	$T_{pred,nom_{hot}}$ (°C)	$T_{pred,nom_{mix}}$ (°C)
1	27.9 ± 0.5	27.6 ± 0.7	33.5 ± 1.0	29.4 ± 0.7
2	30.3 ± 0.6	30.2 ± 1.0	34.9 ± 1.0	32.5 ± 0.8
3	32.4 ± 0.8	32.4 ± 1.2	36.4 ± 1.0	34.8 ± 0.9
4	34.1 ± 0.7	34.1 ± 1.3	37.3 ± 1.0	36.4 ± 0.9
5	36.5 ± 0.6	36.9 ± 1.6	38.5 ± 0.7	38.2 ± 0.7

TABLE 2
Experimental average mass flow rates for all five steady-state runs

Run	m_{hot} (kg/h)	m_{cold} (kg/h)	m_{mix} (kg/h)
1	67.0 ± 5.7	131.5 ± 9.1	275.7 ± 16.4
Standard deviation	1.2	1.3	1.3
2	108.6 ± 8.2	116.7 ± 8.1	292.9 ± 18.9
Standard deviation	1.4	1.2	2.1
3	145.0 ± 11.2	94.7 ± 6.4	298.4 ± 19.5
Standard deviation	2.0	0.8	2.3
4	173.7 ± 12.1	71.7 ± 5.1	293.7 ± 18.4
Standard deviation	1.7	0.8	1.9
5	205.5 ± 16.2	44.0 ± 3.6	272.1 ± 18.6
Standard deviation	3.0	0.7	2.5

TABLE 3
Reproducibility of the inlet conditions

Run	m_{hot} (kg/h)	m_{cold} (kg/h)	T_{hot} (°C)	T_{cold} (°C)
3	145.0 ± 11.2	94.7 ± 6.4	42.8 ± 1.1	22.5 ± 0.9
6	145.6 ± 11.6	99.1 ± 8.6	41.7 ± 0.6	22.8 ± 0.9
7	144.1 ± 9.0	97.3 ± 7.2	41.9 ± 0.8	22.8 ± 0.8
8	144.4 ± 13.2	97.4 ± 6.9	42.1 ± 0.5	23.1 ± 0.6
9	148.0 ± 10.1	96.5 ± 7.6	42.7 ± 0.8	22.7 ± 0.9

To determine which calibration equation was incorrect, material and energy balances were solved simultaneously to predict the mixed stream temperature, which can be compared with the measured values. Table 1 shows the average temperature measured at 270 mm downstream of the T-junction (Tmix2) as well as three predicted mixed stream temperatures calculated by eliminating one of the mass flow rate terms in the energy balance equation. The mixed stream temperature was calculated using Eq. (7), Eq. (8), or Eq. (9) by eliminating the use of the flow rate readings of the cold, hot, or mixed streams, respectively. From this table, it is evident that the only predicted temperature that agrees within error of the measured temperature for all five runs is the one in which the flow rate of the cold stream, m_{cold}, is eliminated from the calculation. This finding indicates that the cold stream flow rate calibration equation was incorrect. Using linear regression analysis between the corrected flow rates of the cold stream obtained from Eq. (3) and the corresponding average voltage readings of the cold stream for the five runs, the correct calibration equation was determined to be

$$m_{cold} = 0.0268\sqrt{V_{cold}} \qquad (18)$$

This equation is valid for voltage readings between 0.5 and 4.7 Volts, corresponding to flow rates of 66 to 210 kg/h. The R^2 value is 0.996, so the calibration equation should be quite accurate.

Students can also use error analysis to explain or discuss trends observed in the experimental data. Table 2 shows the measured average mass flow rate of the hot, cold, and mixed streams with total errors and the corresponding standard deviations for each steady-state run. From this table, it can be seen that the total error in the measured hot and cold streams increase as the flow rates of both streams increases. Since the total error for the experimental values was determined from both the accuracy and precision of the data, the total error increase in the measured values is due to the increase in both the flow rates and the standard deviations in both streams as shown in Table 2. Similar observations and discussions can be made for the mixed stream.

The reproducibility of the study was also examined by repeating one of the runs four times. The time average values of the hot and cold stream flow rates and temperatures are shown for the five runs in Table 3. The flow rate of each stream was readjusted between runs and reset to the same value. Even though the way in which the flow rate is set by the student is very crude (a globe valve and a pressure gauge) and both streams were connected to the domestic cold and hot water supply lines, it can be seen that flow rates for the cold stream and for the hot stream are all within experimental error of each other—verifying the reproducibility of the flow rate settings. Table 3 also shows that the hot and cold temperatures agree within experimental error and the supply temperatures remained relatively constant for all repeat runs.

It should be noted that the experiment outlined in this report is only one of many possible ways in which the students can be asked to analyze this system. A few other examples include: 1) the thermocouples could be setup so that one or more was malfunctioning, with students asked to determine which one(s) are malfunctioning and why; 2) only one or two calibration equations could be given to the students and they then asked to determine the unknown ones; 3) all given information could be correct allowing the students to test material and energy balance principles used to predict a mixed stream flow rate and temperature. Depending on the setup and number of runs conducted, more or less emphasis could be placed on reproducibility of data and/or error analysis. To this end the experiment described here is fairly flexible and allows the instructor(s) to vary the experiment from year to year, while retaining the fundamentals.

CONCLUSIONS

A simple mixing of a hot- and cold-water stream at a T-junction was investigated. The main objective was to use mass and energy balance equations to predict mass flow rates and the temperature of the mixed stream after the T-junction, and then compare these with the measured values. Furthermore, the thermocouple location after the T-junction and the reproducibility of the data were also investigated.

It was found that the predicted mixed stream flow rate calculated using mass balance equations did not agree with the measured mixed stream flow rate for all five runs. It was concluded that one or more given orifice calibration equation must be wrong. In order to determine which orifice calibration equation was wrong, mass and energy balance equations were solved simultaneously to predicte mixed stream temperature. It was found that when only the cold stream flow rate was eliminated from the energy balance, the predicted mixed stream temperature was found to agree with all three measured mixed stream temperatures within experimental error for all five runs. This indicated that the given cold stream orifice calibration was wrong.

The mixed stream temperature measured at 60 mm (Tmix1) had a higher standard deviation error than the temperatures measured at 230 mm (Tmix2) and 400 mm (Tmix3) downstream of the T-junction for all five runs. It was also found that the temperatures measured at Tmix2 and Tmix3 locations had similar absolute and standard deviations and error values. Both observations indicated incomplete mixing at the Tmix1 location. Therefore, to ensure complete mixing and minimize heat losses, the thermocouple should be placed at least 230 mm downstream of the T-junction.

The reproducibility of the experimental data was also studied by repeating one of the runs four times. It was found that the flow rates for hot and cold streams were all within error of each other, verifying the reproducibility of the hot and cold stream flow rate settings.

SUMMARY

In this paper, we proposed a simple experiment of mixing a hot- and cold-water stream at a T- junction to demonstrate how to use steady-state material and energy balance principles in troubleshooting of an existing process and determining the integrity and/or location of the measuring devices, such as thermocouples and orifice meters. This experiment is relatively inexpensive, requires little time to complete and is conceptually simple to understand, making it ideal for the undergraduate students who have a very limited chemical engineering background.

NOMENCLUTURE

C_p	Constant pressure specific heat, J/(kg °C)
g	Gravitational acceleration, m/s^2
m	Mass flow rate, kg/s
h	Enthalpy, J/kg
$\dot{Q}$	Heat transfer rate, W
T	Temperature, °C
T_0	Reference Temperature, °C
U	Velocity, m/s
V	Pressure drop across orifice, Volts
$\dot{W}$	Work input to the system, W
z	Elevation, m
σ	Standard deviation of random variable
ω	Error or uncertainty in a parameter

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To teach entry-level chemical engineering students with limited theoretical and statistical analysis background to write technical reports and apply material and energy balance principles to a critical analysis of real data, it is necessary to use a simple experiment.

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APPENDIX A: SAMPLE DATA, CALCULATIONS AND ERROR ANALYSIS

Sample Data:

The raw data in terms of voltage vs. time and temperature vs. time are shown in Figures A1 and A2.

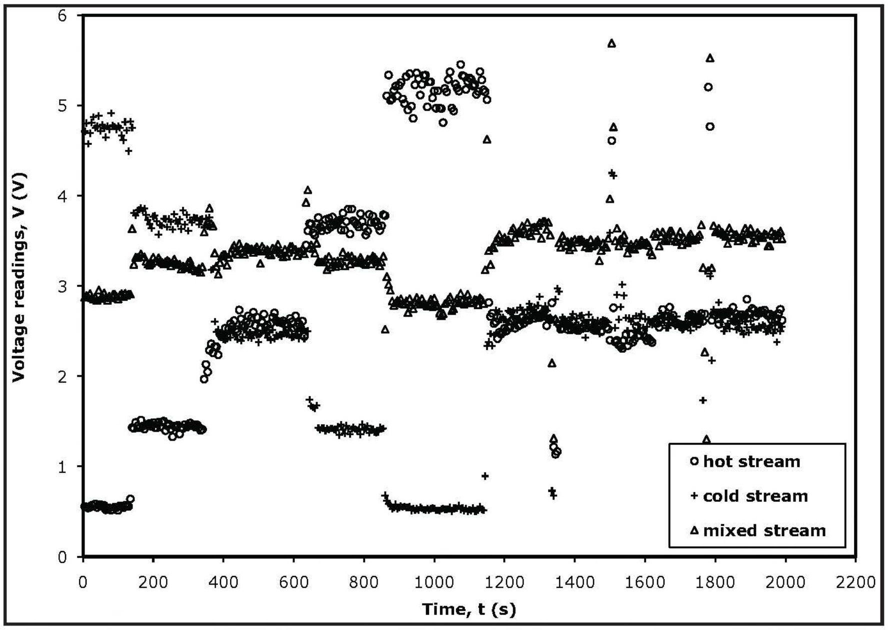

Figure A1. *Voltage outputs from the DP/cells for the hot, cold, and measured mixed streams for all nine steady-state runs.*

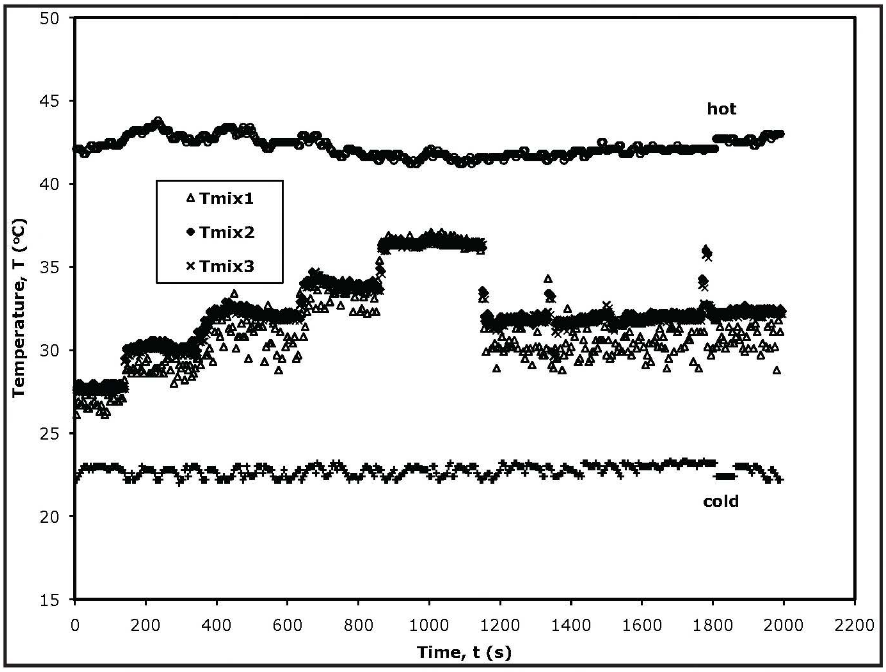

Figure A2. *Temperatures of hot, cold, and measured mixed water streams for all nine steady-state runs.*

Sample calculations:

Using the calibration equations to get mass flow rates from known voltages (Figure A1):

Sample calculations based on the readings recorded at a time of 10 s.

Hot stream voltage = 0.547 V

Cold stream voltage = 4.80 V

Mixed stream voltage = 2.90 V

To get the mass flow rates we use Eqs. (15), (11), and (12)

$$m_{cold} = 0.0168\sqrt{V_{cold}} = 0.0168\sqrt{4.80} = 0.0368\,kg/s$$

$$m_{hot} = 0.0251\sqrt{V_{hot}} = 0.0251\sqrt{0.547} = 0.0186\,kg/s$$

$$m_{mix,meas} = 0.0473\sqrt{V_{mix}} = 0.0473\sqrt{2.9} = 0.0805kg/s$$

The predicted mixed stream flow rate was calculated using the steady-state material balance (Eq. (3)) and average values for flow rates. The average cold stream flow rate for the first run was 0.0365 kg/s, for the hot stream it was 0.0186 kg/s, and for the measured mixed stream it was 0.0766 kg/s. Therefore, for the first run, the predicted mass flow rate of the mixed stream is

$$\mathrm{m}_{\mathrm{mix,pred}} = \mathrm{m}_{\mathrm{cold}} + \mathrm{m}_{\mathrm{hot}} = 0.0365\mathrm{kg/s} + 0.0186\mathrm{kg/s} = 0.0551\mathrm{kg/s}$$

The predicted temperature of the mixed stream was calculated from Eq. (6) using average values for run 1.

$$\mathrm{T}_{\mathrm{pred}} = \frac{\mathrm{m}_{\mathrm{cold}}\mathrm{T}_{\mathrm{cold}} + \mathrm{m}_{\mathrm{hot}}\mathrm{T}_{\mathrm{hot}}}{\mathrm{m}_{\mathrm{mix,meas}}} = \left[\frac{0.0365 \bullet 22.9 + 0.0186 \bullet 42.2}{0.0766}\right] = 21.2$$

Assuming that the cold stream calibration equation is wrong, Eq. (7) can be used to predict the temperature from the average values for the first run.

$$\mathrm{T}_{\mathrm{pred,nom}_{\mathrm{cold}}} = \frac{\left(\mathrm{m}_{\mathrm{mix,meas}} - \mathrm{m}_{\mathrm{hot}}\right)\mathrm{T}_{\mathrm{cold}} + \mathrm{m}_{\mathrm{hot}}\mathrm{T}_{\mathrm{hot}}}{\mathrm{m}_{\mathrm{mix,meas}}} = \frac{\left(0.0766 - 0.0186\right)22.9 + 0.0186 \bullet 42.2}{0.0766} = 27.6^{\circ}\mathrm{C}$$

Assuming that the hot stream calibration equation is wrong, Eq. (8) can be used to predict the temperature from the average values for the first run.

$$\mathrm{T}_{\mathrm{pred,nom}_{\mathrm{hot}}} = \frac{\mathrm{m}_{\mathrm{cold}}\mathrm{T}_{\mathrm{cold}} + \left(\mathrm{m}_{\mathrm{mix,meas}} - \mathrm{m}_{\mathrm{cold}}\right)\mathrm{T}_{\mathrm{hot}}}{\mathrm{m}_{\mathrm{mix,meas}}} = \frac{0.0365 \bullet 22.9 + \left(0.0766 - 0.0356\right)42.2}{0.0766} = 33.5^{\circ}\mathrm{C}$$

Assuming that the mixed stream calibration equation is wrong, Eq, (9) can be used to predict the temperature from the average values for the first run

$$\mathrm{T}_{\mathrm{pred,nom}_{\mathrm{mix}}} = \frac{\mathrm{m}_{\mathrm{cold}}\mathrm{T}_{\mathrm{cold}} + \mathrm{m}_{\mathrm{hot}}\mathrm{T}_{\mathrm{hot}}}{\left(\mathrm{m}_{\mathrm{cold}} + \mathrm{m}_{\mathrm{hot}}\right)} = \frac{0.0365 \bullet 22.9 + 0.0186 \bullet 42.2}{0.0365 + 0.0186} = 29.4^{\circ}\mathrm{C}$$

Error analysis:

The error in the measured flow rates is given as the maximum absolute error in the calibration equation plus two standard deviations. For the mass flow rates the systematic error (accuracy) was ± 5 % for the full scale. For the first run it was determined that the standard deviations were 0.000355 kg/s, 0.000325 kg/s, and 0.000367 kg/s for the cold, hot, and mixed streams, respectively. For the first run these standard deviations were based on 26 data points. Therefore, the total experimental error for the mass flow rates is:

$$\text{Cold Stream}: \omega_{\mathrm{m}_{\mathrm{cold}}} = \pm\left[0.05\mathrm{m}_{\mathrm{cold}} + 2\sigma_{\mathrm{m}_{\mathrm{cold}}}\right] = \pm\left[0.05 \bullet 0.0365 + 2 \bullet 0.000355\right] = \pm 0.00254\,\mathrm{kg/s}$$

$$\text{Hot Stream}: \omega_{\mathrm{m}_{\mathrm{hot}}} = \pm\left[0.05\mathrm{m}_{\mathrm{hot}} + 2\sigma_{\mathrm{m}_{\mathrm{hot}}}\right] = \pm\left[0.05 \bullet 0.0186 + 2 \bullet 0.000325\right] = \pm 0.00158\,\mathrm{kg/s}$$

$$\text{Mixed Stream}: \omega_{\mathrm{m}_{\mathrm{mix,meas}}} = \pm\left[0.05\mathrm{m}_{\mathrm{mix,meas}} + 2\sigma_{\mathrm{m}_{\mathrm{mix,meas}}}\right] = \pm\left[0.05 \bullet 0.0766 + 2 \bullet 0.000367\right] = \pm 0.00456\,\mathrm{kg/s}$$

For the temperatures the systematic error (accuracy) was ± 0.3°C for the full scale. For the first run it was determined that the standard deviations were 0.20°C, 0.19°C, 0.40°C, 0.12°C, and 0.09°C for the cold, hot, mix1, mix2, and mix3 thermocouples, respectively. For the first run these standard deviations were based on 26 data points. Therefore, the total experimental error for the mass flow rates is:

$$\text{Cold thermocouple}: \omega_{\mathrm{T}_{\mathrm{cold}}} = \pm\left[0.3 + 2\sigma_{\mathrm{T}_{\mathrm{cold}}}\right] = \pm\left[0.3 + 2 \bullet 0.20\right] = \pm 0.70^{\circ}\mathrm{C}$$

$$\text{Hot thermocouple}: \omega_{\mathrm{T}_{\mathrm{hot}}} = \pm\left[0.3 + 2\sigma_{\mathrm{T}_{\mathrm{hot}}}\right] = \pm\left[0.3 + 2 \bullet 0.19\right] = \pm 0.68^{\circ}\mathrm{C}$$

$$\text{Mix1 thermocouple}: \omega_{\mathrm{Tmix1}} = \pm\left[0.3 + 2\sigma_{\mathrm{Tmix1}}\right] = \pm\left[0.3 + 2 \bullet 0.40\right] = \pm 1.10^{\circ}\mathrm{C}$$

$$\text{Mix2 thermocouple}: \omega_{\mathrm{Tmix2}} = \pm\left[0.3 + 2\sigma_{\mathrm{Tmix2}}\right] = \pm\left[0.3 + 2 \bullet 0.12\right] = \pm 0.54^{\circ}\mathrm{C}$$

$$\text{Mix3 thermocouple}: \omega_{\mathrm{Tmix3}} = \pm\left[0.3 + 2\sigma_{\mathrm{Tmix3}}\right] = \pm\left[0.3 + 2 \bullet 0.09\right] = \pm 0.48^{\circ}\mathrm{C}$$

Uncertainty in the predicted mixed stream mass flow rates is given by

$$\omega_{m_{\text{mix,pred}}} = \pm\left[\left(\frac{\partial m_{\text{mix,pred}}}{\partial m_{\text{cold}}}\omega_{m_{\text{cold}}}\right)^2 + \left(\frac{\partial m_{\text{mix,pred}}}{\partial m_{\text{hot}}}\omega_{m_{\text{hot}}}\right)^2\right]^{1/2}$$

in this case both partial derivatives are equal to 1 so the uncertainty in the predicted mixed stream mass flow rate becomes

$$\omega_{m_{\text{mix,pred}}} = \pm\left[\left(\omega_{m_{\text{cold}}}\right)^2 + \left(\omega_{m_{\text{hot}}}\right)^2\right]^{1/2} = \pm\left[(0.00254)^2 + (0.00158)^2\right]^{1/2} = \pm 0.0030\,\text{kg/s}$$

The uncertainty in the predicted temperatures from Eqs. (6) to (9) is given by

$$\omega_{T_{\text{pred}}} = \pm\left[\left(\frac{\partial T_{\text{pred}}}{\partial m_{\text{hot}}}\omega_{m_{\text{hot}}}\right)^2 + \left(\frac{\partial T_{\text{pred}}}{\partial m_{\text{cold}}}\omega_{m_{\text{cold}}}\right)^2 + \left(\frac{\partial T_{\text{pred}}}{\partial m_{\text{mix,meas}}}\omega_{m_{\text{mix,meas}}}\right)^2 + \left(\frac{\partial T_{\text{pred}}}{\partial T_{\text{hot}}}\omega_{T_{\text{hot}}}\right)^2 + \left(\frac{\partial T_{\text{pred}}}{\partial T_{\text{cold}}}\omega_{T_{\text{cold}}}\right)^2\right]^{1/2}$$

For the uncertainty in Eq. (6) the partial derivatives are

$$\frac{\partial T_{\text{pred}}}{\partial m_{\text{hot}}} = \frac{T_{\text{hot}}}{m_{\text{mix,meas}}} = \frac{42.2}{0.0766} = 550.9\frac{^\circ\text{C}}{\text{kg/s}}$$

$$\frac{\partial T_{\text{pred}}}{\partial m_{\text{cold}}} = \frac{T_{\text{cold}}}{m_{\text{mix,meas}}} = \frac{22.9}{0.0766} = 299.0\frac{^\circ\text{C}}{\text{kg/s}}$$

$$\frac{\partial T_{\text{pred}}}{\partial m_{\text{mix,meas}}} = -\frac{\left(m_{\text{hot}}T_{\text{hot}} + m_{\text{cold}}T_{\text{cold}}\right)}{\left(M_{\text{mix,meas}}\right)^2} = \frac{-(0.0186\bullet 42.2 + 0.0365\bullet 22.9)}{(0.0766)^2} = -276.2\frac{^\circ\text{C}}{\text{kg/s}}$$

$$\frac{\partial T_{\text{pred}}}{\partial T_{\text{hot}}} = \frac{m_{\text{hot}}}{m_{\text{mix,meas}}} = \frac{0.0186}{0.0766} = 0.2428$$

$$\frac{\partial T_{\text{pred}}}{\partial T_{\text{cold}}} = \frac{m_{\text{cold}}}{m_{\text{mix,meas}}} = \frac{0.0365}{0.0766} = 0.4765$$

Then, the uncertainty in T_{pred} is

$$\omega_{T_{\text{pred}}} = \pm\left[(550.9\bullet 0.00158)^2 + (299.0\bullet 0.00254)^2 + (-276.2\bullet 0.00456)^2 + (0.2428\bullet 0.68)^2 + (0.4765\bullet 0.70)^2\right] = 1.7^\circ\text{C}$$

For the uncertainty in Eq. (7) the partial derivatives are

$$\frac{\partial T_{\text{pred,no}\,m_{\text{cold}}}}{\partial m_{\text{hot}}} = \frac{T_{\text{hot}} - T_{\text{cold}}}{m_{\text{mix,meas}}} = \frac{42.2 - 22.9}{0.0766} = 252.0\frac{^\circ\text{C}}{\text{kg/s}}$$

$$\frac{\partial T_{\text{pred,no}\,m_{\text{cold}}}}{\partial m_{\text{cold}}} = 0$$

$$\frac{\partial T_{\text{pred,no}\,m_{\text{cold}}}}{\partial m_{\text{mix,meas}}} = \frac{m_{\text{hot}}\left(T_{\text{cold}} - T_{\text{hot}}\right)}{\left(m_{\text{mix,meas}}\right)^2} = \frac{0.0186(22.9 - 42.2)}{(0.0766)^2} = -61.2\frac{^\circ\text{C}}{\text{kg/s}}$$

$$\frac{\partial T_{\text{pred,no}\,m_{\text{cold}}}}{\partial T_{\text{hot}}} = \frac{m_{\text{hot}}}{m_{\text{mix,meas}}} = \frac{0.0186}{0.0766} = 0.2428$$

$$\frac{\partial T_{\text{pred,no}\,m_{\text{hot}}}}{\partial T_{\text{cold}}} = \frac{m_{\text{mix,meas}} - m_{\text{hot}}}{m_{\text{mix,meas}}} = \frac{0.0766 - 0.0186}{0.0766} = 0.7572$$

Then, the uncertainty in T_{pred} is

$$\omega_{T_{\text{pred,no}\,m_{\text{cold}}}} = \pm\left[(0\bullet 0.00158)^2 + (-252.0\bullet 0.00254)^2 + (120.1\bullet 0.00456)^2 + (0.5234\bullet 0.68)^2 + (0.4765\bullet 0.70)^2\right]^{1/2} = 1.0^\circ\text{C}$$

> ***This experiment is relatively inexpensive, requires little time to complete, and is conceptually simple to understand, making it ideal for the undergraduate students who have a very limited chemical engineering background.***

For the uncertainty in Eq. (8) the partial derivatives are

$$\frac{\partial T_{\mathrm{pred,nom_{hot}}}}{\partial m_{\mathrm{hot}}}=0$$

$$\frac{\partial T_{\mathrm{pred,nom_{hot}}}}{\partial m_{\mathrm{cold}}}=\frac{T_{\mathrm{cold}}-T_{\mathrm{hot}}}{m_{\mathrm{mix,meas}}}=\frac{22.9-42.2}{0.0766}=-252\frac{^{\circ}\mathrm{C}}{\mathrm{kg/s}}$$

$$\frac{\partial T_{\mathrm{pred,nom_{hot}}}}{\partial m_{\mathrm{mix,meas}}}=\frac{m_{\mathrm{cold}}\left(T_{\mathrm{hot}}-T_{\mathrm{cold}}\right)}{\left(m_{\mathrm{mix,meas}}\right)^{2}}=\frac{0.0365\left(42.2-22.9\right)}{\left(0.0766\right)^{2}}=120.1\frac{^{\circ}\mathrm{C}}{\mathrm{kg/s}}$$

$$\frac{\partial T_{\mathrm{pred,nom_{hot}}}}{\partial T_{\mathrm{hot}}}=\frac{m_{\mathrm{mix,meas}}-m_{\mathrm{cold}}}{m_{\mathrm{mix,meas}}}=\frac{0.0766-0.0365}{0.0766}=0.5234$$

$$\frac{\partial T_{\mathrm{pred,nom_{cold}}}}{\partial T_{\mathrm{cold}}}=\frac{m_{\mathrm{cold}}}{m_{\mathrm{mix,meas}}}=\frac{0.0365}{0.0766}=0.4765$$

Then, the uncertainty in T_{pred} is

$$\omega_{T_{\mathrm{pred,nom_{hot}}}}=\pm\left[\left(0\bullet 0.00158\right)^{2}+\left(-252.0\bullet 0.00254\right)^{2}+\left(120.1\bullet 0.00456\right)^{2}+\left(0.5234\bullet 0.68\right)^{2}+\left(0.4765\bullet 0.70\right)^{2}\right]^{1/2}=1.0^{\circ}\mathrm{C}$$

For the uncertainty in Eq. (9) the partial derivatives are

$$\frac{\partial T_{\mathrm{pred,nom_{mix}}}}{\partial m_{\mathrm{hot}}}=\frac{\left(m_{\mathrm{cold}}+m_{\mathrm{hot}}\right)T_{\mathrm{hot}}-\left(m_{\mathrm{hot}}T_{\mathrm{hot}}+m_{\mathrm{cold}}T_{\mathrm{cold}}\right)}{\left(m_{\mathrm{cold}}+m_{\mathrm{hot}}\right)^{2}}$$

$$=\frac{\left(0.0368+0.0186\right)42.2-\left(0.0186\bullet 42.2+0.0368\bullet 22.9\right)}{\left(0.0368+0.0186\right)^{2}}=231.41\frac{^{\circ}\mathrm{C}}{\mathrm{kg/s}}$$

$$\frac{\partial T_{\mathrm{pred,nom_{mix}}}}{\partial m_{\mathrm{cold}}}=\frac{\left(m_{\mathrm{hot}}+m_{\mathrm{cold}}\right)T_{\mathrm{cold}}-\left(m_{\mathrm{hot}}T_{\mathrm{hot}}+m_{\mathrm{cold}}T_{\mathrm{cold}}\right)}{\left(m_{\mathrm{cold}}+m_{\mathrm{hot}}\right)^{2}}$$

$$=\frac{\left(0.0186+0.0368\right)22.9-\left(0.0186\bullet 42.2+0.0368\bullet 22.9\right)}{\left(0.0368+0.0186\right)^{2}}=-117.0\frac{^{\circ}\mathrm{C}}{\mathrm{kg/s}}$$

$$\frac{\partial T_{\mathrm{pred,nom_{mix}}}}{\partial m_{\mathrm{mix,meas}}}=0$$

$$\frac{\partial T_{\mathrm{pred,nom_{mix}}}}{\partial T_{\mathrm{hot}}}=\frac{m_{\mathrm{hot}}}{m_{\mathrm{cold}}+m_{\mathrm{hot}}}=\frac{0.0186}{0.0368+0.0186}=0.3357$$

$$\frac{\partial T_{\mathrm{pred,nom_{cold}}}}{\partial T_{\mathrm{cold}}}=\frac{m_{\mathrm{cold}}}{m_{\mathrm{cold}}+m_{\mathrm{hot}}}=\frac{0.0368}{0.0368+0.0186}=0.6643$$

Then, the uncertainty in T_{pred} is

$$\omega_{T_{\mathrm{pred,nom_{hot}}}}=\pm\left[\left(231.41\bullet 0.00158\right)^{2}+\left(-117.0\bullet 0.00254\right)^{2}+\left(0\bullet 0.00456\right)^{2}+\left(0.3357\bullet 0.69\right)^{2}+\left(0.6643\bullet 0.71\right)^{2}\right]=0.7^{\circ}\mathrm{C}$$

TABLE A1
Corrected Mass Flow Rates and Corresponding Voltage Readings for the Cold Stream.

run	Corrected cold stream mass flow rate, $m_{cold, corrected}$ (kg/s)	Average cold stream voltage, V_{cold}
1 0.	058	4.7
2 0.	051	3.7
3 0.	043	2.5
4 0.	033	1.4
5 0.	019	0.5

Since the calibration equation for the cold stream mass flow rate is incorrect, it was necessary to find a new, corrected calibration equation. This was done using the material balance and solving for the cold stream mass flow rate. For the first run

$$m_{cold,corrected} = m_{mix,measured} - m_{hot} = 0.0766 - 0.0186 = 0.058 kg/s$$

The average voltage reading for the corresponding run was 4.7 V for the cold stream. Table A1 shows the data for all five runs with different flow settings.

Regression analysis was done between the corrected mass flow rate and the square root of the voltage reading. The new calibration equation is

$$m_{cold} = 0.0268\sqrt{V_{cold}}$$

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