A PROCESS DYNAMICS AND CONTROL EXPERIMENT

for the Undergraduate Laboratory

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While the variety of experiments contained in typical chemical engineering laboratory courses continues to broaden, it remains important to include experiments or projects involving process dynamics and control. And, in fact, modern hardware and software make such experiments more and more possible, realistic, and interesting. The experiment described here employs relatively simple and inexpensive equipment to demonstrate several important aspects of process dynamics and control, both model-free and model-based analyses of process dynamics data, and two related controller tuning methods.

This experiment permits the students to observe the response of a flow system to impulse injection of a tracer, to collect large amounts of data quickly, and to process the data rapidly using Excel and LabVIEW or QuickBASIC programs. It also permits students to observe, in action and with short time constants, the operation of a PID feedback control system. The students see the actuator (a stepper-motor driven valve) move, see dye enter and pass through a glass flow system, see the transducer (a spectrophotometer) respond to changes in the measured variable, and see the controller (a LabVIEW program) respond to changes in the measured variable and drive the actuator.

From a pedagogical standpoint, the experiment—described in enough detail to be accurately reproduced—provides a comprehensive treatment of a PID controlled flow system, including both standard and more advanced topics. It demonstrates also modern data acquisition and data processing techniques, including the use of LabVIEW. It is designed for use in either a junior- or senior-level laboratory, but should be preceded by or taught in parallel with a course in process dynamics and control.

APPARATUS

The apparatus is shown schematically in Figure 1. To remove dissolved air, house water is stored in a polyethylene carboy and supplied to the experiment by a magnetic-drive

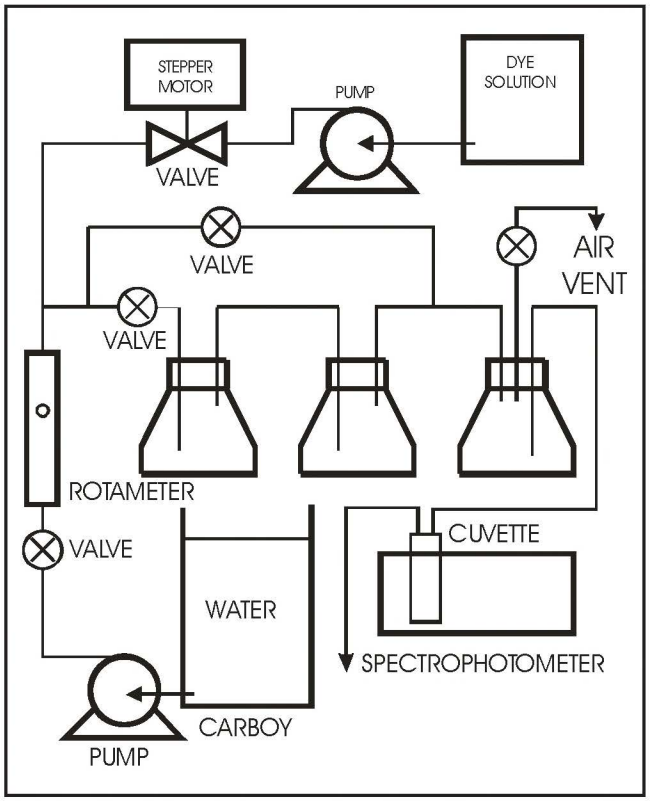

Figure 1. *Schematic diagram of the apparatus, showing the three flasks and valving of the flow system, the water feed system, the dye feed system with stepper-motor controlled valve, and the spectrophotometer with a flow cuvette.*

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centrifugal pump. A rotameter measures the flow rate, which is controlled by a manual metering valve. The flow system consists of three Erlenmeyer flasks, each of volume about 290 ml. The flasks are closed by Teflon stoppers sealed by o-rings, and are connected by 1/4 inch stainless tubing. A ball valve with a connection for a syringe allows impulse injection of dye solution at the entrance to the flow system. Two stainless ball valves allow three flow patterns, namely: (a) flow through one flask only, (b) flow through three flasks in series, and (c) partial flow through the first two flasks in series and full flow through the last tank. The tubing connecting the first two tanks is designed to prevent air accumulation in these tanks, while the last tank is designed to accumulate air, which can be vented via a valve before a run. This avoids the passage of air bubbles to the downstream spectrophotometer. A ball valve connected to a tee at the entrance to the flow system allows a dye solution (typically 10 ml of methylene blue dye at a level of 50 mg/L) to be injected using a syringe.

The transient dye concentration in the effluent water from the flow system is measured by a spectrophotometer (Milton Roy, Model Spec 20). A test tube with an o-ring-sealed Teflon stopper holding inlet and outlet tubes is inserted into the spectrophotometer and serves as a flow cuvette. The spectrophotometer is set at a wavelength of 640 nm, with the 0 to 1 volt analog output signal connected to a National Instruments A/D board (PCI-6043E).

Methylene blue solution (10 mg/L) held in a 20 L polyethylene carboy is pumped to stepper-motor (Arrick Robotics) driven needle valve. This valve controls the flow of the dye solution to a mixing tee upstream of the three-flask flow system. The stepper-motor is driven by signals from a LabVIEW VI that implements a standard PID (Stephanopoulos[1]) control algorithm, corresponding to the following discrete-time algorithm

Valve position = Base position + K_C[Present error + $(1/T_I)$ Integral of past errors + T_D Derivative of error]

Here K_C is the controller gain, T_I is the integral time, T_D is the derivative time, and error is the set point less the measured variable. The valve position is measured in stepper-motor steps (400 steps = 1 turn). The algorithm departs slightly from ideality in that the valve motion is limited to 100 positive or negative steps in a single iteration. When the system is close to the steady state and changes in valve position are small, this feature will have no effect.

The PID algorithm is iterated every four seconds, with a typical run involving 300 iterations, corresponding to 20 minutes. Users can change the set point and the three controller parameters at any time during the run, and a full record of the controller parameters and state variables is written to a spreadsheet file at the end of the run.

OPERATION

There are three modes of operation.

Impulse Response Mode

Prior to the run the spectrophotometer is zeroed and then set to 100 percent transmittance when the dye concentration is zero. The ball valves are set to give single-tank flow, three-tanks-in-series flow, or parallel flow. The rotameter is used to set the water flow rate to a value between 300 and 1200 ml/min. At time zero about 10 ml of methylene blue solution (100 mg/L) is injected rapidly through a ball valve at the entrance to the flow system, and at the same time a LabVIEW program (VI) is started. The program samples the spectrophotometer output signal at a selectable rate of roughly 2 samples per second, for a time long enough that the dye concentration has returned to zero. The program converts the transmittance signal to concentration, and also calculates the mean residence time from the concentration vs. time signal. The data are also written to a spreadsheet file for later plotting and processing.

Control Mode

With the flow rate and the flow system configuration set and the dye solution pump running, the LabVIEW PID control program is started. The students first set the controller parameters (K_C, T_I, T_D), and then the set point (typically about 50 percent Spec 20 transmittance, corresponding to 0.6 dimensionless concentration units). The program samples the transmittance signal every four seconds, converts it to dimensionless concentration, implements the PID algorithm, and plots the concentration, error, and valve position as functions of time. (Full valve travel is 3200 steps, equivalent to 8 turns, and the program limits the number of steps to 100 at each iteration.) At any time during the 20 minute run (300 points at 4 seconds per point) the students can change the set point or the controller gains or the flow system configuration. At the end of the run complete data on controller parameters, dye concentration, and valve position are written to a spreadsheet file for later plotting and analysis.

Step Response Mode

With the needle valve opened manually four turns, the system is allowed to reach steady state, and 400 baseline concentration points are acquired. Then the valve is manually opened two more turns, and 400 additional points are acquired. Subtracting the average baseline concentration from the step transient produces the desired step response, as shown in Figure 2.

MODELING AND PARAMETER ESTIMATION

The program used to acquire the impulse injection data also computes the mean residence time, based on the equation

$$\tau = \int_0^\infty tCdt \Big/ \int_0^\infty Cdt \qquad (1)$$

where τ is the mean residence time, C is effluent stream dye concentration and t is time. If Q is the volumetric flow rate

and V is the volume of the flow system, then

$$V = Q\tau \qquad (2)$$

Here the volume of the system is defined as the volume accessible to dye and enclosed within boundaries outside of which no dye can diffuse or move.

Since Q is known, the students can compare the calculated value of V with the known volume, 290 ml for flow through a single flask, and 870 ml for flow involving all three tanks. Note that this analysis is not based on the assumption that the flasks are well mixed, and in fact is not based on a model for the flow system. But linearity of the system with respect to dye concentration measurement, and that no dye can penetrate upstream of the injection point, are assumed.

In contrast, if we have some information about the structure of the flow system, we can attempt to estimate the value of one or more parameters of a model of the system. Such a model is shown in Figure 3. The model consists of three well-mixed tanks, corresponding to the three flasks in the flow system, and connected in the same way. The state variables of the model are the concentrations (x_1, x_2, and x_3) in the tanks, and the measured variable is the dye concentration (x_3) in the last tank. The undetermined parameters of the model, to be estimated by finding a least-squares fit to the data,[4] are b_1, the amount of dye injected, b_2, the overall flow rate, and b_3, the flow rate through the lower leg of the flow system.

The state equations of the model, used in the parameter estimation step, are as follows:

$$dx_1/dt = -(b_2/V_f)X_1,\ x_1(0) = (b_1/V_f)(b_2/b_3) \qquad (3)$$

$$dx_2/dt = (b_2/V_f)(x_1 - x_2),\ x_2(0) = 0 \qquad (4)$$

$$dx_3/dt = (b_2/V_f)x_2 - (b_3/V_f)x_3,$$
$$x_3(0) = (b_1/V_f)(b_3 - b_2)/b_3 \qquad (5)$$

Here x_1, x_2, and x_3 are the dye concentrations in the three flasks, V_f is the volume of each flask, b_1 is the amount of dye injected, b_2 is the flow rate through the first two flasks and b_3 is the total flow rate through the whole system.

The best (least-squares) parameter values are determined by a QuickBASIC program implementing a standard Gauss-Newton non-linear regression algorithm.[4] The program also computes the parameter correlation matrix and the variance-covariance matrix and the confidence limits for the estimated parameter values. Note that the amount of dye actually injected is not known, while the total flow rate is known and can be compared to the estimated value. Note also that parameter b_2, the flow rate through the first two flasks, cannot be measured directly with the existing apparatus, which illustrates the power of using modeling and parameter estimation methods for indirect measurement of quantities that cannot be directly measured.

RESULTS

Impulse Response Runs

Figure 4 shows the concentration vs. time data, and also the best fit based on the model, for an impulse response run in which all the flow passes through a single flask only. Samples were taken every 0.40 seconds. The data correspond closely to the single exponential expected for impulse injection of tracer into a well-mixed tank. The first few points (not shown or fitted) were close to zero, due to the plug flow in the connecting tubing of the flow system. The concentration curve (solid line)

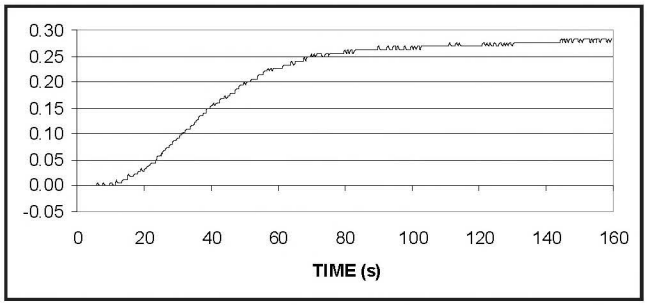

Figure 2. *Shifted step response for three-tank flow configuration, valve opened 800 steps from base position.*

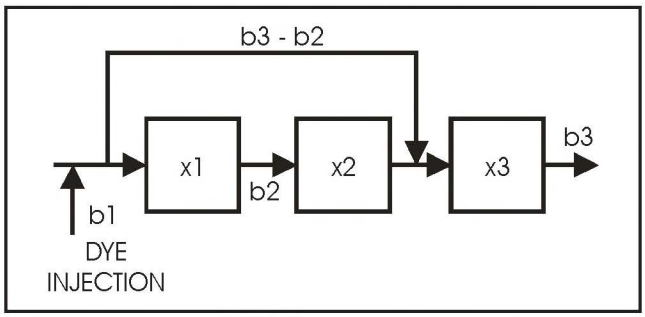

Figure 3. *Schematic diagram of a model of the flow system. The parameters to be estimated are b_1, the amount of dye injected, b_2, the flow rate through the first two flasks and b_3, the flow rate through the system. The state variables are dye concentrations x_1, x_2, and x_3.*

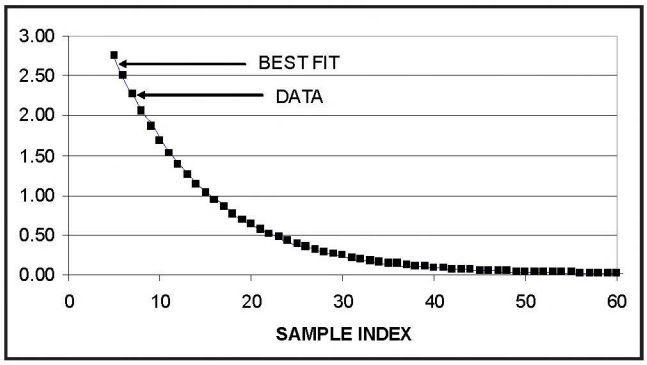

Figure 4. *The experimental concentration vs. time data for an impulse injection run in which the lower ball valve was closed, thus allowing the total flow to pass through only one flask in series. Also shown, as a solid line, is the best fit based on the model.*

based on the model fits the data almost perfectly, and is also an almost perfect exponential. Parameter b_2, representing the flow rate through the first two flasks, was less than 10% of the total flow, but not the expected value of zero. Parameter b_3, the total flow rate, was 956 ml/min, reasonably close to the measured value of 860 ml/min. These results reflect the normal uncertainty associated with parameter estimates.

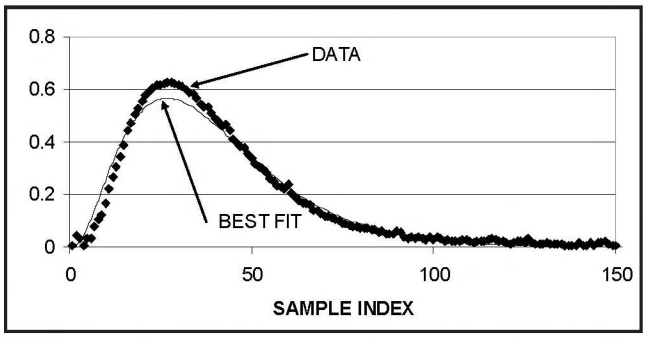

Figure 5. *The experimental concentration vs. time data for an impulse injection run in which the upper ball valve was closed, thus allowing the total flow to pass through three flasks in series. Also shown is the best fit based on the model.*

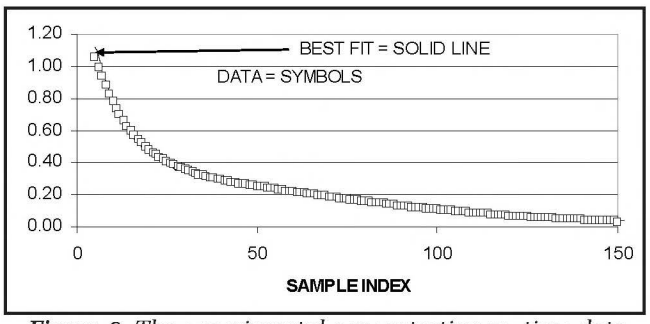

Figure 6. *The experimental concentration vs. time data for an impulse injection run in which both ball valves were open, thus allowing some water to flow through three flasks in series, and the rest to pass through only the last flask. Also shown is the best fit based on the model.*

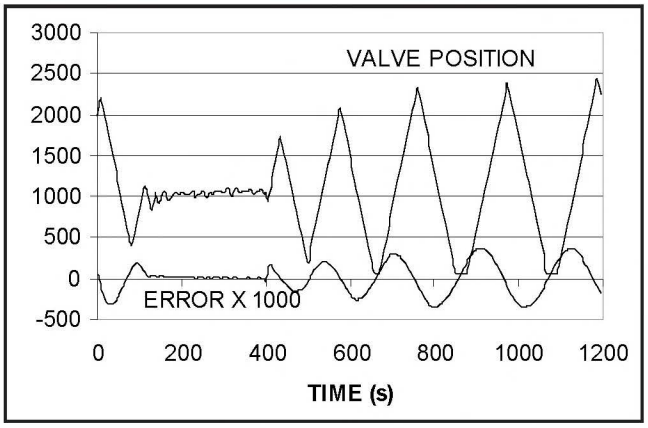

Figure 7. *Error and valve position for PI control run, with $K_C = 5000$ and $T_I = 30$. Initially flow was through a single tank and the error approached zero relatively rapidly. At time = 400 seconds, the flow configuration was changed to three tanks in series, the system became unstable, and limit cycle oscillations resulted.*

Figure 5 shows the concentration vs. time data for an impulse injection run in which all the flow passes through three flasks in series, and shows also the best fit based on the model. Note that the fit is quite good, but not perfect. The estimated flow through the first two flasks is essentially equal to the total flow rate, in accord with the physical situation. The estimated system flow rate is reasonably close to the actual flow rate, as expected.

Figure 6 shows the concentration vs. time data for an impulse response run in which the flow passes partly through three flasks and partly through only one flask (parallel flow). The initial shape is very close to the single exponential corresponding to tracer injection into a single well-mixed vessel. Later dye that has passed through the first two flasks appears in, and increases, the spectrophotometer signal. The small delay near time zero corresponds to plug flow in the tubing that connects the flasks to each other and to the spectrophotometer. The run duration was 290 seconds, during which 200 data points were acquired and processed. There is no evidence of random behavior that might arise from turbulent poorly mixed flow in the flasks, although such behavior does appear at very low flow rates.

Figure 6 also shows the best fit based on the model. In general the fit is quite good. The best-fit value of b_1, representing the amount of dye injected, could not be checked since the amount of dye injected was not known. The value of b_2, the flow rate through the first two flasks, was 378 ml/min. The value of the overall flow rate, namely b_3=780 ml/min, was reasonably close to the measured value of 850 ml/min. The fair, but not perfect, agreement illustrates for the students realistic aspects of the power and limitations of modeling and parameter estimation methods. The students see also how quantities such as the lower leg flow rate, not directly measurable, can be estimated by using a model and parameter estimation.

PI Control Runs

A PI control run was made with gains $K_C = 5000$, $T_I = 30$, and $T_D = 0$ and (initially) flow through a single flask. As shown in Figure 7, the error exhibited a rapidly damped oscillation in the first part of the run, corresponding to an essentially stable system. (Note, however, a low-amplitude, high-frequency valve oscillation, probably reflecting the discrete-time nature of the PID algorithm.) Following a flow configuration change to three flasks in series, the system was no longer stable, began an increasing oscillation, and rapidly entered a large-amplitude, low-frequency limit cycle in which the valve position eventually reached its lower limit. This behavior shows clearly the potentially destabilizing effect of adding time lag to a feedback loop.

The fact that the system shows an oscillatory instability as the gains are increased is consistent with a root locus analy-

sis,[1] based on a transfer function that corresponds to three well-stirred tanks (not of equal volume) under PI control. As an example of such a system we choose an open-loop transfer function

$$G(s) = K(s+0.5)/s(s+1)(s+2)(s+3) \qquad (6)$$

This transfer function corresponds to the schematic root locus plot in Figure 8, which shows that the corresponding closed-loop system is stable for small positive gains K, exhibits an exponentially decaying oscillation as the gain is increased, and begins an exponentially increasing oscillation as the gain increases beyond a critical value. This is in qualitative agreement with the observed behavior, except that nonlinearities produce a limit cycle instead of an exponentially increasing oscillation.

Controller Tuning

While the gain parameters for a system equipped with a PID controller can be selected by trial and error, there are well-developed methods for calculating values that are in some sense optimal, or at least satisfactory. Perhaps the best known methods are those of Ziegler and Nichols.[2, 3]

In the open-loop Ziegler-Nichols version, the response of the system to a step change in the control variable is recorded and used to calculate values for the gain K_C, the integral time T_I, and the derivative time T_D. These parameters appear in the controller transfer function G_C as follows:

$$G_C(s) = K_C\left[1+(1/T_I)(1/s)+T_D s\right] \qquad (7)$$

A typical step response, corresponding to flow through three tanks in series, is shown in Figure 2. This response was generated by manually opening the control valve from 5 turns to 7 turns, and then shifting the concentration origin to zero. Based on the times corresponding to concentration increases of 28.3% and 63.2% of the steady state change, namely 28 and 45 seconds, the PID parameters were calculated as K_C = 4771, T_I = 38.8, and T_D = 9.7.

The PID controller parameters determined by the open-loop Ziegler-Nichols method were used in a control run, with the valve position and error as functions of time shown in Figure 9. The error, initially relatively large, rapidly returned to zero, corresponding to quite good control. After 300 seconds, dye solution was injected rapidly at the entry to the flow system. This produced a rapid decrease in the error. The error then became positive, followed by a rapid decrease to essentially zero. At about 750 seconds a larger amount of dye was injected, with qualitatively similar results. In general this shows that the Ziegler-Nichols open-loop tuning method is, at least in this case, quite effective. Very similar results were obtained using controller parameters determined by the Ziegler- Nichols closed-loop method.

In the closed-loop Ziegler-Nichols procedure, the controller involved proportional action only, with T_I = 10,000 (which effectively removes any integral action) and T_D = 0. As shown in Figure 10, the gain K_C was initially 12,000 and was later decreased to 9,000, at which point the error still oscillated. Then K_C was decreased to 6,000, at which value the oscillation disappeared. The ultimate gain was estimated at 7,500, and the ultimate period was 110 seconds. Based on these results the controller parameters were calculated as K_C = 4500, T_I = 55, and T_D = 13.75. We note that there is reasonable agreement between the open-loop and closed-loop

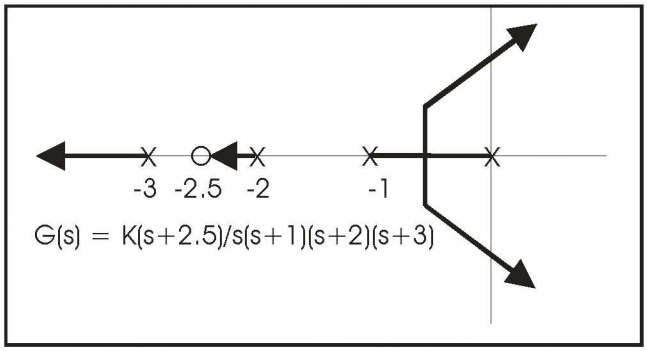

Figure 8. *Schematic root locus plot for a PI control system with open loop transfer function G(s) = K(s + 0.5)/s(s + 1)(s + 2)(s + 3).*

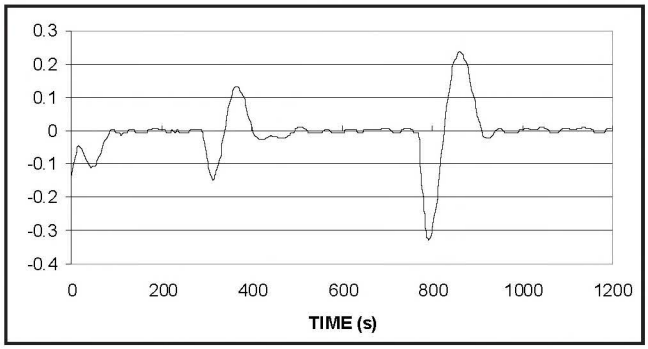

Figure 9. *Error data for a PID control run with gains K_C = 4771, T_I = 38.8 and T_D = 9.7, corresponding to Ziegler-Nichols open-loop tuning. The flow configuration was three tanks in series.*

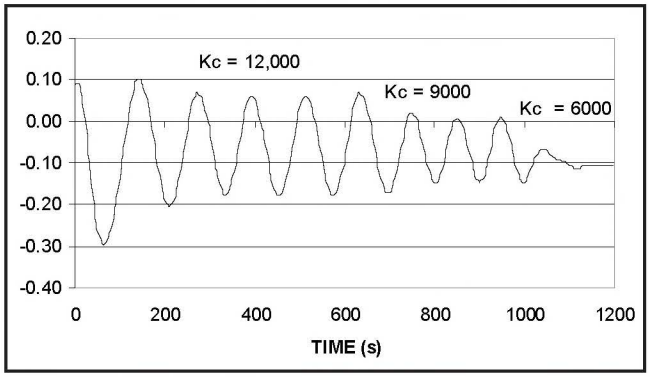

Figure 10. *Oscillatory behavior for the closed-loop system with three tanks in series, for proportional control with K_C = 12,000, then 9,000, and finally 6,000.*

Ziegler-Nichols procedure results. The open-loop procedure, however, requires only a single step-response run, while the closed-loop procedure requires a series of relatively long oscillatory steady-state runs.

DISCUSSION

The process dynamics and control teaching experiment described above is based on a relatively simple apparatus. It illustrates two basic and important aspects of the subject, namely the acquisition and processing of impulse response data, and the operation and tuning of a feedback controller. The impulse response data are analyzed using a nonlinear regression method to determine three parameters of a model of the flow system. In the control studies, the students use both the open-loop and closed-loop Ziegler- Nichols controller tuning procedures to estimate the parameters of the PID controller. Then they observe the behavior of the system under PID control using these parameter values.

The experiment allows the students to observe closely and in detail the interactions of the actuator, plant, transducer, and controller components of a single-variable feedback control system. In particular the opening and closing of the control valve, the resulting changes in dye concentration in each of the glass flasks, and the response of the concentration measuring spectrophotometer are clearly visible. And the key internal variables of the PID controller program, implemented in the modern LabVIEW language, can also be followed as the system responds on a convenient time scale of tens of seconds. This provides an optimal environment for students to develop a practical—as opposed to purely theoretical—understanding of a realistic feedback control system.

The experiment also embodies several more general and relatively more sophisticated concepts, including the use of modern data acquisition software (LabVIEW), and nonlinear regression methods to estimate the parameters of a model of the flow system. The students see how a dynamic model of a flow system can be used in the estimation of several parameters of a model, and see also that the parameter estimates are subject to errors because the model rarely represents perfectly the modeled system.

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