THE SOCCER BALL MODEL:
A Useful Visualization Protocol for Scaling Concepts in Continua

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The major characteristic that sets an engineer apart from every other profession in the world is his/her ability to apply the concepts of scaling/up-scaling to a variety of situations. What do we mean by scaling? Well, take for instance a chemist working in the laboratory designing a new drug for a company. Would this chemist be your first choice to take that laboratory synthesis and convert it to a process that produces thousands of tons of that drug per year? Probably not; however, a chemical engineer would be an excellent candidate. Similarly, if building an airplane, scientists (physicists, material, computational) would not be the first choices that come to mind, in spite of the obvious useful roles of their professions. An aeronautical engineer would most likely be the selection that makes everybody comfortable. The same can be said for building structures (bridges, buildings, etc.) where civil engineers are the masters, and for the scaling of industry where industrial/managerial engineers are very skillful. The list is long, but these few examples illustrate the basic concept: Engineers are masters of scaling/up-scaling. Therefore, it is imperative when training engineering students, that they fully grasp the concept of scaling/up-scaling to be able to implement it for practical applications, such as the ones mentioned above.

One important class of up-scaling in engineering education is the different scales involved in describing quantities related to the physics of transport (mass, momentum, energy). In many high school or college-level courses, students are introduced to velocity, density, energy, etc., from a discrete scale point of view.[1] In many engineering applications, however, when studying the physics of transport, it is necessary to develop conservation equations for system properties,

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such as total mass, energy, and momentum for a continuum, or microscopic scale.[2] To accomplish this, the concept of a continuum scale must be introduced to students. Since most students have only been exposed to the physical and chemical concepts related to total mass, energy, and momentum, from a discrete scale point of view, the concept of a continuum scale can be very challenging.

In making the transition from a discrete scale to a continuum scale, one very important pedagogical aspect to keep in mind is that students already have substantial knowledge related to calculating the total mass, velocity, and momentum of a single particle (discrete domain). So from the students' learning point of view, how does the instructor use their previous experience and knowledge with the discrete domain to scale it up to the continuum domain?

Most textbooks do not address this issue. In fact, many of them have suppressed or hidden the process associated with the up-scaling[b,3] on the assumption that all steps and concepts are familiar to the learner, when in fact they are not. This can be frustrating to students and does not enable them to fully understand the importance of the idea of a continuum. Moreover, some textbooks[4] have approached the problem from the point of view of the definition of an intensive property, such as density, and from the traditional definition for the discrete case:

$$m_p = \rho_p V_p \qquad (1)$$

where m_p is mass of the particle, ρ_p is density of the particle, and V_p is volume of the particle. They have then simply extended this definition to a continuum control volume, $V_{c(t)}$ as follows:

$$m_s \equiv \int_{V_c(t)} \rho dV \qquad (2)$$

where m_s is the total mass of the system under study. Based on Eq. (1) and Eq. (2), it seems that as suggested in Figure 1, two domains exist: A non-applicable (for the system description), or "old" domain (discrete domain) and a "new" domain (continuum domain). As Figure 1 shows, the discrete, or "old," domain is valid for very small scale systems (order of molecules), whereas the continuum, or "new," domain adequately describes the mass of the system for domains of a larger or continuum (microscopic) scale. It is interesting to note that the so-called old domain in Figure 1 is at the molecular level and the concepts learned by students during, for example, high school or college physics are not necessarily at this scale. The molecular scale is a discrete domain, however, and this characteristic offers a bridge for student learning that is effectively used in the Soccer Ball Model (SBM) protocol described in this paper.

The pedagogical challenge described in Figure 1 is that the "old" domain is the domain in which the students are most comfortable and more knowledgeable with the concepts. Students, in general, are unfamiliar with the new domain indicated in Figure 1. Many teaching approaches (in the literature) focus on the new domain and mostly forget the level of knowledge that students already have on the old domain. This situation is probably very familiar to most students, unfortunately, as oftentimes when learning new concepts they are told to "forget" everything they already know; this type of learning approach completely nullifies the knowledge that the students have already acquired. Another option that instructors sometimes use is to force students to imagine a new system where the boundary (or boundaries) are no longer well defined. This, then, requires students to apply "old concepts" to the "new (suddenly introduced)" system. These two options illustrate the many disadvantages for the students when they are not engaged in the process

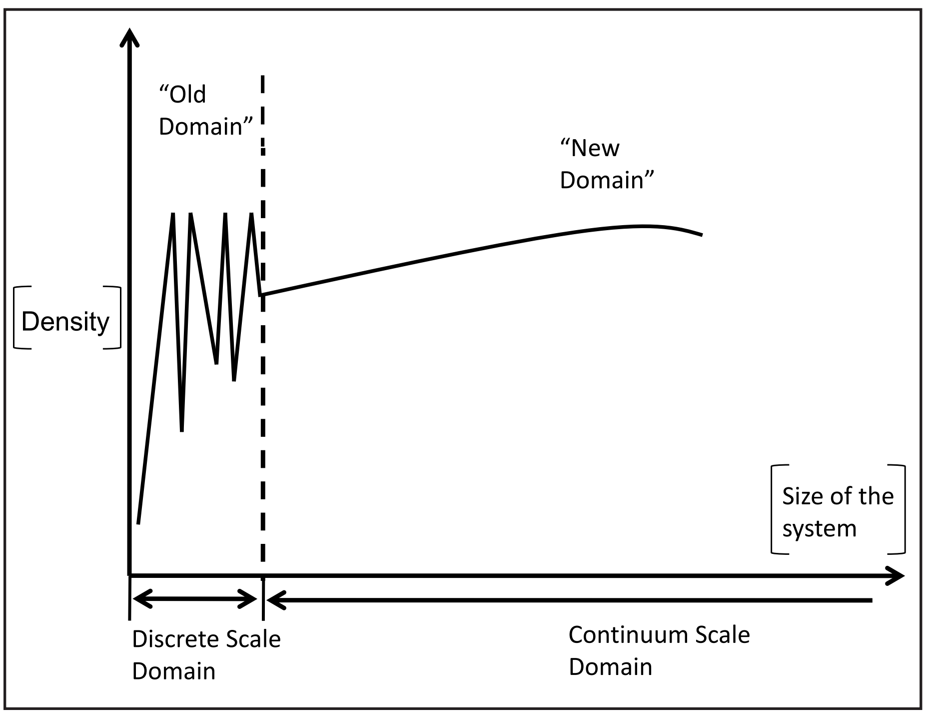

Figure 1. *Sketch of the material density as a function of the size of the system indicating the two scales or domains of interest.*

b *The word up-scaling here is used to indicate the change of the description of a property from one scale to another, such as, for example, from the microscopic to the macroscopic scales.*

of transforming and adapting what they already know. This suggests the need for adopting a procedure in which students are fully engaged in the process of learning (up-scaling), then coaching them on how to move from one scaling level to the next. Moreover, such a process allows students to build on what they already know about the discrete point of view, and to integrate this knowledge with the new "view" of matter, *i.e.,* the microscopic or continuum scale.

In addition to having an introduction to several scientific concepts from the discrete point of view, students have an adequate background in many complementary subjects including calculus, integral concepts, and algebra. It appears that the instructor could take advantage of this strong scientific and mathematical background to help students in catalyzing the transformation from one scale to the other one in an effective way from the students' learning point of view. In other words, instead of hiding the details about the scaling-up process, by giving the final answer, the instructors could identify one or several activities in which students are exposed to and can learn and reflect on the many aspects involved in the process.[5] In this contribution, we propose a visual[c] process to help with the transformation of scales (domains), *i.e.*, from discrete to continuum, by using soccer balls in conjunction with geometrical domains, mathematical principles, and physical properties. The student is exposed to a very powerful set of pedagogical activities to construct a learning environment that is both practical and effective. An introduction to this environment is given in the next section.

From the learning environment point of view, the SBM protocol is an effective Principal Object of Knowledge, or POK, a tool introduced in the Colloquial Approach[6,7] and later adapted to include other learning environments.[8,9] POKs are tools that allow the facilitator to focus students' learning on a collection of topics or variables conducive to visualizing the process of understanding the different aspects. In this sense, the SBM presents scaling, packing, geometrical, mechanical, and mathematical ideas or concepts in an efficient manner for the process of students' learning.

DESCRIPTION OF THE SYSTEM(S): THE SOCCER BALL MODEL ELEMENTS

Soccer[d] is the world's most popular sport. It is played on the beaches of Brazil, on the grassless surfaces of Argentina, Uruguay, Chile, Africa, as well as on the nap-inviting fields of Europe and North America. Therefore, soccer balls are geometrical objects that are popular among college students in a large number of countries in the world. The International Federation of Association Football, FIFA,[10] (international government of the sport) has five ball sizes shown in Figure 2a. The largest one is number five and it is the official size used in every soccer game at the professional level. The smaller sizes (numbers four, three, two) are used in games depending on player ages, and the smallest ones (number one) are mostly given as souvenirs.

One can observe from Figure 2a that one of the attractive features of the set of soccer balls (decreasing size from the largest one to the smallest one) is the fact that they are all like objects of the same geometry. Although the balls are made of a shell with air at a given pressure, in the soccer ball model it is assumed that they are all made of the same material as the shell (see Figure 3). This assumption usually promotes

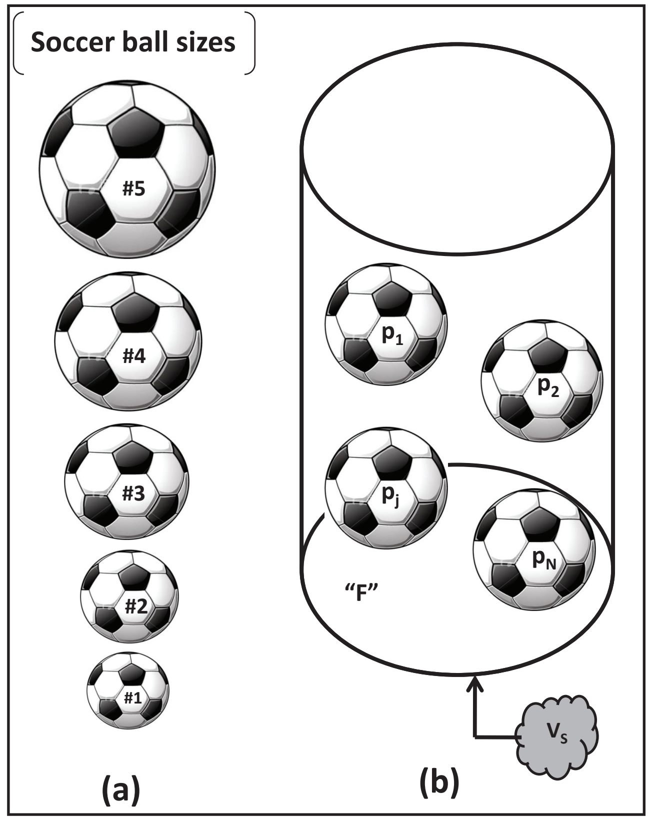

Figure 2. *Elements of the soccer ball model (SBM). (a) Set of the five sizes of soccer balls approved by the FIFA. (b) Container of a given volume, V_s.*

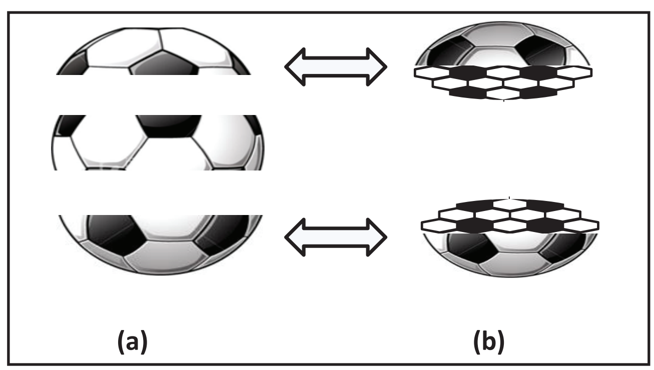

Figure 3. *Visualization of the "soccer ball material."*

c *The research on brain-based learning suggests that vision is the most powerful tool for the brain to add new knowledge (Medina, 2008).*

d *We have used the name mostly used in the United States, however in the rest of the world this foot-based sport is simply called "football."*

strong discussions of why this can be proposed and allows the instructor to bring previous vs. new student knowledge to the discussion.

In addition to N balls of a given size, "k" (k=1, 2, 3, 4, 5), the soccer ball model uses a container of a given volume, V_s (t) (see Figure 2b). This container could be either rigid (vessel) or flexible/deformable (bag), and it can be of different geometries, *i.e.*, rectangular, cylindrical, or spherical. For simplicity, a rigid, cylindrical vessel is assumed for the analysis (see Figure 4). The idea of control volume is discussed in connection with the vessel of cylindrical shape. In general, students are introduced to this idea and also to the concept of dimensions associated with the control domains. In fact, they are made aware that these domains could be of one dimension (line), two dimensions (surface), and three dimensions (volume)[e].

Figure 4 shows a typical situation that is helpful for the scaling-up analysis of this contribution. The vessel identified before is partially filled with N soccer balls of a given size. The system (vessel + soccer balls) can be viewed as a composite, or a two-phase system with one phase made completely of the N soccer balls and the other one made of the "fluid" filling the void space between the soccer balls. In many classes students are presented with a transparent vessel containing soccer balls to show the different "phases" and spaces. If the experiment is conducted in a regular classroom, the fluid can be associated with "air"; however, a discussion is conducted for several different possibilities. The fluid phase is denoted by "F" and the mass associated with it by m_F. The mass associated with the N soccer balls inside the vessel is denoted by m_{SB}. Since

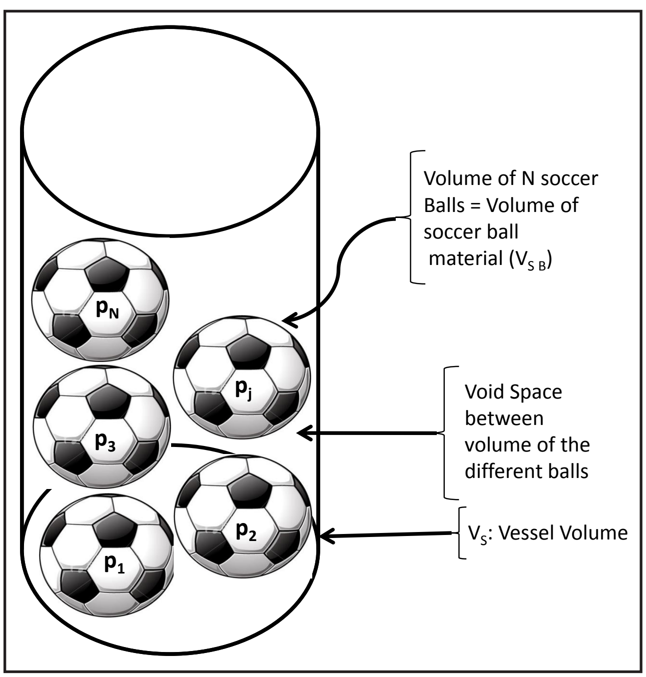

Figure 4. *Sketch of the different components associated with the container identified in Figure 2.*

in engineering many different types of practical systems exist[f], students are introduced to a variety of systems that may have these types of characteristics where the soccer balls can easily be related to "particle models" in a fluid inside a container. One interesting characteristic from the didactic point of view is that these "particles" are discrete objects that the students can see and touch. Once the system in Figure 4 is understood, then a procedure for the mathematical formulation for the total mass of such a system can be developed. The process should start with a system such as the one shown in Figure 4, then steps are made where the students can be coached until the formulation (or scale-up) to a continuum scale can be reached. This is the focus of the next section.

LEARNING PROCESS: TRANSFORMATION OF SCALES

To compute the total mass of the system depicted in Figure 4, let's start by stating some assumptions. We will assume that the volume of the vessel of cylindrical shape is given by V_s. Also, we will assume that a set of "N" soccer balls of the same size, SB_N^k, has a volume, V_{pj} (j=1, 2,...,N), density, ρ_j (j=1, 2,...,N), and mass, m_j (j=1, 2,...,N). Since they are discrete objects, one can easily compute the mass of ball "j" as in the classical college physics textbook[1], *i.e.*:

$$m_j \equiv \rho_j V_{pj},\ j = 1, 2, \ldots, N \tag{3}$$

Because of the assumptions stated above, it is immediately recognized that the total mass of the soccer balls can be computed as:

$$m_{SB} = \sum_{j=1}^{N} m_j = \sum_{j=1}^{N} \rho_j V_{pj} \tag{4}$$

and, therefore, the total mass of the system, m_s, with control volume v_s^g can be computed (for the volume of the container) as follows:

$$m_S = m_{SB} + m_F \tag{5}$$

where m_F is the mass of the "F" material. Then in view of Eq. (4), one can express Eq. (5) as:

$$m_S = \sum_{j=1}^{N} \rho_j V_{pj} + m_F \tag{6}$$

Now, one question arises: How can we reduce the mass associated with "F" (m_F) and simultaneously increase the mass

e The idea of domain is connected to the domain concept of a mathematical function, which students are familiar with from calculus courses.

f In general, colloidal and non-colloidal suspensions are very good candidates, but other systems, such as packed or fluidized beds can also be discussed.

g Students are reminded that the N balls must fill up the system completely. The idea of fractions is possible or, alternatively, the choice of V_S is discussed.

of the soccer balls (m_{SB}) while maintaining the volume of the whole system as constant (*i.e.,* V_s=constant*)*? To answer this question, one should recognize that within the container there are spaces (*i.e.*, void spaces that do not include soccer ball material) and they are filled with a mass of the "F" material (for example, air, see above) that is located between the different balls (see Figure 4). The rest of the spaces within the container are occupied by the soccer balls.

Coaching Point 1: The instructor may want to discuss with the students several examples of particle *packing* systems: marbles of different sizes and sand are excellent examples. The discussion should be focused on the role played by the size of the particles and the void spaces in a given container to help connect the previous knowledge with the analysis of the situation. The instructor should strongly refuse to give answers, and instead act as a facilitator being ready to offer counter examples to the situations brought up by the students. The discussion should lead to the conclusion that *by reducing the particle size, the void spaces are also reduced*. Now as a corollary: What would the effect of this reducing process be on the number of soccer balls? Should N increase or decrease? The hypothesis identified in the exercises/discussions of coaching point 1 may be tested by using the soccer ball model. Here, for example, the number 5 soccer balls should be used as a first step. Those balls, N in total, should be loaded into the container. Both m_{SB} and m_F should be determined or estimated. This is a very useful exercise[h] to acquire a solid idea of the system's characteristics. The instructor could assign vessels of the same volume but of different geometries and ask students if N is the same, or what would change.

Coaching Point 2: The instructor may want to coach the students in calculating the mass of particles in a given volume. The idea of voids and porosity of a packed bed can be easily connected to the problem. Experiments to measure the properties should be discussed. This exercise will produce intense discussions among students regarding very relevant aspects of the different geometries (see coaching point 1, above). Now after the concept identified in coaching point 1 has been understood, students should be able to check it by using the soccer ball model. By using the idea of the size of soccer balls, the process sketched in Figure 5, one should change the number 5 soccer balls to number 4, again measure the mass of soccer ball material and the mass related to the void space, m_F. Once the process or experimental protocol has been identified and tested by students, the next question is at what iteration should it be stopped? The idea of an approximation in engineering becomes useful to address this question. Recall what is intended; to minimize the mass of the void spaces (m_F) up to a point where:

$$m_S \approx m_{SB} \tag{7}$$

Coaching point 3: The instructor may want to discuss with students at this point the implications or approximations if the protocol were to be implemented in the laboratory. Some of the relevant aspects may include:

1. How do we stop the iteration process to produce the desired approximation in Eq. (7)? *Hint:* The idea of the sequence and the comparison of the mass of the system in iteration k with the k-1 would be helpful: $\left\| m_s^k - m_s^{k-1} \right\| < \varepsilon$
2. The step in the sequence (*i.e.,* "k") may be determined by the accuracy of the instrument being used in the measurements.
3. How valid is the approximation in Eq. (7) for the purposes of reducing m_F and increasing m_{SB}?

By stressing the various geometrical and experimental aspects of the protocol, students gain a very useful hands-on and concrete view of the transformation proposed in the process shown in Figure 5, where the unloading and reloading of the vessel with the different-size soccer balls is sketched. Students soon realize that the set of soccer balls is incomplete for the purposes of perhaps reaching a valid approximation in order for Eq. (7) to hold. This is another great advantage so they can develop possibilities for other systems that will help them to achieve the results. In this sense, the soccer ball model is just

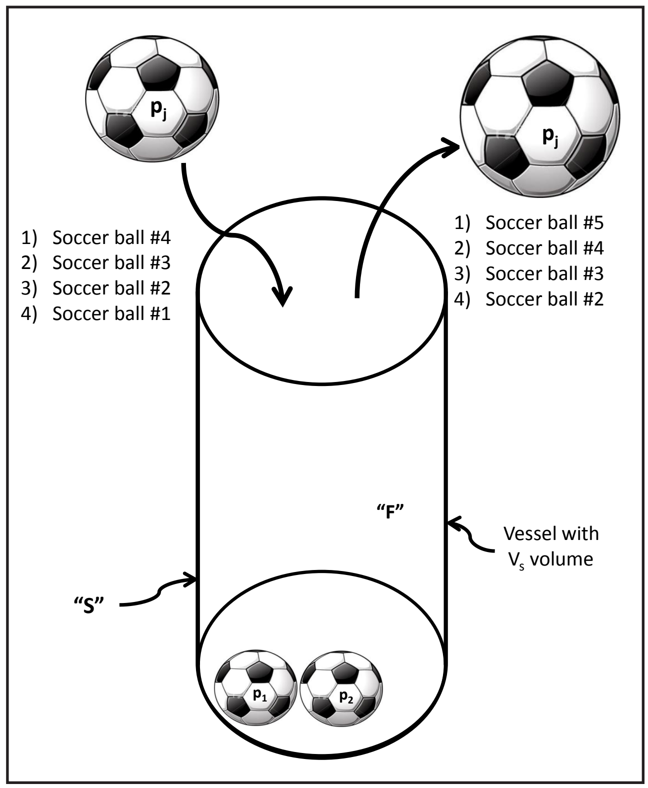

Figure 5. *Sketch of the "Protocol" of reducing the mass of the "F" phase and increasing the mass of "soccer ball material."*

h *A very powerful visualization of this protocol can be achieved by using actual soccer balls and containers of different geometries.*

a pedagogical promoter, or initiator of a process that allows students to visualize the transformation in the scales.

At the end of the process when the approximation given in Eq. (7) is reached, the mass of the system is given by

$$m_S \approx \sum_{j=1}^{N} \rho_j V_{pj} \tag{8}$$

since m_F is very small it can be neglected compared to m_{SB} for all practical purposes. Now the next question is to check how Eq. (8) can be improved. One excellent possible solution is to continue using small objects (smaller than the smallest soccer ball) as most likely students have proposed, and going to sizes such as, for example, grains of sand and even molecular sizes. Mathematically, this implies

$$m_S = \lim_{\substack{N\to\infty \\ V_{pj}\to 0}} \sum_{j=1}^{N} \rho_j V_{pj} \tag{9}$$

Eq. (9) can be slightly modified to bring it closer to a mathematically useful definition. First we want to map the geometrical situation in the vessel to a mathematical-based domain with incremental volume ΔV_j (see Figure 6). It is useful to discuss with the students the dimensions of the volume of this tiny domain (with respect to the volume of the vessel)[11] with the mathematical concept of incremental volume. From this, now Eq. (9) becomes:

$$m_S = \lim_{\substack{N\to\infty \\ \Delta V_j\to 0}} \sum_{j=1}^{N} \rho_j \Delta V_j \tag{10}$$

Eq. (10) is nothing but a representation of the Riemann sum,[11] that in the limit produces the Riemann integral, *i.e.*,

$$\lim_{\substack{N\to\infty \\ \Delta V_j\to 0}} \sum_{j=1}^{N} \rho_j \Delta V_j \equiv \int_{V_c(t)} \rho dV \tag{11}$$

From Eq. (10) and Eq. (11) now we can write:

$$m_s = \int_{V_c(t)} \rho dV \tag{12}$$

It is very straightforward to conclude that Eq. (12) allows us to compute the total mass of the system from a continuum point of view whose control volume $V_c(t)=V_s$. This equation (valid for a continuum) is derived directly from the discrete objects (*i.e.*, particles=soccer balls) and therefore every physical concept that was valid for a discrete domain is also valid for the continuum domain. By using the visualization protocol as described in this section, we have introduced a different "scale" in the computation of a physical property, *i.e.*, for this case the mass (total) of the system. The total mass of the system (for a single component system, the total mass coincides with the mass of the component) is the primary variable or property that allows us to compute others that are proportional to it (see the section below). Therefore, the transformation from a discrete scale point of view to a continuum scale point of view is relatively straightforward. Students never have to deny that what they learned in the discrete scale is valid for the continuum scale. It is, at the end, a different mathematical description of the same property since the scale has changed.

EXAMPLES AND APPLICATIONS: OTHER VARIABLES OF INTEREST

The learning protocol described in the previous section may be applied to other variables that are relevant for the formulation of conservation principles, such as linear and angular momentum and energy.[12] The steps are identical as for the case of total mass. First, one should start with the mathematical definition of the property for the case of discrete variables and then apply the process identified in the prior

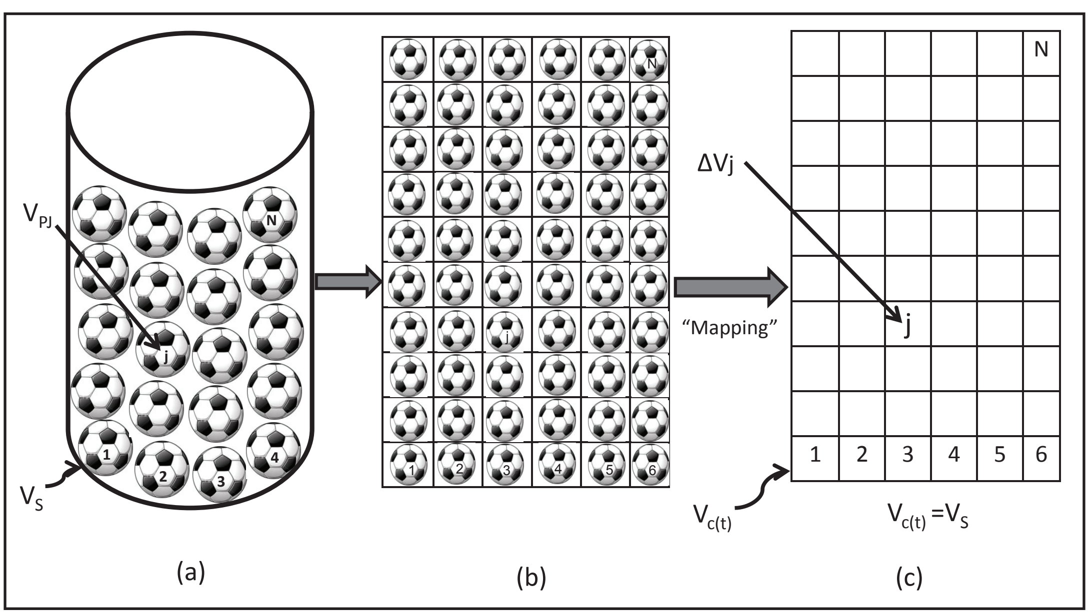

Figure 6. *Mapping of the container volume to a geometrical domain of size V_s. (a): Container filled up with N soccer balls. (b) Side view of the space occupied by the N soccer balls. (c) Geometrical domain showing N incremental volumes of size ΔV_j and with a density ρ_j.*

section to reach the proper mathematical equation for the new property. For example, the linear momentum, $\vec{p}$, for a discrete particle is:

$$\vec{p} = m_p \vec{v}_p \tag{13}$$

for each particle of mass, m_p, and velocity, $\vec{v}_p$. From a continuum point of view, (by using the protocol previously described) we can conclude,

$$\vec{p} = \int_{V_c(t)} \rho \vec{v} dV \tag{14}$$

There is a "shortcut" approach by realizing the mass of the system is given by Eq. (12) and then, by replacing the velocity of the particle by the one of the medium, one arrives to Eq. (14) from the "suggested" form given by Eq. (13). Didactically, this is consistent with the fact that students have a protocol in mind of how the transformation works and is similar to the "mathematical" tricks used frequently in analysis courses to obtain results in a quicker manner. Similarly, energy, E, for the discrete point of view is given by:

$$E = \vec{p} \bullet \vec{v}_p \tag{15}$$

By using the strategy identified above, Eq. (15) is transformed into:[13]

$$E = \int_{V_c(t)} \rho \left[\vec{v} \bullet \vec{v} \right] dV \tag{16}$$

Note: Students may want to use the relation,

$$\vec{p} = \vec{v} \int_{V_c(t)} \rho dV \tag{17}$$

The velocity field $\vec{v} = \vec{v}\left(\vec{x}\right)$ could, however, be a function of the position inside the control volume, $V_c(t)$, and therefore Eq. (17) will not capture this situation. Eq. (14) and also Eq. (16) will capture non-uniform velocity fields, *i.e.*, Eq. (16) is the most general equation for describing the energy of the system for a continuum scale.

More complicated functions or properties can be expressed from a continuum point of view. For example, the moment around a point, *i.e.*, torque, $\vec{M}$, is given by:

$$\vec{M} = \vec{r}_p \times \vec{F} \tag{18}$$

where $\vec{r}_p$ is the position vector of the force, $\vec{F}$. It is known from mechanics that

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(m_p \vec{v} \right) = \vec{R} \tag{19}$$

From a continuum point of view Eq. (19) can be expressed as,

$$\vec{R} = \frac{d}{dt} \int_{V_c(t)} \rho \vec{v} dV = \tag{20}$$

If $\vec{R}$ is the net force applied to the system, *i.e.*, the one enclosed within the control volume, $V_c(t)$, then from Eq. (18) and Eq. (20):

$$\vec{M} = \frac{d}{dt} \int_{V_c(t)} \rho \vec{r}_p \times \vec{v} dV = \vec{R} \tag{21}$$

in the case that $\vec{r}_p = \vec{r}_p\left(t\right)$. Caution must be kept in mind regarding the interpretation of the meaning of the derivative, $\frac{d}{dt}$ in Eq. (19); also, the formulation of Eq. (21) and similar ones requires a careful analysis and discussion that are not part of the scope of this contribution.[12, 14] The protocol of the soccer ball model is actually a helpful tool, from a didactic as well as from the conceptual point of view, since, in practice, all key variables for the description of the conservation principles in a continuum scale can be systematically derived by using such a protocol; or, alternatively, shortcuts based on the protocol are possible.

IMPACT ON STUDENT LEARNING

The SBM protocol was introduced some years ago[7] and it has been systematically implemented in various core courses in fluid mechanics, heat transfer, and transport and reactions, both at the undergraduate and graduate level. The comments by students in course exit interviews have indicated the healthy action of the protocol in helping students build an excellent level of knowledge based on the previous level they bring to the classroom as well as avoid misconceptions. In addition, the protocol has been extremely effective for introducing macroscopic or integral balances from a continuum point of view without much difficulty from the students' point of view. Furthermore, the connection between mathematical concepts learned in calculus and engineering applications, such as the change of scales, is effectively integrated by using elements of the SBM.[3] This, in turn, assists the students in understanding the relevancy of the mathematical tools in engineering applications and enhances the appreciation of their power in, for example, simulating engineering processes.

We believe the protocol of the SBM is an effective tool in removing the students' frustration in understanding a very different description (from the students' point of view) of matter, momentum, energy, and related concepts from a new and more sophisticated scale, *i.e.*, the continuum scale.

SUMMARY AND CONCLUDING REMARKS

This contribution summarizes some of the typical approaches used to introduce students to scaling/up-scaling for variables and properties related to conservation principles in continua. The key aspect is the introduction of a new learning protocol, the soccer ball model, that engages students in every step of the process of transforming scales from a discrete level to build a continuum. The soccer ball model approach allows students to use what knowledge they have already acquired in previous courses from the discrete point of view, to apply it in a systematic manner, and to obtain the description of properties such as mass, energy, and momentum; these properties are used in conservation equations for the continuum point of view. The protocol identified in the learning environment of the soccer ball model is powerful since students never lose track of the discrete nature of the objects when engaging in building a continuum. They reach this level at the end of the protocol and simultaneously they have been able to develop an excellent idea of the continuum with an equation to compute the given property or properties of the system.

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