The Hydrodynamic Stability of a Fluid-Particle Flow:
INSTABILITIES IN GAS-FLUIDIZED BEDS

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The last few lectures of an undergraduate class in fluid mechanics offer instructors an opportunity to teach students some advanced topics that go beyond the traditional course material.[1] Fluid-particle flows, where both the fluid and particle are in motion, are prevalent in many industries including the chemical, materials, and energy industries.[2] In the pharmaceutical and biotechnology industries, which are hiring unprecedented numbers of chemical engineers, nearly all manufacturing facilities involve multiple processing steps that include fluid-particle flows. In spite of the industrial importance of fluid-particle flows, they are rarely covered in any depth in a fluid mechanics course.[3] Moreover, it is difficult to find examples of fluid-particle flows where undergraduates have the necessary background to handle the equations and analysis that is necessary if more than a survey of the material is to be achieved.

The hydrodynamic stability of a fluid in motion is a fundamental concept in fluid mechanics.[4] In an undergraduate fluid mechanics class, students are usually introduced to hydrodynamic stability during discussions of the transition from laminar to turbulent pipe flows,[5] but a detailed understanding of hydrodynamic stability is not critical for most single-phase flow examples. In multiphase flows, however,flow instabilities, density waves, and nonuniform flows are generic. Thus controlling and understanding flow instabilities is crucial for numerous industries that process fluid and particles.

Here we present a fluid-particle example, a gas-fluidized bed, that has been taught at Rutgers in the fluid mechanics class. It relies on a student's knowledge of the Navier Stokes equations together with Taylor series and complex numbers to perform a fluid-particle stability analysis. At Rutgers, students have already learned Taylor series and complex numbers in a previous math class by the time they take the fluid mechanics class in their junior year. The fluid- particle flow problem presented for analysis has been simplified as a single-phase compressible fluid acted upon by a force representing the fluid-particle drag force, and analytical solutions can be obtained for this simplified system. Thus the model looks like the Navier Stokes equations with an extra term. The problem could be easily implemented in a Fluid Dynamics or Transport Phenomena course in the chemical or mechanical engineering curriculum or an Applied Math course in Fluid Dynamics. In general, we would like to provide students with a fundamental understanding of fluid-particle flows and linear stability theory.

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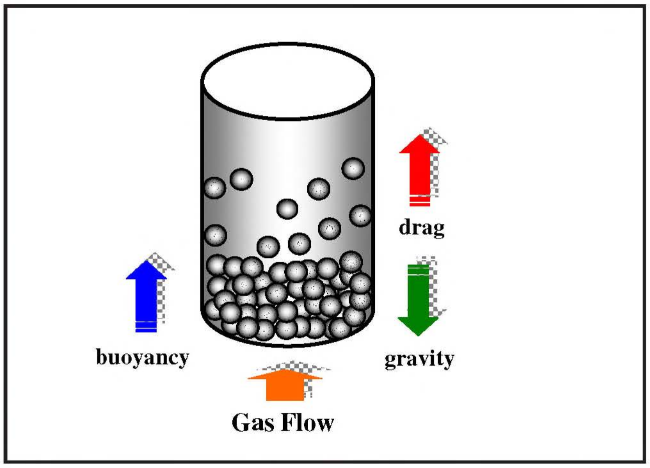


Figure 1. *Fluidized bed.*

In fluid mechanics, stable flow is best described as flow that will be maintained in spite of small disturbances or perturbations to the flow. The flow is unstable if a small disturbance will lead to the flow to progressively depart from the initial base state.[6] The study of hydrodynamic stability thus involves determining when the state of fluid flow becomes unstable to small perturbations, and how instabilities evolve in space and time.[4, 7]

In stability theory, flow behavior is first investigated by performing a linear stability analysis of steady state solutions satisfying appropriate equations of motion and boundary conditions. The stability of such a system is determined by examining its reaction to all possible infinitesimal disturbances to basic steady flow. These results provide the groundwork for further investigation of development of instabilities and evolution of unstable waveforms. Since these methods of analysis involve the linearization and numerical integration of nonlinear partial differential equations of motion, this can lead to many technical difficulties in all but the simplest of flow configurations, and thus is difficult for undergraduate students. To avoid these difficulties, the following problem demonstrates how the stability of a two-phase flow system can be examined using a single-phase compressible flow model, which has been shown to capture the salient features of instability development in the physical system it represents.

PHYSICAL PROBLEM

The gas-fluidized bed consists of a vertical column containing particles supported by a porous bottom (distributor) plate (Figure 1). When a gas is introduced to the column through the distributor, the particles remain stationary until the drag force exerted by the upward flow of gas is balanced by the weight of the bed. At this point, the particles become mobilized, and the bed transitions from being packed to fluidized. In some cases, the bed can expand uniformly at points beyond the minimum fluid velocity u_{mf} with relatively little particle motion (see Figure 2a depicting uniform or *particulate* fluidization). For most cases, however, uniform fluidization is restricted to a narrow fluidization velocity range bounded by u_{mf} and the commencement of bubbling, u_{mb}. At this point, the bed becomes hydrodynamically unstable to small perturbations and lends itself to the formation of vertically traveling voidage waves that can become spatially amplified in the bed and bring about complex and turbulent flow behavior (see Figure 2b depicting bubbling or *aggregative* fluidization).

In the fluidization research, instability behavior in gas-fluidized beds has been examined by hydrodynamic stability analysis since the early 1960s. Flow instabilities in these systems are in the form of "traveling waves." The physical manifestation of the traveling wave solution in a fluidized bed takes the form of particle free voidage waves (*e.g.*, bubbles, slugs, and other waveforms), as well as dense particle-cluster formations, which can move violently throughout the bed and dramatically impact process performance and safety[2] (see Figure 2b). Since fluidized beds are of tremendous importance in industry, the onset and behavior of the unstable flow regime must be well characterized by analysis of the equations governing fluid and particle flow.

Continuum arguments have been used to develop equations of continuity and motion for describing the behavior of the fluid and particle phases in a similar way to the development of the Navier-Stokes equations for Newtonian single phase flows.[8] The multiphase continuum approach has been used quite successfully for predicting the onset and propagation behavior of instabilities in gas-fluidized beds. Recently, it has been shown that the salient features of instability development in gas-fluidized beds predicted using the multiphase continuum approach are also captured using a single-phase flow model for a compressible fluid acted upon by a density dependent force provided by the drag force.[9, 10] This simplified model takes a form similar to the Navier Stokes equations for fluid flow. While this problem is quite significant in itself for gaining physical insight into the development of density waves in fluidized beds, it also presents an opportunity for chemical engineering students to develop analytical skills for examining the hydrodynamic stability of a fluid-particle flow using a simple flow model.

MODEL EQUATIONS

The underlying assumption of the Johri & Glasser[9, 10] model is that a nonuniform suspension of particles fluidized by a gas can sometimes behave (in the continuum) like a Newtonian compressible "fluid" whose motion can be related to the solids in a fluidized bed. From this point on the term "fluid" will be used in this context where "gas" refers to the fluidization medium. Based on the assumption that the inertial and viscous force terms in the gas phase equation are negligible, these authors simplified the multiphase model to equations of continuity and motion for a single fluid having variable density. Continuum equations of continuity and motion for the fluid are respectively written as[10]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \underline{v}\right) = 0 \qquad (1)$$

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) = \underline{F} - \nabla \cdot \underline{\underline{\sigma}} + \rho \underline{g} \qquad (2)$$

where the density of the Newtonian fluid (ρ) varies linearly with the solids volume fraction ϕ as $\rho = \rho_s \phi$, and ρ_s is the absolute particle density. The fluid velocity vector is represented by $\underline{v}$; $\underline{\underline{\sigma}}$ is the fluid phase stress tensor; and $\underline{g}$ is the gravity force vector. The density dependent force $\underline{F}$ represents the drag force exerted on the particle assembly by the gas flow. Eqs. (1) and (2) represent equations of continuity and motion for a compressible fluid and are exactly what students would be exposed to in a course in fluid mechanics except for the additional density dependent force, $\underline{F}$.

Continuum arguments provide constitutive relations for the various terms. Johri & Glasser[10] adopted a suitable closure for $\underline{\underline{\sigma}}$ motivated by the work of Anderson & Jackson,[11] which takes a form analogous to that for a Newtonian fluid:

$$\underline{\underline{\sigma}} = P\underline{\underline{I}} - \mu\left[\nabla \underline{v} + \left(\nabla \underline{v}\right)^T - \frac{2}{3}\left(\nabla \cdot \underline{v}\right)\underline{\underline{I}}\right] \qquad (3)$$

where μ is the viscosity of the fluid (assumed to be constant in this analysis), and P is the pressure, which is dependent on particle volume fraction ϕ, or, in this case, ρ . This pressure term is analogous to the pressure of an ideal gas, which is a function of gas density. We will examine flow only in the vertical dimension (x), in which case there is no variation in the other two directions (y and z) thus equations 1 and 2 are written as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho v\right) = 0 \qquad (4)$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = -\frac{\partial P}{\partial x} + F + \mu\left(\frac{4}{3}\right)\frac{\partial^2 v}{\partial x^2} - \rho g \qquad (5)$$

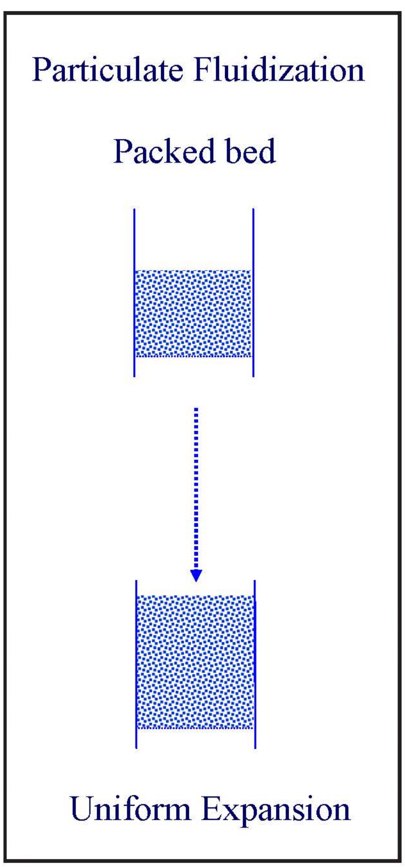
where $v=v_x$ and $F=F_x$. Linear forms for F and P are adopted and these represent the simplest possible forms capable of capturing the hydrodynamic instability:

$$F = A\rho + B; \qquad P = E\rho \qquad (6)$$

where A, B, and E are appropriately assigned constants consistent with experimental evidence of gas-fluidized bed behavior.

LINEAR STABILITY ANALYSIS PROCEDURE

As stated in the Introduction, hydrodynamic stability of a system is first investigated by linear stability analysis (LSA)



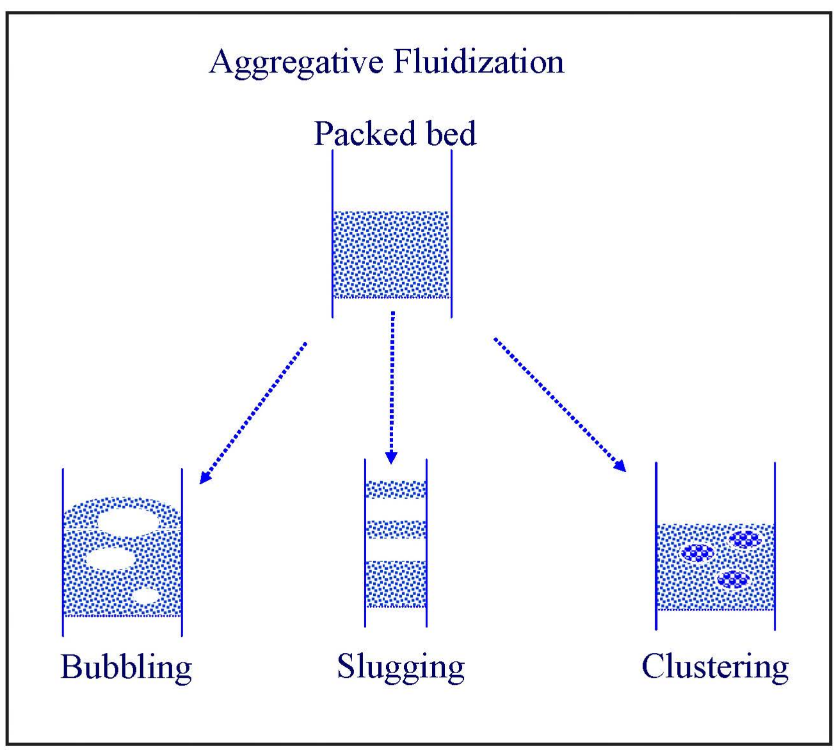


Figure 2a. *(left) Particulate fluidization.*
Figure 2b. *(above) Aggregative fluidization.*

of steady state solutions satisfying the governing equations. We therefore begin with a linear stability analysis of the steady state solution. Students should perform each of the following steps (in their entirety) either individually or in small groups of two to three students. Discussion is strongly encouraged during the analysis to provide the students with insight into the system's physical behavior. Topics for discussion are provided within the text.

Steady State Solution: Prove that the simplest solution to the set of coupled nonlinear partial differential Eqs. (4) and (5) represents a spatially uniform state of static "fluid" where the density dependent force F is balanced by the gravitational force of the fluid. In particular, show that under these conditions $v_0=0$, $\rho=\rho_0$ and $F_0=\rho_0 g$, where $\rho_0=\rho_s\phi_0$ and the subscript '0' is used to designate conditions at steady state. Find numerical solutions for the steady state values of ϕ_0 and F_0 in dilute beds having ρ_0=220 and 440 kg/m^3 and dense beds with ρ_0=1100, 1210, and 1320 kg/m^3 when ρ_s=2200 kg/m^3. Find the constant B which is chosen in accordance with $F_0=\rho_0 g$ for each of these bed conditions and write functional forms for the linear closure for F using parameter values from Table 1.

Linearization: Impose perturbations ρ' and v' on the steady state solution representing infinitesimal changes in density and velocity:

$$\rho=\rho_0+\rho' \qquad v=v_0+v'$$

Rewrite Eqs. (4) and (5) in terms of the perturbation variables, and perform a Taylor series expansion about the steady state solution. Since the perturbations are assumed to be both small and smoothly varying in space and time, their derivatives are also small. By neglecting terms in the series involving powers of perturbation variables greater than one, and eliminating products of perturbation variables, the students should obtain the following linearized equations in perturbation variables ρ' and v':

$$\frac{\partial\rho'}{\partial t}+\rho_0\frac{\partial v'}{\partial x}=0 \tag{7}$$

$$\rho_0\frac{\partial v'}{\partial t}=-P_0'\frac{\partial\rho'}{\partial x}+\left[F_0'-g\right]\rho'+\mu\left(\frac{4}{3}\right)\frac{\partial^2 v'}{\partial x^2}=0 \tag{8}$$

where:

$$F_0'=\left(\frac{dF}{d\rho}\right)_{\rho=\rho_0} \qquad P_0'=\left(\frac{dP}{d\rho}\right)_{\rho=\rho_0} \tag{9}$$

We seek a solution to Eqs. (7) and (8) in the form of plane waves since in real fluidized beds the development of the density waves can be observed in the bed:

$$\rho'=\hat{\rho}\exp(st)\exp(i\kappa) \qquad v'=\hat{v}\exp(st)\exp(i\kappa) \tag{10}$$

where $\hat{\rho}$ and $\hat{v}$ are (complex) amplitudes of the perturbations in density and velocity respectively, and κ is the wavenumber of the disturbance (in one dimension x), having real components, whose wavelength $\lambda=2\pi/|\kappa|$. In general, s is complex, $s=\sigma\pm\omega i$, where the imaginary part is used to determine wavespeed (c) according to the relationship $c=\omega/\kappa$, and the real part determines the growth or decay rate of the wave with time. If σ is positive, the perturbations grow in time and the base state is unstable, and if σ is negative, the perturbations decay and the base state is stable [see Eq. (10)]. That is, for a positive σ the base state solution will not be observed in practice.

Computational Analysis: By combining Eqs. (7) and (8), we can reduce the linearized PDE's to a single algebraic equation in s by performing the following steps: take the $\partial/\partial x$ of Eq. (8); substitute into the resulting equation using the expression for $\partial v'/\partial x\partial t$, $\partial^3 v'/\partial x^3$ obtained from continuity Eq. (7) and its derivatives to eliminate v'. The student should obtain a single differential equation in the density perturbation variable ρ':

$$-\frac{\partial^2\rho'}{\partial t^2}=-P_0'\frac{\partial^2\rho'}{\partial x^2}+\left[F_0'-g\right]\frac{\partial\rho'}{\partial x}-\left(\frac{4\mu}{3\rho_0}\right)\frac{\partial^3\rho'}{\partial t\partial x^2}=0 \tag{11}$$

A solution for ρ' in the form of Eq. (10) and its derivative forms are then introduced into Eq. (11) to obtain a quadratic expression in s whose roots are given by

$$s=\frac{2\mu\kappa^2}{3\rho_0}\left[-1\pm\sqrt{1-\frac{9\rho_0^2P_0'}{4\mu^2\kappa^2}-\frac{9\rho_0^2\left(F_0'-g\right)i}{4\mu^2\kappa^3}}\right] \tag{12}$$

The resulting growth rate s is thus a function of parameter values ρ_0, μ, F_0', and P_0' and the wavenumber of the disturbance κ. Moreover, s is complex indicating disturbances propagate through the bed in the form of traveling waves. From Eq. 12, it is clear that we have analytically solved the problem, and numerical analysis used in most multiphase flow simulations is avoided. This will greatly reduce the mathematical difficulty and help students to focus on the stability theory and the problem itself instead of the numerical analysis. Note that a sign error was made in Eq. (23) of Johri & Glasser[10] and Eq. (18) of Johri & Glasser[9] where the last term under the square root should be – not +. This error resulted in negative computed wavespeeds. Johri & Glasser[10] discusses the implications of wavespeed direction with respect to fluid flow in order to physically justify their findings. Correct signage, however, [as shown in Eq. (12) of this manuscript] results in computed wavespeeds of equal magnitude to Johri & Glasser results, but in the direction of fluid flow and physically realizable.

RESULTS

Since we are interested in distinguishing waves that become amplified in the bed from those that are damped out, the student should proceed to plot the real part of the growth rate σ versus wavenumber κ using parameter values from Table 1. These values were chosen to represent glass beads a few hundred micron in diameter fluidized by air. Here, closures for F and P from Eq. (6) are used where $F_0'=A$ and $P_0'=E$. To examine the density effect, the students should compare the linear stability of steady state solutions having low fluid densities, $\rho_0=220$ and 440 kg/m^3 (representing dilute fluidized beds) with steady states having high fluid densities, $\rho_0=1100$, 1210, and 1320 kg/m^3 (representing particle dense fluidized beds). Results for a high density fluidized bed ($\rho_0=1100$ kg/m^3) are shown in Figure 3 where the real part of the growth rate σ is plotted as a function of wavenumber κ. Students should independently generate linear stability curves for each ρ_0 value condition using Mathematica, MatLab or equivalent.

As shown in Figure 3, the curve has positive growth rate σ for a range of wavenumbers beginning at $\kappa=0$. The growth rate then goes through a maximum at σ_m, and then decreases to zero at a critical wavenumber κ_c. Physically, this represents the boundary between disturbances, which become amplified as they propagate through the bed from those that are damped out. Note that the use of linear closures results in the system becoming *more unstable* as fluid density is *increased*, that is, the critical wavenumber κ_c and maximum growth rate σ_m both increase with an increase in ρ_0. This is because the inertial terms, which drive the instability, increase with an increase in density.

Points for discussion: What do the density dependent force terms physically represent? Use Figure 1 to illustrate that as particles move closer together in the bed to form a more densely packed region, the interstitial gas velocity increases between particles resulting in an increase in particle "drag." How significant is the effect of closure in the dilute and dense flow regimes? How might the magnitude and direction of this force term serve to damp out or amplify unstable voidage waves? Discuss the physical significance of competing density effects with respect to stability. Why would an increase in the pressure gradient (as opposed to pressure) serve to stabilize the bed? How might one conceive of the origin and growth of low density cluster-like insta-

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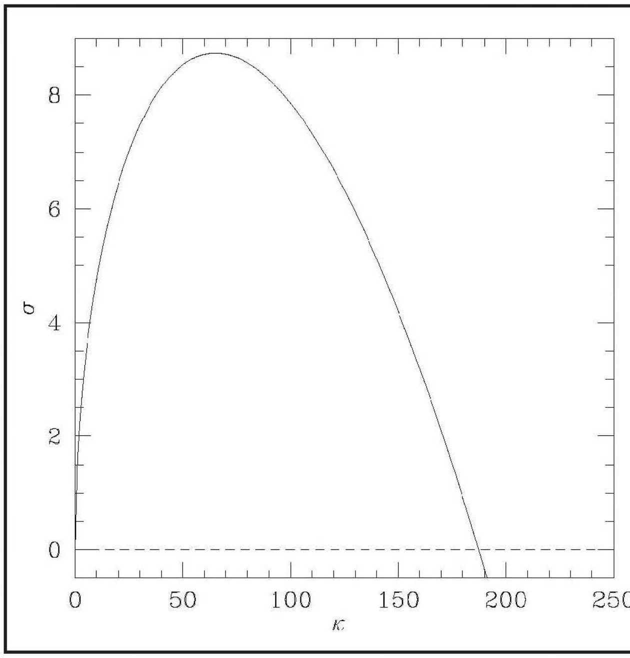

Figure 3. *The real part of the growth rate σ (with units of 1/s) versus the vertical wavenumber κ (with units of 1/m), computed by a linear stability analysis about the uniform state using linear closures for F and P evaluated at ρ_o= 1100 kg/m^3.*

TABLE 1
Parameter Values for the Linear Closures

ρ_0	1100kg/m^3
μ	0.665 kg/(m.s)
A	14.7m/s^2
E	0.03J/kg
c_b	0.173m/s

TABLE 2
ChE 303 Linear Stability Survey
Number of students in each category (total student number = 28).

	Strongly Disagree		Neutral		Strongly Agree
Category	**1**	**2**	**3**	**4**	**5**
Q1			14	11	3
Q2			8	13	7
Q3		3	9	10	6
Q4	1		10	8	9

Q1: I learned a great deal in the lecture.

Q2: The lecture helped me understand that just because a solution is obtained using a momentum balance doesn't mean it will be observed in practice.

Q3: I feel I had adequate math background to understand the mathematical concepts put across in the lecture.

Q4: I recommend teaching this material to the class next year.

bilities versus that of bubble-like high density instabilities, and how is this analogous to the behavior of a compressible fluid? What would be the physical manifestation of unstable waveforms in low density and high density flow of a compressible fluid? What is the role of the density dependent force, and what effect does its closure form have? The reader is referred to Johri a& Glasser[9, 10] for further discussion of the physical situation.

EVALUATION

The stability theory discussed in this paper has been taught in a chemical engineering course: Transport Phenomena I, at Rutgers University in 2005 and 2006. To spur students' interest in the stability theory, we played experimental videos in the beginning of the class to show the development of the density waves, such as bubbles and slugs in fluidized beds. Such videos are available on a CD from Rhodes.[12] Student feedback in 2006 was obtained by issuing a questionnaire (see Table 2, previous page), in which students had to state to what extent they agreed with four statements on a scale ranging from 1, "strongly disagree," to 5, "strongly agree." Generally, we obtained positive feedback from students. A fair number of students felt that they learned a lot from this lecture (Q1 and Q2 in Table 3), and would recommend teaching this material to the class next year (Q4 in Table 3). Some of the comments from students included "I really enjoyed this class. It really sparked my interest in chemical engineering," "I think the explanations were valuable and showed a great deal of importance," and "It was good because it connected several courses. It is always good to see applications that span different classes." Most students believed that they had adequate math background to understand the mathematical concepts put across in the lecture (Q3 in Table 3). Several students, however, also pointed out that one lecture is not enough to fully understand the stability theory material. Such comments included "Maybe there was less time for all that material," and "It is a good beginning to understanding the material that will grow more in depth." We will focus on this point in future classes.

CONCLUSION

We have presented a simple example of an industrially relevant fluid-particle flow problem, which introduces students to methods of linear stability analysis involving nonlinear partial differential equations. This example demonstrates how the stability of a two-phase flow system can be examined using a simplified single-phase compressible flow model, which has been shown to capture the salient features of instability behavior. Students are expected to perform each step of the analysis, and points for classroom discussion have been noted to provide physical insight into the mechanistic features associated with unstable flow behavior and the physical manifestation of unstable waveforms.

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