This paper describes a simple, yet innovative, experimental project that was used as a term project in two core undergraduate-level chemical engineering (ChE) courses at the University of Massachusetts Lowell (UML). The project aimed to illustrate the critical nature of experimentation in developing mathematical models for the analysis of laboratory experiments and for the design and operation of pilot- and commercial-scale operations. A general tank-draining problem was selected for the project because the draining of water from a container is a simple physical situation that most students are familiar with from everyday life. The project was assigned as a part of a freshman-level Introduction to Chemical Engineering (ICE) course and a sophomore-level Energy Balance and Introduction to Thermodynamics (EBIT) course.

To develop a mathematical model that correctly describes a system, one must employ conservation laws for mass, energy, and momentum. However, the application of conservation equations alone may not lead to an effective model. Rather, a combination of experiments and conservation principles is needed. The experimentally determined part of the model is called the constitutive equation. In this paper, we will deal only with the conservation of mass and energy equations, and will show the development of a constitutive equation using an experimental hands-on project.

Educational researchers acknowledge that students learn more effectively by doing than by passively listening to a lecture.\(^1\) However, most engineering classes are taught in a lecture format that involves only passive listening. As discussed by Felder and Silverman,\(^2\) this traditional teaching style does not match the learning styles of all students, leading some students to become bored, inattentive, and discouraged, to perform poorly, and to change to other curricula. There have been several major efforts to improve the teaching methods that are used in traditional engineering classes. For instance, Farrell, et al.,\(^3\) developed a semester-long project for freshman ChE students at Rowan University. Their project involved the application of engineering measurements and calculations, mass balances, and process simulation to the human respiratory system. Barak and Dori\(^4\) described the incorporation of an information technology-supported project-based learning method into three undergraduate chemistry courses at the Technion Israel Institute of Technology. Students used computerized molecular modeling as a tool to construct models of complex molecules. This project encouraged understanding of chemical concepts, theories, and molecular structures. Participating students performed much better on exams compared to classmates who did not participate in the project. Hohn\(^5\) incorporated a hands-on experiment into a freshman ChE course at Kansas State University. This experiment allowed students to investigate a simple phenomenon—a carbonated

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**Nese Orbey** is an associate professor at the Department of Chemical Engineering, University of Massachusetts Lowell (UML) where she has received the Departmental Excellence in Teaching award and Excellence in Innovative and Transformational Education award. She obtained her Ph.D. from McGill University, Montreal, Canada. Her research interests are in polymer rheology and structure-property relations of polymers/nanocomposites. Before joining UML, she worked as a technology manager at QinetiQ North America (formerly Foster-Miller, Inc.).

**Marisel De Jesús Vega** received her B.S. in chemical engineering from University of Puerto Rico, Mayagüez campus. She recently completed her Doctoral degree in chemical engineering at University of Massachusetts Lowell. Her research interests are microfluidic devices, microfabrication and materials.

**Fulya Sudur Zalluhoglu** received her B.S. in chemical engineering from the Middle East Technical University, Turkey. She completed her Doctoral degree in chemical engineering at the University of Massachusetts Lowell. Her research interests are materials, microencapsulation and sol-gel chemistry.

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beverage going flat—using engineering analyses.\textsuperscript{[5]} Students listed the activity as useful and fun, and indicated that they valued the hands-on nature of the project.\textsuperscript{[5]} Hanyak and Raymond\textsuperscript{[6]} developed a team-based project for teaching a ChE energy balances course at Bucknell University. The project resulted in enhanced teamwork, communication, and understanding of the course material. Apostolidis, et al.\textsuperscript{[7]} incorporated a computational fluid dynamics (CFD) project in a ChE course on fluid mechanics at the University of Delaware. They reported an overwhelmingly positive student response. By applying CFD to a contemporary cardiovascular problem, students in the course gained knowledge and appreciation about microscopic fluid mechanics and CFD simulations.

Use of a tank-draining system to explain major concepts in ChE in an introductory freshman-level course was first discussed by T.W.F. Russell and M.M. Denn at the University of Delaware.\textsuperscript{[8]} In this paper, we describe how this approach was incorporated into course content as a hands-on term project to augment understanding of concepts simultaneously presented in lectures. Tank draining is a simple physical process that is easy to model and, therefore, has been modeled often in various chemical engineering education formats. Commonly, it is included as an experiment in upper-level laboratory courses, such as chemical engineering laboratory\textsuperscript{[9-11]} or process modeling and control laboratory.\textsuperscript{[12]} In this format, students will have already completed courses in thermodynamics and transport operations, and the experiment reinforces concepts that were covered in these classes. Students use existing experimental setups equipped with sensors, flowmeters, and control units. In the chemical engineering laboratory course, the tank-draining experiment is used to demonstrate an application of the principles of conservation of energy and momentum as described in Kunz’s textbook.\textsuperscript{[13]} Muske\textsuperscript{[12]} used the tank-draining experiment to reinforce previously introduced process dynamics, simulation, and control concepts. Hesketh, et al.\textsuperscript{[9]} used a tank-draining experiment at the freshman level wherein students used an existing experimental setup to measure tank-drainage rate as a function of hydrostatic pressure and time. The governing mass and energy balance equations were provided and the students were not required to carry out derivations or modeling.

Our approach is fundamentally different from the aforementioned courses in that we use the tank-draining experiment to introduce the concepts of mass and energy conservation in two lower-level courses rather than to reinforce previously taught concepts. In addition, instead of using an existing experimental setup, the students must design their own setup. They are asked to apply mass and energy balances to derive the model equations and to compare their results with the experimental data they obtain. The expected Accreditation Board for Engineering and Technology (ABET) student outcomes from this term project are a, b, e, g, and k as listed in Table 1.

### COURSE STRUCTURE

#### Sophomore students

The EBIT course is a 3-credit, 3-hour-per-week course that is required for all ChE students at UML. This course is offered during the 13-week spring semester of the sophomore year. Students enroll in the course upon successful completion of the Material Balances course. The EBIT course is an introduction to thermodynamics and application of simultaneous mass and energy balances to reacting and nonreacting systems. Enrollment is about 100 students, and the course is taught in parallel in three sections. One of the sections is for students in the UML honors program. The honors section covers the same syllabus as the regular sections, but with a more in-depth study of the concepts. This project was assigned during the Spring 2014 and Spring 2015 semesters, with a class enrollment of 28 students in total.

#### Freshman students

The ICE course is another one-semester 3-credit, 3-hour-per-week course that is required for all incoming ChE students. This course is offered in the fall and spring semesters, with the class size being kept around 60 students. The course is designed to give students an overview of the ChE curriculum and to solidify their interest in the profession at an early stage in their education. For the first 6 weeks of the 13-week semester, all enrolled students attend lectures that provide a general introduction to ChE, discuss the rules of technical writing, and describe the use of Microsoft Excel spreadsheets for basic calculations. For the remaining

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>ABET Student Outcomes</th>
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<tbody>
<tr>
<td></td>
<td>Not Achieved</td>
</tr>
<tr>
<td>An ability to apply knowledge of mathematics, science, and engineering (ABET student outcome a)</td>
<td>0%</td>
</tr>
<tr>
<td>An ability to design and conduct experiment, as well as to analyze and interpret data (ABET student outcome b)</td>
<td>0%</td>
</tr>
<tr>
<td>An ability to identify, formulate, and solve engineering problems (ABET student outcome e)</td>
<td>5%</td>
</tr>
<tr>
<td>An ability to communicate effectively (both written and oral) (ABET student outcome g)</td>
<td>5%</td>
</tr>
<tr>
<td>An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice (ABET student outcome k)</td>
<td>5%</td>
</tr>
</tbody>
</table>
7 weeks of the semester, students are assigned to different projects under a professor from the department. Sixteen students worked on this project in the Fall 2014 semester and 10 students in the Spring 2015 semester. Other available projects were related to alternative options in our program, such as biological engineering, nuclear engineering, and nanomaterials engineering.

THE PROJECT

A general tank-draining problem was selected to illustrate the simple application of mass and energy balances and the development of a constitutive model. The tank-draining problem is the lab-scale equivalent of a water tower. Almost all municipalities have a water tower (a large tank containing water) that is located on the highest ground in the area, to ensure a supply of water to residences and businesses in the event of power failure. Flow from the tank depends on the elevation, the amount of water in the tank, and the opening and valve systems. Thus, the tank-draining project has an actual engineering application besides providing a simple system for analysis.

Figure 1 illustrates schematically the draining tank under analysis. For simplification of the mathematics, a cylindrical tank of constant cross-sectional area, A, was selected. The tank is open to the atmosphere and contains water at an initial height of \( h_0 \). The height of the water at any time is denoted by \( h \). The volume, \( V \), of water in the tank at any given time can be obtained by \( A \cdot h \). There is no inflow to the tank. The tank empties by gravity-driven flow through a small hole (orifice) of constant cross-sectional area, \( A_0 \), at a mass flow rate of \( \dot{m} \) (or a volumetric flow rate of \( q \)). The water temperature and water density, \( \rho \), are assumed to be constant during the experiment. Students were asked to find a mathematical relation for how the flow rate through the orifice varies with the height of the liquid in the tank, using the principles of conservation of mass and energy and the experimental data.

**Sophomore students**

For sophomore students, the project consisted of four specific steps.

**Step 1: Experiment.** The objectives of the experimental part of the project were to observe how the rate \( q \) changes with the height of the liquid flow and to obtain reproducible height versus time data as the tank is draining. Working in groups of two, students planned the experimental setup and procedure. They discussed their plans with instructors before starting the experiment. Students had to choose the size of the tank and the size and location of the orifice. They were very creative in selecting the tank, which included buckets, pails, and water or soda bottles of various sizes, and plastic poster tubes. A sample experimental setup is shown in Figure 2.

Students needed to calibrate their tank so that they could record the height of water remaining in the tank over time. They almost invariably took a video of the tank using their cell phone and recorded data from the video. Students determined that the tank size should be at least 1 gallon (3.78 L), to enable...
accurate recording of the data. Use of a tank size smaller than 1 gallon resulted in very fast draining of the water. Most students carried out the experiment in the laboratory, although a few students carried it out in their dorm. Time at prescribed heights was most often measured, but some students preferred to measure the height of the water at regular time intervals. Each group repeated the experiment between three and five times to ensure reproducibility. Obtained data were averaged and plotted as scattered points (Figure 3a). Students observed that the flow rate increased with increasing orifice size, but decreased with decreasing water height as the tank drained.

**Step 2: Application of the conservation of mass equation.** Students were asked to apply the general mass balance Eq. (1) to their system.

\[ \dot{m}_{\text{input}} - \dot{m}_{\text{output}} = \dot{m}_{\text{accumulation}} \]  

When there is no mass flow into the system, Eq. (1) simplifies to Eq. (2).

\[-\rho q = \frac{d(pV)}{dt} \]  

Because temperature is constant, the density terms cancel out. Volume can be expressed in terms of the height of the water and the cross-sectional area. Thus, Eq. (2) reduces to:

\[-q = \frac{d(Ah)}{dt} = A \frac{dh}{dt} \quad \text{or} \quad \frac{dh}{dt} = -\frac{q}{A} \]  

There are two unknown quantities in Eq. (3): the height of the water \( h \) and the output volumetric flow rate \( q \). To solve this equation, we need to develop a mathematical relation that describes how \( q \) varies with \( h \). This constitutive equation can be substituted in Eq. (3) and integrated to predict height versus time. Validity of the postulated constitutive equation can be checked by comparing the predicted height as a function of time from the constitutive equation with the experimental data.

**Step 3: Development of a constitutive relation between \( q \) and \( h \).** Students were asked to postulate constitutive relations between \( q \) and \( h \) and to use experimental data for verification. This step was done iteratively. First, looking at the data in Figure 3a, the students intuitively postulated a relation. The two simplest relations that can be postulated are that \( (1) \) \( q \) is independent of \( h \) (i.e., \( q = a \)), and \( (2) \) \( q \) varies linearly with \( h \) (i.e., \( q = a \cdot h \), where \( a \) is a constant). Inserting these postulated relations into Eq. (3), students obtained a simple differential equation that can be solved to obtain height versus time. To generalize, a power-law relation between the volumetric flow rate and height in the form of \( q = a \cdot h^n \) can be assumed. When \( n = 0 \), the relation reduces to the postulate where \( q \) is independent of \( h \). When \( n = 1 \), the relation becomes the postulate where \( q \) varies linearly with \( h \). Substituting the generic relation into Eq. (3) results in:

\[ \frac{dh}{dt} = -kh^n \quad \text{or} \quad \frac{dh}{h^n} = -kd t \]  

where \( n \) and \( k (=a/A) \) are constants to be determined using the experimental data.

Solutions of Eq. (4) for different values of \( n \) are given in Table 2. Solutions for \( n = 0 \) and \( n = 1 \) are plotted in Figure 3a.

**TABLE 2**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( q(h) )</th>
<th>Solution of Eq. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( k )</td>
<td>( h = h_0 - kt )</td>
</tr>
<tr>
<td>1/2</td>
<td>( kh^{1/2} )</td>
<td>( h^{1/2} = \frac{h_0^{1/2}}{2} - \frac{k}{2} t )</td>
</tr>
<tr>
<td>1</td>
<td>( kh )</td>
<td>( h = h_0 \exp{-kt} )</td>
</tr>
<tr>
<td>( n \neq 1 )</td>
<td>( kh^n )</td>
<td>( \frac{h^{1+n}}{1-n} = \frac{h_0^{1+n}}{1-n} - kt )</td>
</tr>
</tbody>
</table>

\[ \text{Figure 3a. The height versus time relationship – experimental data (scattered points) and predictions of postulated mathematical models for different values of } n \text{ (lines and scattered lines).} \]
When\( n = 0 \), the solution of Eq. (4) results in a linear relation between \( h \) and \( t \). Although the first few data points follow a linear relation, the data quickly deviate from this initial linear behavior, consistent with our understanding of the physical phenomena. Specifically, \( q \) is fastest at the beginning of the experiment due to a greater pressure head, but decreases with time as the height of the water and the pressure decrease. When \( n = 1 \), Eq. (4) results in an exponential decay of \( h \) with respect to time.

At short times, the model predictions and experimental data are in agreement. However, at later times, the model predictions overestimate the value of \( h \) at a given time compared to the experimental data. In addition, an exponential decay means that an infinite time is needed to drain the tank completely, in contrast to the experimental observation. Thus, the experimental data lie between the predictions of postulates for \( n = 0 \) and \( n = 1 \). Students were asked to assume different values of \( n (0 < n < 1) \) and to solve Eq. (4) until the best fit with the experimental data was obtained. This was an iterative procedure. When a postulate did not agree with the data, the way in which the data disagreed was used as an aid to postulate the next value of \( n \). The value of the constant \( k \) can be calculated by using the slope of the first three data points for each value of \( n \) where the plot shows linear behavior. The mathematical relation that gave the closest agreement with the experiment was found to be for \( q = k \cdot h^{1/2} \) (Figure 3b).

**Step 4: Application of the conservation of energy equation.** Students were asked to apply the general energy balance Eq. (5) to the draining of a tank, to find a theoretical relation between \( q \) and \( h \), which they compared with the experimental data from Step 3. The general energy balance equation is:

\[
(H + PE + KE)_{in} - (H + PE + KE)_{out} + Q + W = \frac{d}{dt}[m(U + PE + KE)]
\]

where subscript \( cv \) denotes the control volume. The datum plane is taken to be the tank exit and with no mass input and no heat and work terms, this equation reduces to the following form:

\[
\frac{d}{dt}[m(U + PE + KE)]_{out} = -(H + PE + KE)_{out}
\]

The internal energy and enthalpy remain constant during the process and the change of kinetic energy of the control volume \((KE)_{out}\) is negligible compared to the change in potential energy, therefore Eq. (6) further reduces:

\[
\frac{d}{dt}[\rho Ah (PE + KE)] = -[\rho q (KE)]_{out}
\]

The kinetic energy of the exit stream can be expressed as:

\[
KE_{out} = \frac{1}{2}v_{out}^2 = \frac{1}{2}\left(\frac{q}{C_0 A_0}\right)^2
\]

In this relation, \( C_0 \) is the orifice coefficient needed to correct for the velocity difference between the water and the orifice. Students were learning this concept simultaneously in their Fluid Mechanics class. Considering the center of mass, the potential energy of the control volume can be expressed as \( g \cdot h / 2 \) where \( g \) is the gravitational acceleration. Finally, \( \frac{dh}{dt} \) can be substituted from Eq. (3), resulting in the following equation:

\[
q^2 = 2(C_0 A_0)^2 g h \quad \text{or} \quad q = C_0 A_0 \sqrt{2gh}
\]

Eq. (9) shows that the simultaneous solution of the mass and energy balance equations predicts that the volumetric flow rate \( q \) is proportional to \( h^{3/2} \), which was the experimentally obtained result.
The tank-draining term project was assigned at the end of week 7, after the concept of the conservation of energy had been introduced during the students’ regular lectures. Working in teams of two, students provided an interim report of their findings from Steps 1 to 3 at 4 weeks after the project was assigned. Each team met with the instructors to discuss the results in their interim report. The final report, which included the analysis in Step 4 and any revisions needed to the interim report, was due 2 weeks after the interim report. A few teams needed to repeat their experiments. Students presented their projects to the class at the end of the semester. The timetable is summarized in Table 3.

### TABLE 3
**Timetable used in the sophomore class**

| Weeks 1-4 | • Step 1 Obtain experimental data  
|          | • Step 2 Apply conservation of mass  
|          | • Step 3 Postulate and evaluate constitutive equations  
|          | • Submit Interim report  
| Weeks 5-6 | • Meet with instructors  
|          | • Revise data/analysis if needed  
|          | • Step 4 Conservation of energy  
|          | • Submit final report  
| Week 7   | • Presentations  

The questionnaire for sophomore students included three sections. The first section contained six questions that students answered on a scale from 1 to 3, indicating that they disagreed (1), agreed (2), or strongly agreed (3) with the statement. The second section contained two yes/no questions. The last section evaluated the degree to which the anticipated ABET outcomes were achieved.

### COURSE EVALUATIONS

Students were very enthusiastic about designing and carrying out the experiment. Most students in both years preferred to work in the laboratory; hence, teaching assistants were able to obtain immediate feedback. Students made positive comments while they were doing the experiment. Some of these comments were:

- “This is my first time but I think it is simple.”
- “I like it because it is actually kind of fun.”
- “It is simple and no need to buy something new just for the project.”
- “I like it a lot because it is very simple.”

Students filled out a questionnaire at the end of the semester, and their feedback was overwhelmingly positive. Questionnaires and feedback are summarized below.

### Sophomore students

The questionnaire for sophomore students included three sections. The first section contained six questions that students answered on a scale from 1 to 3, indicating that they disagreed (1), agreed (2), or strongly agreed (3) with the statement. The second section contained two yes/no questions. The last section evaluated the degree to which the anticipated ABET outcomes were achieved.

Questions for the first part are listed below, followed by a summary of the students’ responses to these questions in Figure 4.

A. Was doing the experiment and analysis out of class more effective than a lecture?
B. Did you learn the concepts covered more effectively compared to an in-class lecture experience?
C. You worked in teams of two. Did the interaction with another student help you to learn better?
D. Was this design project useful in explaining how modeling is done in chemical engineering?
E. Was this project useful in demonstrating that experiments are required in chemical engineering analysis?

F. Was this particular problem more demonstrative of the application of mass and energy balance (for systems in which there is no chemical reaction) compared to other processes you have studied, such as a crystallization unit, dryer, distillation column, etc.?

Most students responded positively (score of 2 or 3) to these questions. Compared to the lecture format, most students strongly agreed or agreed that the hands-on approach was more effective (63% and 32%, respectively) and enabled them to learn more effectively (42% and 53%, respectively). Students strongly agreed (69%) or agreed (20%) that working with another student helped them to learn better. About 58% of students strongly agreed and 37% agreed that the design project was useful in explaining how modeling is done in ChE. All students found the project to be useful in demonstrating that experiments are necessary in ChE analysis. Finally, 74% of students strongly agreed and 21% agreed that the project was more demonstrative of the application of mass balance than other processes studied before.

The two yes/no questions were as follows:

G. Would it have been better to apply and solve the mass and energy balance before you made the experiment?

H. Did the experimental data and model predictions agree reasonably well?

Student responses showed that 74% thought that it was better to carry out the experiments before applying and solving the mass and energy balance. All students indicated that their experimental data and model predictions agreed reasonably well. More than 67% of students indicated that the desired ABET outcomes were fully achieved (Table 1). At the end of the questionnaire, the students were also asked to provide additional comments if they wanted to. Some comments received are presented below:

- “Great experience to see modeling on small scale experiments.”
- “Fun and interesting.”
- “I learned more than sitting through lecture because it is hands-on.”
- “Challenging yet fair.”
- “I feel like we should have more experiments like this because I learned more and this class should have a lab component to it.”
- “Great experiment.”

**Freshman students**

Questions A to G above were also asked of the freshman class, with question F being modified to omit the energy balance part (which was not covered at the freshman level). Student responses to these questions are shown in Figure 5.
Most students responded positively (score of 2 or 3) to the experience. Compared to an in-class lecture experience, all students strongly agreed or agreed that the hands-on approach was more effective (90% and 10%, respectively) and enabled them to learn more effectively (80% and 20%, respectively). Furthermore, 80% of students strongly agreed and 10% agreed that working with another student helped them to learn better. All students strongly agreed or agreed that the design project was useful in demonstrating how modeling is done in ChE (71% and 29%, respectively) and the necessity of experiments in ChE analysis (83% and 17%, respectively). Finally, 69% of students strongly agreed and 23% agreed that the project was more demonstrative of the application of mass balance than other processes studied before.

The yes/no questions were the same for the sophomore and freshman students. However, the energy balance part was omitted from question G, and an additional question (I) was added, for freshman students.

I. Did the project help you to have a better understanding of the mass balance concept?

Student feedback for question G showed that more than 60% of students thought the design project style for the course was better than the lecture style in terms of understanding and learning the material balance concepts. About 87% of the students responded to question H indicating that they were able to predict a model that was in agreement with the experimental data. Seventy-nine percent of the students indicated that the project helped them have a better understanding of the mass balance concept (question I). At the end of the questionnaire students were asked to write comments and some of these comments are listed below.

- “A hands-on approach solidified abstract concepts I had previously known. It’s almost always better to learn with a real-life application.”
- “I think hands-on approach is more effective than lecture, but a little lecture is also necessary to understand the experiment.”
- “The most important thing I learned is to know exactly what you are looking for before getting in the lab.”
- “I learned the real-life application of calculus.”
- “I learned the math behind the experiment and how to conduct a proper model.”
- “I learned to make accurate measurements for the first time.”
- “Make sure the equations are fully explained to students.”
- “More time and more detail would be better.”
- “Make it mandatory to set up a lab time to make sure the tank/procedure are acceptable.”

Overall, students gave very positive feedback to the project and enjoyed combining the theory with a hands-on experiment. In addition, as explained in the project description section, the students had a chance to design the experiments by selecting their own experimental setup. They used their creativity and engineering judgment to construct or select a tank with the limited resources they had in their dormitories or homes.

The students were motivated and felt like engineers because they designed the experiments themselves. Several students commented that the combination of designing the experimental setup and developing the mathematical model gave them a better perspective on scale-up of chemical processes.

CONCLUSIONS

We employed the classic chemical engineering experiment of gravity drainage of a tank as a hands-on project in two undergraduate chemical engineering lecture courses with two aims: to introduce the principles of conservation of mass and energy; and to show the development of a constitutive equation. Students selected or constructed a tank with a constant cross-sectional area and designed their own experimental setup. The use of this experiment to introduce these concepts at the freshman and sophomore level, rather than at a higher level as a conceptual reinforcement exercise, is a novel approach. Most students responded positively to the project and indicated that compared to the lecture format, the hands-on approach was more effective and enabled them to learn more effectively.

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